Scattering of Protons by Protons*

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The data of Worthington, McGruer, and Findley (WMF) on scattering of protons by protons show an apparently real contribution to the cross section arising from scattering in the p state. Results of phase shift analysis of these data are presented in Table I and in Fig. 1. A satisfactory fit is obtained in terms of s and p waves only, and there is no evidence for a contribution from higher orbital angular momenta or from p waves of "noncentral" character. The s part of the scattering anomaly is analyzed in terms of the f function, and substantial agreement with older results is obtained. The energy dependence of the p wave phase shift is consistent with the assumption of a Yukawa force of range 1.176×10^{-13} cm, and its magnitude is accounted for tentatively in terms of a combination of "ordinary" and "Majorana" potentials as represented by the factor $(0.4+0.6P_M)$ in the nuclear Hamiltonian.

THE experimental work of Worthington, McGruer, and Findley,[1] on the scattering of protons by protons, described in the preceding paper, is the first to cover an extended energy range with sufficient accuracy to exhibit clearly the effects of scattering in states of angular momentum greater than zero. The error of ±0.3 percent in the differential cross section is small enough not to obscure the effect of p scattering, which appears to contribute about 1 percent to the cross section. Also, the observations have been extended through the range of small angles within which the relative contribution of p waves is largest. This paper is a summary of a detailed analysis of these data.

Table I presents the results of phase shift analysis of the differential cross sections given in Table V of WMF. During the course of the experiments, preliminary analysis of the data was made in terms of s waves alone, and an "average" K_0 was found for each energy by inspection of the large angle results. In this part of the work, it was found that the numerical computation could be done rapidly and conveniently by using Eq. (4.5) of Jackson and Blatt[2] in its rigorous form. These calculations showed that the uncorrected data could not be accounted for in terms of s scattering alone. Experimental corrections, as described by WMF, were subsequently applied with the result that an apparently real departure from pure s scattering remained, which had an angular dependence describable approximately in terms of *negative* p phase shifts (see Fig. 11 of WMF). The angular dependence of the deviations is, of course, sensitive to the assumed value of the "average" K_0. Therefore, a least-squares procedure was adopted in which a best fit was obtained by independent variation of K_0 and K_1. In every case, the "average" K_0 was sufficiently near the final value that the necessary changes in the theoretical cross section were linear in the increment in K_0. Also, quadratic terms in the contribution of the p waves are negligible. Therefore, the least-squares analysis could be based directly on the results of the preliminary calculations. Weight factors were introduced, proportional to the reciprocal squares of the nonsystematic uncertainties given in Table V of WMF. With this choice of weights, the effects of the nonsystematic errors in the determination of K_0 and K_1 were obtained by the conventional methods of propagation of errors. The uncertainties given in Table I also include an estimate of the contributions of systematic errors.[3] The authors feel that this straightforward procedure of fitting the data by the method of least squares reduces subjective elements to a minimum. It can be easily extended to allow simultaneous calculation of three or more phase shifts without excessive numerical work and allows a systematic appraisal of the improvement in fit obtained by inclusion of terms in higher phase shifts.

TABLE I. Results of the phase shift analysis of the differential p-p cross sections given in reference 1.

E (Mev)	K_0 (degrees)	K_1 (degrees)
1.855	44.212±0.023	−0.049±0.020
1.858	44.218±0.028	−0.057±0.024
2.425	48.318±0.029	−0.075±0.018
3.037	50.971±0.040	−0.082±0.022
3.527	52.475±0.046	−0.094±0.023
3.899	53.257±0.057	−0.109±0.020
4.203	53.808±0.081	−0.074±0.023

Examination of Fig. 1, which presents the results of the calculations described above, shows that a very satisfactory fit is obtained in terms of K_0 and K_1 alone, and that there is definitely a part of the cross section attributable to scattering in the p state. An attempt was made to improve the fit by introduction of a d wave contribution. It was found that no statistically significant improvement could be obtained in this way. Moreover, a *negative* K_2 was obtained, and one feels that one is not likely to have repulsive forces in the d

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[1] Worthington, McGruer, and Findley (WMF), Phys. Rev. **90**, 899 (1953).

[2] Jackson and Blatt, Revs. Modern Phys. **22**, 77 (1950).

[3] A more detailed account of the treatment of experimental errors is given in the doctoral thesis of H. H. Hall, University of Wisconsin, August, 1952 (unpublished).

state. For these reasons, one cannot reliably attribute any part of the anomaly to d scattering.

It has been shown that strong noncentral forces may modify the p wave anomaly in an essential way.[4] We have considered the possibility that strong scattering in p states of different total angular momentum might, through interference, produce the observed small effect. Least-square analysis was applied, using directly formula (1) of BKT,[5] and it was found that improvement of the fit could not be obtained by inclusion of quadratic terms in the three phase shifts. Therefore, such interference apparently does not occur, and the data can only be explained in terms of *small* phase shifts, in which case, as is well known, the effects of noncentral forces are not distinguishable from central field scattering. This situation is not changed if one at the same time introduces a contribution due to d waves. These considerations have led to the conclusion that the data are most reasonably interpreted in terms of the phase shifts given in Table I.

Final calculations included relativistic corrections arising from the transformation of cross section[6] and scattering angle from laboratory to center-of-mass coordinates. This correction is at most 0.2 percent in absolute magnitude, and the least-squares values of the phase shifts are not significantly changed if this correction is omitted. No attempt has been made to include relativistic corrections arising from dynamic effects.

The f function method has been used to obtain the range and depth of the singlet potential from the energy dependence of the s wave phase shift. The f function was expanded in the form[7]

$$f=f^{(0)}+f^{(1)}E+f^{(2)}E^2+f^{(3)}E^3.$$

For each potential well shape studied, the appropriate values of $f^{(2)}$ and $f^{(3)}$ were obtained from the paper of Jackson and Blatt.[2] This expansion was then fitted to the experimental values of f by a least-square adjustment of $f^{(0)}$ and $f^{(1)}$. Table II contains the results of this work, expressed in terms of the scattering length and effective range of Jackson and Blatt. The goodness

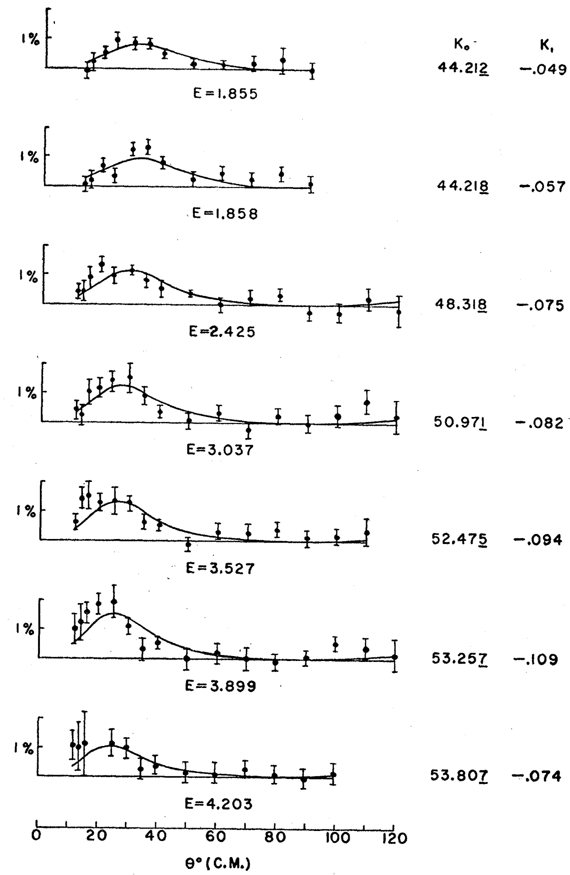

FIG. 1. Comparison of experimental data to results of phase-shift analysis. The ordinate is the percentage difference between the observed cross section and the calculated "pure s" cross section based upon the values of K_0 listed at the right.

of fit is essentially independent of the well shape assumed in the selection of $f^{(2)}$ and $f^{(3)}$. For the Yukawa well, the range and depth corresponding to the best fit (see Fig. 2) are $(1.176\pm0.006)\times10^{-13}$ cm and (46.5 ± 0.5) Mev, respectively. These may be compared to the corresponding numbers obtained by YSHBB from analysis of previous experiments, namely, $(1.16\pm0.006)\times10^{-13}$ cm and (47.7 ± 0.5) Mev. These values do not strictly agree, within the given uncertainties. However, when one remembers that the present results are the work of a "single group of observers," in a restricted energy range, it is not inconsistent to conclude that the agreement is satisfactory.

The energy dependence of the p phase shifts is exhibited graphically in Fig. 3. The solid curve is a theoretical estimate, based upon Taylor's approximation,[8] of the phase shift to be expected for a central Yukawa potential of range 1.176×10^{-13} cm and depth adjusted to correspond to the observed magnitude of K_1. The energy dependence of K_1 is evidently consistent with the assumed range of the Yukawa potential. The

TABLE II. Scattering length and effective range determined by least-squares analysis for several well shapes.

Shape	Scattering length	Effective range
Yukawa	-7.75×10^{-13} cm	2.79×10^{-13} cm
Exponential	-7.70	2.67
$P=Q=0$	-7.69	2.65
Square	-7.65	2.57

[4] Breit, Kittel, and Thaxton (BKT), Phys. Rev. **57**, 255 (1940).
[5] R. S. Wright has remarked that the third term of Eq. (1) BKT may be written:

$$(12/\eta^2)[-\tfrac{3}{4}\sin^2(\delta_1-\delta_2)-\tfrac{1}{3}\sin^2(\delta_0-\delta_2)](3\cos^2\theta-1).$$

[6] O. Chamberlain and C. Wiegand, Phys. Rev. **79**, 81 (1950).
[7] Yovits, Smith, Hull, Bengston, and Breit (YSHBB), Phys. Rev. **85**, 540 (1952).
[8] Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1060 (1939).

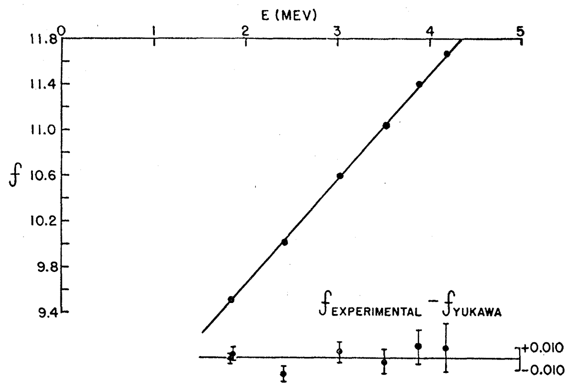

FIG. 2. Results of analysis of s wave anomaly in terms of the f-function.

point at 4.203 Mev is not entirely consistent with the other observations, but as explained by WMF, experimental difficulties at this highest energy were such that this run is considered to be less reliable than the others for the determination of K_1. This unreliability arises from the absence of data at angles which are critical for the determination of K_1 and is not reflected in the statistical uncertainty indicated by the vertical bar in Fig. 3. It has been assumed that this discrepancy can reasonably be ignored. This assumption is supported by the measurements of Zimmerman and Kruger[9] at 5.86 Mev, who obtained $K_1=-0.36\pm0.22°$, in excellent agreement with the trend established by the present data, exclusive of the point at 4.203 Mev.

The strength of the triplet interaction corresponding to these phase shifts may be correlated with other properties of the two-nucleon system in terms of the phenomenological potential:[10]

$$V=(1-\alpha+\alpha P_M)\left\{V_c\frac{e^{-r/r_c}}{r/r_c}+V_t\frac{e^{-r/r_t}}{r/r_t}S_{12}\right\},$$

in which P_M is the Majorana exchange operator. Under the assumption of charge independence, the range and strength of the central and tensor parts may be obtained uniquely from Table VI of Feshbach and Schwinger. The p scattering of protons then serves to determine the parameter α. Assuming the values 1.176×10^{-13} cm for the range of the central force and 1.704×10^{-13} cm for the triplet effective range,[11] we find $\alpha=0.62$. This result is insensitive to the specific choice of the ranges of the central or tensor forces, provided the potential satisfies the conditions imposed by the known properties of the n-p system. The value 0.62 is not inconsistent with other estimates based upon high energy data.[12]

On the basis of this work, it is tentatively suggested that the potential function,

$$V=V_0(1-\alpha+\alpha P_M)\left\{[1-\tfrac{1}{2}g+\tfrac{1}{2}g(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]\frac{e^{-r/r_c}}{r/r_c}+\gamma S_{12}\frac{e^{-r/r_t}}{r/r_t}\right\},$$

where $r_c=1.176\times10^{-13}$ cm, $\gamma=0.8481$, $r_t=1.529\times10^{-13}$ cm, $g=-0.0834$, $V_0=39.83$ Mev, and $\alpha=0.62$, gives

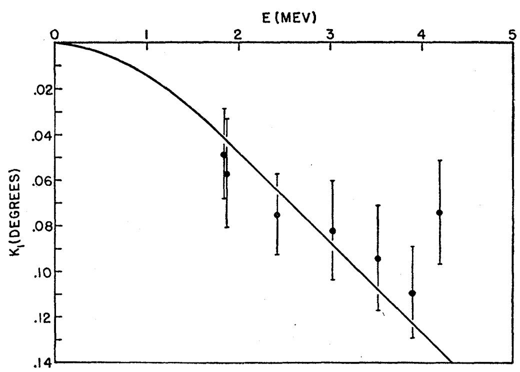

FIG. 3. Energy dependence of p phase shifts.

an adequate description of the basic low energy properties of the two-nucleon system. It may be remarked that this interaction does not satisfy the requirements of saturation in complex nuclei.[13]

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[9] E. J. Zimmerman and P. G. Kruger, Phys. Rev. **83**, 218 (1951); E. J. Zimmerman, doctoral thesis, University of Illinois, 1951 (unpublished).
[10] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 199 (1951).
[11] Burgy, Ringo, and Hughes, Phys. Rev. **84**, 1160 (1951).
[12] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950).
[13] R. G. Sachs (private communication).