The Multiplicity of Neutrons from the Interaction of μ^- Mesons at Rest in Pb, Bi, Sn, and Al

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The average multiplicity $\langle\nu\rangle$ of neutrons emitted from a nucleus which has absorbed a μ^- meson has been determined for several materials: $\langle\nu\rangle = 2.14 \pm 0.13$ for Pb, 2.32 ± 0.17 for Bi, 1.54 ± 0.12 for Sn, 0.95 ± 0.17 for Al. From $\langle\nu\rangle$, by using compound nucleus theory, the average excitation given the nucleus was found to be about 20 Mev for all the nuclei studied.

The μ^- meson interactions were classified according to the number n of neutrons observed per event and from the observed distributions in n some properties of the functions $I(\nu)$, which give the probability of events in which exactly ν-neutrons are emitted, have been determined. The results indicate higher values of neutron multiplicity and excitation energy than have been calculated by Tiomno and Wheeler, and Rosenbluth but are consistent with the assumption that a large part of the rest energy of the μ^- meson is carried out of the nucleus by a light neutral particle.

INTRODUCTION

An experiment has been performed to study the emission of neutrons resulting from the nuclear interaction of μ^- mesons at rest in Pb, Bi, Sn, and Al. The mean value $\langle\nu\rangle$ of the multiplicity of neutrons emitted in such an interaction has been measured. The spread in the distribution of multiplicities ν about the mean value, which is given by the standard deviation $\sigma = [\langle\nu^2\rangle - \langle\nu\rangle^2]^{\frac{1}{2}}$, has also been determined from the data.

The experiment was performed in a tunnel under about 2000 g cm^{-2} of shale.[1] This depth was chosen in order to reduce the uncertainty in the identification of the charged particles which were observed to enter the apparatus and stop in the absorber. The 2000 g cm^{-2} above the apparatus was sufficient to reduce to a negligible value the intensity of N-component particles incident from above the earth's surface, so that it could be assumed that the charged particles studied were μ-mesons or their secondaries.

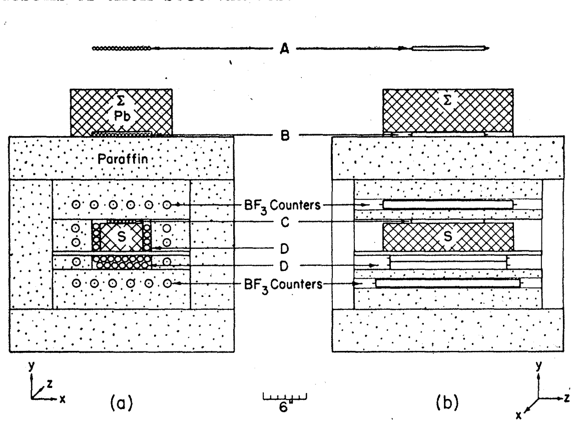

FIG. 1. Geometry of experiment.

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[1] Genesee black shale: 60 percent SiO_2, 15 percent Al_2O_3, 25 percent Ca, Fe, and Mg oxides; density 2.65 g cm^{-3}.

I. APPARATUS

The experimental arrangement is shown in Fig. 1 (*a* and *b*). A, B, and C were trays of Geiger counters forming a coincidence telescope which included a solid angle of 0.14 steradian; D was a tray of Geiger counters operated in anticoincidence with A, B, and C.

The Pb filter Σ between A and B was 172 g cm^{-2} thick. The absorber S was 4 in. thick (dimensions 4 in. $\times$6 in.$\times$18 in.) and consisted of 113 g cm^{-2} of Pb, 99.5 g cm^{-2} of Bi, 74.0 g cm^{-2} of Sn, or 27.4 g cm^{-2} of Al.

S was surrounded by a paraffin moderator in which were imbedded 18 enriched BF_3 proportional counters for detecting thermal neutrons. The eighteen counters were connected in parallel, and the group of counters operating together had a plateau about 150 volts long, with a slope of about 5 percent per 100 volts. The construction and filling procedure and the operating characteristics of these counters have been described in detail elsewhere.[2] The spacing of the counters in the moderator was such as to give a high efficiency for counting neutrons thermalized in traversing $\frac{1}{2}$ in.–$3\frac{1}{2}$ in. of paraffin. Thus, the neutrons detected were those of moderate energy, about 1–10 Mev, which are evaporated from excited nuclei.

The requirement was made that one and only one channel in each of trays A, B, and C, and none in tray D be discharged. Events which satisfied this requirement gave rise to a master pulse, ABC$-$D, which was delayed by 5 microseconds after the Geiger counter telescope discharge.

The anticoincidence tray D was designed to have a high efficiency. This tray consisted of a double layer of counters below S and rows of counters extending up the sides of S, as shown in Fig. 1. Thus, tray D covered a large solid angle and had almost no gaps caused by counter walls or interstices between counters. Also, whenever a pulse occurred in tray D, a gate was promptly generated which prevented the recording of

[2] Tongiorgi, Hayakawa, and Widgoff, Rev. Sci. Instr. 22, 899 (1951).

TABLE I. Measured and corrected numbers of interacting μ^- mesons and neutron coincidences.

Material	Running time (hr)	ABC	ABC−D uncorrected	ABC−D corrected	M (number of μ^- mesons undergoing nuclear interaction)	N (total number of neutrons recorded in coincidence with ABC−D)
Pb	813	$12.06\times10^{+4}$	4823	4175	1837	303
Bi	617	$8.94\times10^{+4}$	3112	2620	1153	207
Sn	975	$14.03\times10^{+4}$	4399	3623	1594	182
Al	997	$14.55\times10^{+4}$	2453	1661	457	30
No absorber ($S=0$)	631	$9.17\times10^{+4}$	501	486	—	0

any ABC−D pulse for 400 microseconds after the pulse in D. This gave the counters in tray D time to recover from each pulse, so no spurious anticoincidences caused by the missing of pulses in D during counter dead time could be recorded. The use of the 400-microsecond suppressing gate introduced an inefficiency in the apparatus of about 1 percent for counting true events ABC−D. This inefficiency did not affect the results; it simply meant that the counting rate was reduced by this small amount. Because of the geometrical arrangement and the artificial dead time, the efficiency of tray D was at least 99.7 percent and was more likely about 99.9 percent.

The master pulse ABC−D triggered a 320-microsecond sweep and intensifier in a five-inch cathode-ray tube. Neutron counter pulses which occurred during the duration of the sweep, i.e., between 5 and 325 microseconds after an event ABC−D, were displayed on the sweep,[3] which was automatically photographed. The BF_3 proportional counters had essentially no dead time, and multiple pulses could be distinguished on the sweep with a resolving time of about two microseconds.

The events studied thus were the interactions of single charged particles which were able to traverse the 2000 g cm^{-2} of earth above the apparatus, 172 g cm^{-2} of Pb and 26 g cm^{-2} of paraffin, and which were stopped in the absorber S.

II. OBSERVATIONS

The numbers of events ABC and ABC−D were recorded. For each event ABC−D, the number n of neutrons detected in coincidence with the master pulse was found from the photograph of the oscilloscope trace, so that the distribution in n of the events, as well as the total number N of neutron coincidences, was obtained.

TABLE II. Distribution of μ^- interactions with regard to the number n of observed neutrons.

Material	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n>5$
Pb	1557	262	15	2	0	1	0
Bi	964	174	13	1	1	0	0
Sn	1423	161	9	1	0	0	0
Al	427	30	0	0	0	0	0

[3] Cocconi, Tongiorgi, and Widgoff, Phys. Rev. **79**, 768 (1950).

Corrections to the measured rates ABC−D and N for chance events and for background without absorber (i.e., with $S=0$) were calculated. The chance anticoincidence rates were obtained with account taken of the contributions of unrelated events in A, B, and C, and related events such as a true double coincidence in chance coincidence with a count in the third tray. The chance anticoincidence rate was found to be less than 1 percent of the observed rate ABC−D except for $S=0$, in which case it was 3 percent of the observed rate. Chance coincidences of neutrons with the trigger ABC−D constituted 0.1 percent of all coincident neutrons for Pb, Bi and Sn, and 0.3 percent of all coincident neutrons for Al. The rates ABC−D were actually corrected for chance events, but the corrections to N were considered negligible.

The correction for background was made by measuring the rates of events ABC−D and N with $S=0$ and subtracting these from the rates observed with absorbers in S. The events ABC−D recorded with $S=0$ included: (1) side showers which discharged A, B, and C, and not D, (2) counts caused by the inefficiency of tray D which resulted from counter dead time and gaps between counters, (3) mesons which were scattered in Σ so that they discharged A, B, and C, but missed D, (4) chance events, (5) knock-on electrons produced in the apparatus which discharged the telescope trays missed by their parent μ-mesons, and (6) mesons which stopped in the residual material, such as the counter walls. The subtraction of this background is not entirely justified *a priori*, since some of the contributing effects changed when an absorber was present. The corrections for chance events were made separately for the background rates and for the rates with absorbers before subtracting the background. An analysis of the other effects indicates that the systematic uncertainty in the number M of interacting μ-mesons which is introduced by subtracting the background amounts to about 5 percent for Pb, Bi and Sn, and about 10 percent for Al. The numbers of events ABC−D corrected for chance events and for background with $S=0$ are given in column 5, Table I. The rate of N with $S=0$ was found to be zero.

It was necessary to reduce the corrected number ABC−D to the number of negative mesons stopping in S and further to correct this number for those which

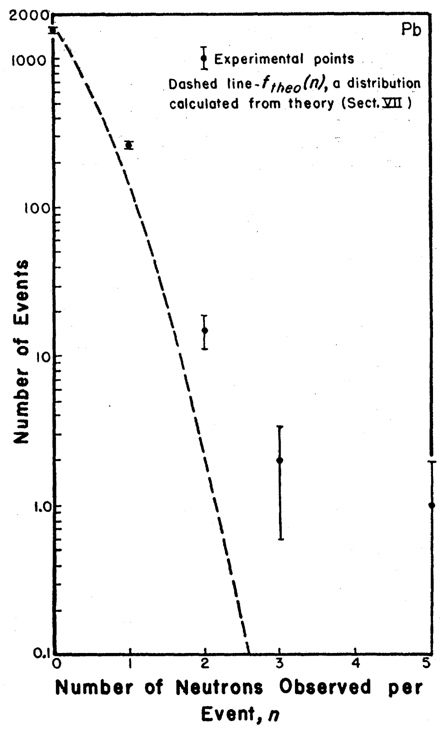

FIG. 2. Distribution in n of μ^- meson interactions in Pb.

underwent spontaneous decay, since only mesons which interact with nuclei can lead to the emission of neutrons. Owen and Wilson[4] have measured the ratio of positive to negative μ-mesons at sea level, as a function of momentum. The ratio is not very sensitive to the momentum in the range 2–10 Bev/c; for mesons of about 4 Bev/c they found $\mu^+/\mu^- = 1.263 \pm 0.019$. It was mesons of such energy that were able to penetrate the 2000 g cm^{-2} above the apparatus and that were then stopped in the absorber. Thus, of all the mesons stopped, 44.2 ± 0.7 percent were negative.

For Pb, Bi, and Sn, the probability of spontaneous decay of μ^- mesons is negligible. However, 40 ± 3 percent of the μ^- mesons stopping in Al decay.[5]

Except in the case of Bi, for which the data were obtained in a single run, two separate runs were taken with each material, and the background was measured in six separate runs in order to check that the apparatus gave consistent results over the long period of time during which it was in use. The good agreement between runs made at rather widely separated times showed that the system did behave in a consistent fashion.

Table I gives the numbers of events ABC and ABC$-$D, the number M of μ^- mesons undergoing nuclear interaction in S, and the total number N of neutrons observed in coincidence with events ABC$-$D.

Table II gives the distribution in n of the μ^- capture processes, where n is the number of neutrons observed per event. These data are also plotted in Figs. 2–5.

III. CONSIDERATION OF PROCESSES, OTHER THAN μ^- MESON ABSORPTION, WHICH MIGHT HAVE CONTRIBUTED TO THE ANTICOINCIDENCE RATE

Some of the events included under M may be nuclear interactions involving phenomena other than μ^- absorption. Such extraneous interactions would give a different distribution in n from that which arises from the nuclear capture of μ^- mesons at rest. The following processes were considered: (1) Interactions of N-component particles (neutrons, protons, π mesons) which penetrated the 2000 g cm^{-2} of earth above the apparatus, in addition to the paraffin and the absorber Σ, and stopped in S. (2) Interactions in flight in the apparatus of μ-mesons which failed to discharge Tray D. (3) Interactions of N-component particles produced locally by interactions in flight of μ-mesons.

Exact calculations of the contributions of these processes could not be made on account of uncertainties in the available data concerning such interactions and difficulty in evaluating the efficiency of the experimental arrangement for detecting such events. The experiment was designed to give a small probability for detecting events other than μ-meson absorp-

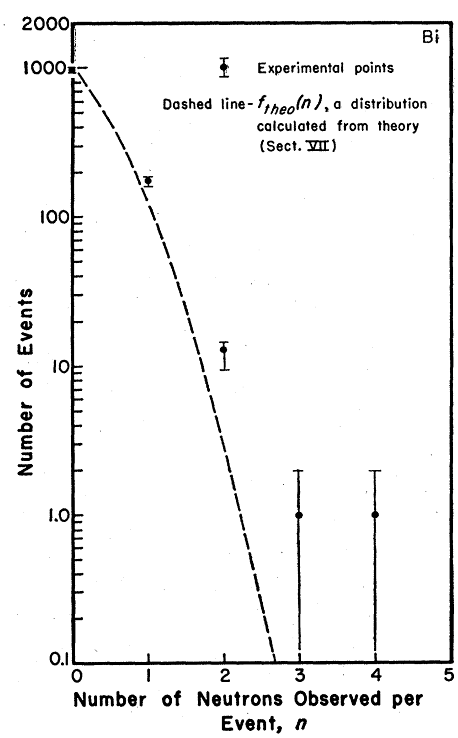

FIG. 3. Distribution in n of μ^- meson interactions in Bi.

[4] B. G. Owen and J. G. Wilson, Proc. Phys. Soc. (London) A64, 417 (1951).

[5] Keuffel, Harrison, Godfrey, Reynolds, Phys. Rev. 87, 942 (1952); H. K. Ticho, Phys. Rev. 74, 1337 (1948); G. E. Valley and B. Rossi. Phys. Rev. 73, 177 (1948).

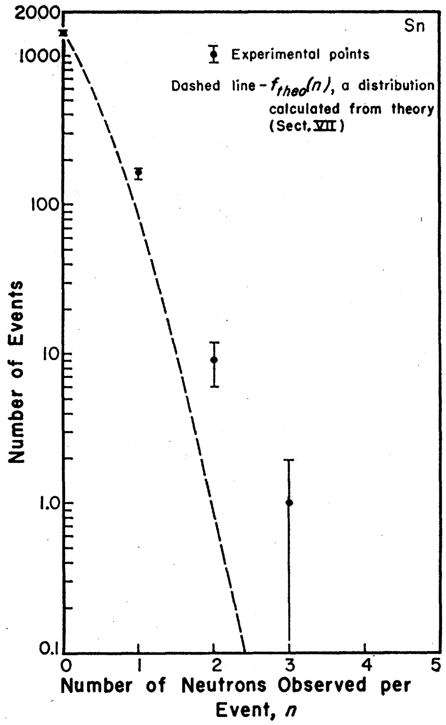

FIG. 4. Distribution in n of μ^- meson interactions in Sn.

tions, by the choice of the depth underground, by the use of the filter Σ, by requiring that one and only one particle discharge trays A, B, and C, and by making the anticoincidence tray D of high efficiency. The discrimination of the apparatus against the extraneous phenomena listed above is indicated by the following considerations: (1) N-component particles from above the earth's surface had only a very small probability of reaching S, since they have an absorption mean free path of 160–200 g cm^{-2} in materials of low Z.[6] Of those that do penetrate the 2000 g cm^{-2} of earth above the apparatus, only about 2 percent are charged particles, capable of discharging A, B, and C.

(2) A μ-meson in flight could give a master pulse ABC$-$D only by going through A, B, and C, unaccompanied by any charged particles, and by failing to discharge D, either on account of the small inefficiency of D or because it underwent in Σ or S an interaction[7,8] resulting in large angle scattering so that it missed tray D. This would have required a scattering in S of more than 30°, which is improbable according to the available evidence.[7,9]

(3) Consider now the N-component particles produced locally by energetic μ-mesons: it is very unlikely that any protons were produced above the apparatus which were capable of penetrating far enough from their point of origin to reach S, since a proton would have needed to have an energy of about 500 Mev to cross Σ and about 200 Mev to penetrate the paraffin above S, even if it escaped nuclear interaction in crossing these absorbers. An event initiated by a neutron could have been detected only if the neutron happened to be accompanied by a single charged particle which gave a pulse ABC$-$D. In order for a π-meson interaction in S to be detected either (a) the π must have come from above the apparatus unaccompanied by its parent μ or other charged particles and have gone through Σ without multiplying or being absorbed, or (b) a μ must have discharged trays A and B and interacted in flight in Σ, producing a π which went through tray C alone and was captured in S.

The effects of all the above processes were further limited by the anticoincidence requirement, since only events of fairly low average neutron multiplicity, 10 or less per event, could be detected with reasonable probability because of the fact that events of higher neutron multiplicity also yield appreciable numbers of charged particles which would have had a high probability of discharging tray D.

Estimated upper limits for the effects calculated indicated that they would certainly have a negligible influence on the distribution in n of observed events for $n\leq 2$ and would be too small probably by an order of magnitude to account for all events observed with $n\geq 3$. The largest contribution is that of the locally produced π-meson secondaries of μ-mesons interacting in flight. Because of the uncertainty of the calculations, it is impossible to rule out completely the possibility that at least some of the high n events observed were due to the extraneous processes considered here; however, most of them were more likely due to μ^- meson capture. No correction has, therefore, been made for the contribution of the other processes. It should be pointed out that such corrections, even if the estimated upper limits were used, would have a small effect, about equal to the statistical error, on the values of $\langle\nu\rangle$, the average neutron multiplicity, and hence on the mean excitation energy. The effect on σ, or the width of the neutron multiplicity distribution function, is small also, since this quantity, as well as $\langle\nu\rangle$, is determined mainly by the events with $n\leq 2$. Thus, the effect of the corrections is important chiefly in calculating the tail of the distribution in neutron multiplicity, which cannot be obtained from the data with any statistical accuracy anyway.

IV. EFFICIENCY OF THE NEUTRON DETECTOR

To obtain from the data of Table I, the average multiplicity of neutrons emitted after the absorption of μ^- mesons in each of the four materials studied, and from this, to deduce the average excitation energy given the nuclei, one needs to know the efficiency of the neutron detector. From the efficiency and the distribution

[6] G. D. Rochester and W. G. V. Rosser, Repts. Progr. Phys. 14, 227 (1951).
[7] E. P. George and J. Evans, Proc. Phys. Soc. (London) A63, 1248 (1950).
[8] G. Cocconi and V. Tongiorgi, Phys. Rev. 84, 29 (1951).
[9] E. Amaldi and G. Fidecaro, Helv. Phys. Acta 23, 93 (1950).

in n of observed events, given in Table II, one can obtain some information about the shape of the distribution functions $I(\nu)$, giving the probability of events in which exactly ν neutrons are produced. It should be emphasized that n is the number of neutrons observed in an event, while ν is the number of neutrons emitted.

The efficiency E can be written: $E=E_ik$, where E_i is the probability that a neutron produced in S will enter a BF_3 counter and give rise to a pulse and k is the probability that the neutron pulse will occur while the oscilloscope sweep is on.

The time distribution of the recorded neutrons was obtained from the sweep photographs. The data fit an exponential curve, which gave a mean life for thermal neutrons in the apparatus of $\tau=160\pm10$ microseconds. The duration of the sweep was 320 microseconds, and its start was delayed by 5 microseconds from the discharge of the Geiger counter telescope. Then k is given by

$$k=\int_5^{325} e^{-t/160}\frac{dt}{160}=0.85.$$

The factor E_i was measured experimentally by finding the counting rate $r(x, y, z)$ produced in the BF_3 counter arrangement by a calibrated source of neutrons. The source was placed at various points (x, y, z) in S, with the absorbing material in place. The functions $r(x, y, z)$ were thus obtained with Pb, Sn, and Al in S, and with $S=0$. If R is the known intensity of the source, the average efficiency of the system for detecting neutrons is given by

$$E=kE_i=k\frac{1}{R}\int_S r(x, y, z)f(x, y, z)dxdydz,$$

with

$$\int_S f(x, y, z)dxdydz=1.$$

The function $f(x, y, z)$ gives the spatial distribution in S of stopped mesons, which were the neutron sources in the experiment. This function depends on the angular distribution of the particles which stopped in S, the thickness of absorbing material above the apparatus for various directions of incidence, and the varying effective area of the counter telescope for rays of different directions. For the apparatus shown in Fig. 1, only the variation in the z direction need be considered, since the extension of S in the x and y directions was small. E was evaluated from the measured $r(x, y, z)$ and the calculated $f(z)$ by numerical integration.

The value of the efficiency which is thus obtained is that for detecting neutrons of the same energy as those emitted by the standard source. It is to be expected that the efficiency of a given counter arrangement will depend on neutron energy, because of the dependence of E_i on energy.

A theoretical estimate was made, for each material studied, of the distribution in energy of neutrons evaporated from a compound nucleus which was assumed to have an excitation of 20 Mev.[10] The calculated energy distributions were compared with the measured energy distribution of neutrons from a RaαBe source.[11] Except in the case of Al, the neutrons resulting from μ^- absorption are expected to have energies substantially smaller than RaαBe neutrons: the calculated distributions indicate that most of the evaporated neutrons have energies between 0 and 3 Mev, while the RaαBe neutrons have an energy range between 1 and 11 Mev, with a peak between 3 and 5 Mev.

In order to investigate and to take account of the energy dependence of the efficiency, measurements were made with two sources: a one-mC RaαBe source which was calibrated at Cornell by comparing it with a 10-mC RaαBe source calibrated within ±5 percent at the Argonne National Laboratory, and a MsThγBe source constructed at Cornell which gives neutrons whose average energy is 0.827 ± 0.030 Mev.[12] The yield of the latter source was known with an accuracy of ±25 percent.

The neutron detecting system was found to be insensitive to neutron energy in the energy range covered by the two sources, within the accuracy of their cali-

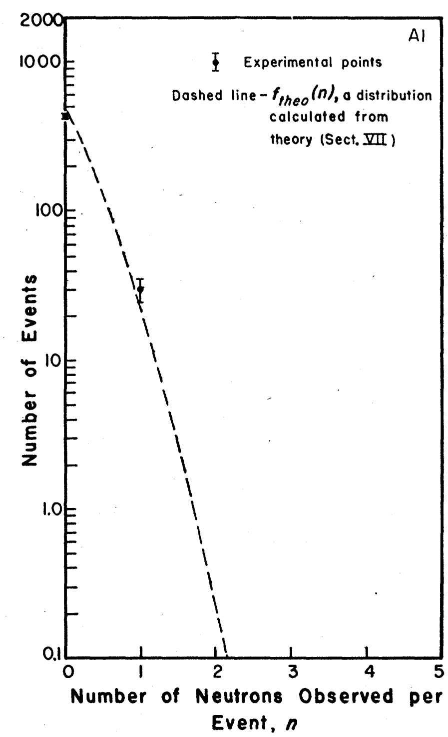

FIG. 5. Distribution in n of μ^- meson interactions in Al.

[10] J. M. Blatt and V. Weisskopf, U. S. Office of Naval Research Technical Report No. 42, ONR (NR-026-001) (unpublished).
[11] H. L. Anderson, Preliminary Report No. 6, Nuclear Science Series, NRC, 1949 (unpublished).
[12] A. O. Hanson, Phys. Rev. **75**, 1794 (1949).

TABLE III. Efficiency of the neutron detector (measured with the RaαBe source). The efficiency for evaporated neutrons from Pb and Sn may be up to 20 percent higher than the value given in the table, while the value given for Al is probably an overestimate by a small amount.

Material	Efficiency (%)
Pb	7.7
Sn	7.4
Al	6.9
No absorber	6.7

brations. It was decided that the values of efficiency obtained with the RaαBe source should be used, since there was less uncertainty in the RaαBe calibration. A systematic uncertainty of about 20 percent must be assumed, on account of possible energy dependence. Since the system is thought to be more efficient for low energy neutrons than for high energy ones, this 20 percent uncertainty is taken to mean that the efficiency for measuring the evaporated neutrons may be up to 20 percent higher than the efficiency for measuring RaαBe neutrons, except in the case of Al, for which the value of E measured with RaαBe neutrons is probably an overestimate.

The values of efficiency which have been used are listed in Table III. The efficiency for Bi was assumed to be the same as that for Pb.

V. AVERAGE MULTIPLICITIES AND MULTIPLICITY DISTRIBUTION FUNCTIONS

Functions $f(n)$, giving the expected number of events with a given n, are related to the multiplicity distributions $I(\nu)$ by the expression

$$f(n)=M\sum_{\nu=n}^{\infty}I(\nu)\binom{\nu}{n}E^n(1-E)^{\nu-n}.$$

The first moment of $I(\nu)$ is $\langle\nu\rangle$ and is given by the expression

$$\langle\nu\rangle=\sum_{\nu=0}^{\infty}\nu I(\nu)=\frac{\sum_{n=0}^{\infty}nf(n)}{ME}=\frac{N}{ME}=\frac{\langle n\rangle}{E},$$

where $\langle n\rangle$ is the average number of neutrons observed per μ^- capture.

The width of the main part of the multiplicity distribution is determined by the standard deviation which is found from the data by using the expression

$$\sigma=[\langle\nu^2\rangle-\langle\nu\rangle^2]^{\frac{1}{2}}=\frac{1}{E}[\langle n^2\rangle-\langle n\rangle^2-\langle n\rangle(1-E)]^{\frac{1}{2}}.$$

Higher moments and their differences are required to determine the high multiplicity tail of the distribution. The values of $\langle\nu\rangle$ and σ obtained from the data are given in Table IV, with their statistical errors. $\langle\nu\rangle$ is determined mainly by events with $n=1$ and σ mainly by events with $n=2$. The large errors in σ arise because only 9 to 15 events with $n=2$ were observed for each absorber, except in the case of Al, where none were observed. The value of σ given for Al is an upper limit obtained by assuming a statistical error of $^{+1}_{-0}$ in the number of events with $n=2$ and a negligible statistical error in the number of events with $n>2$. Since Pb and Bi are quite similar, the data for these two were combined to give $\langle\nu\rangle$ and σ with smaller statistical errors.

Higher moments depend entirely on events with $n>2$, of which there were no more than three with any given absorber. Thus, the tails of the distributions are not determined by the data. However, the efficiency for recording two specific neutrons in an event is about 1/200, while, for Pb and Bi, two or more neutrons were observed in about 1/90 of the meson absorptions, which implies that three and more than three neutrons are frequently emitted.

Several other authors have reported measurements of the neutron multiplicity resulting from μ^- capture in Pb. The values of $\langle\nu\rangle$ and $\langle\nu^2\rangle$ obtained in this experiment agree within the statistical errors with those of Crouch and Sard,[13] who have obtained, in a similar experiment, $\langle\nu\rangle=2.15\pm0.15$ and $\langle\nu^2\rangle=5.3\pm1.9$. The value of σ from the Crouch and Sard data is not statistically significant, so these data do not give any information on the shape of the distribution function. Groetzinger, Berger, and McClure,[14] by using a magnetic lens to select μ^- mesons, have measured $\langle\nu\rangle=1.96\pm0.72$ (probable error). Conforto and Sard,[15] also using a magnetic lens, obtained a considerably lower value, $\langle\nu\rangle=1.47\pm0.13$. The last-named experimenters are inclined to attach more weight to the measurements of Crouch and Sard, since their experiment was designed to give relative, rather than absolute, values of the average multiplicity. Neither of the two magnetic lens experiments gave values for $\langle\nu^2\rangle$ or σ.

Conforto and Sard also reported preliminary results on neutron multiplicities for Ca and Mg which indicate values smaller than that for Pb by about a factor of five. They stated that the average multiplicity is almost certainly less than half that for Pb. Because of the preliminary nature of their results, it is difficult to make a comparison with our measurement of $\langle\nu\rangle$ for Al. Also, at values of Z below about 20, one might expect to find rather large variations in neutron multiplicity because

TABLE IV. First moments and standard deviations of the multiplicity distributions $I(\nu)$, calculated from the data of Tables I and II.

	Pb	Bi	Pb+Bi	Sn	Al
$\langle\nu\rangle$	2.14 ± 0.13	2.32 ± 0.17	2.22 ± 0.10	1.54 ± 0.12	0.95 ± 0.17
σ	1.80 ± 0.51	1.83 ± 0.54	1.81 ± 0.38	1.39 ± 0.32	$0.2\,^{+0.8}_{-0.2}$

[13] M. F. Crouch and R. D. Sard, Phys. Rev. **85**, 120 (1952).
[14] Groetzinger, Berger, and McClure, Phys. Rev. **81**, 969 (1951).
[15] A. M. Conforto and R. D. Sard, Phys. Rev. **86**, 465 (1952).

of the effect of proton competition.[16] However, the absorption of a μ^- meson leads to a decrease in charge in the target nucleus, so there will, in general, be an excess of neutrons in the compound nucleus, and the average multiplicity from Mg and Ca should not be much less than that found for Al.

In addition to the values of $\langle\nu\rangle$ and σ measured in this experiment, some other information about $I(\nu)$ is available. First of all, $I(\nu)$ must be such that the emission of zero neutrons is possible, in which case the excitation energy of the nucleus is carried off by gamma-rays or protons. One can also set an upper limit for ν. If the whole rest mass of the meson went into exciting the nucleus, about 12–16 neutrons could be emitted, though it is unlikely that so many would be, since there would be considerable competition from proton emission at this high excitation energy. The upper limit must be at least five, since five neutrons were observed in one case. Thus an upper limit between 5 and 12 is reasonable.

Within the limits given above, many forms of the function $I(\nu)$ can be found which fit the data, but they are all somewhat similar. As an example, let us assume a rectangular distribution as follows:

$$\begin{aligned} I(\nu)&=a \quad \text{for} \quad \nu=0,1;\\ I(\nu)&=b \quad \text{for} \quad \nu=2,3;\\ I(\nu)&=c \quad \text{for} \quad \nu=4,5,6,7,8;\\ I(\nu)&=0 \quad \text{for} \quad \nu>8, \end{aligned} \tag{1}$$

where a, b, and c are constants determined from the data by using $\langle\nu\rangle$ and $\langle\nu^2\rangle$ and normalizing $I(\nu)$. The values thus obtained for a, b, and c are given in Table V.

The data for Al can be fitted equally well by assuming either that one neutron is emitted in every case, i.e., $I(1)=1$, or that $I(\nu)$ is the same for $\nu=0$, 1, 2 and zero for $\nu>2$, i.e., $I(0)=\frac{1}{3}$, $I(1)=\frac{1}{3}$, and $I(2)=\frac{1}{3}$. A linear combination of the two functions would, of course, also give a good fit; in fact, any function which has a maximum at $\nu=1$, equal values for $\nu=0$ and 2, and which is practically zero for $\nu>2$, would fit the data.

We have insufficient data to obtain $I(\nu)$ in more detail. The third moments are determined only within ± 70 percent, so that, if the assumed expression for $I(\nu)$ has more than three arbitrary parameters, these parameters cannot be determined from the data with any significance.

Although many other forms of $I(\nu)$ may be found which fit the data, any reasonable form, having one maximum and becoming negligible rapidly for $\nu>8$, gives a probability of about $\frac{1}{3}$ to $\frac{1}{4}$ for events with $\nu\geq 3$ for Pb, Bi, and Sn, and a probability of about $\frac{1}{2}$ for events with $\nu\geq 2$ for these materials. These results may be contrasted with the predictions made in Sec. VII from the calculations of Tiomno and Wheeler, and Rosenbluth.

[16] J. Heidmann and H. A. Bethe, Phys. Rev. **84**, 274 (1951).

VI. AVERAGE EXCITATION ENERGIES

A calculation was made, by using compound nucleus theory,[10] of the average number $\langle m(\mathcal{E})\rangle$ of evaporated neutrons expected, as a function of excitation energy $\mathcal{E}$. The quantity $\langle m(\mathcal{E})\rangle$ was calculated for each isotope of the elements considered, and the results were averaged with account taken of the relative isotope abundance.

The probability of proton emission was considered to be negligible. In the case of Al, where proton emission is more likely than in the heavier elements, the effect of taking proton emission into account is to reduce $\langle m\rangle$ by less than 5 percent. The calculation with proton emission taken into account was done by following the method of Heidmann and Bethe.[16]

Experimentally determined values of the mass differences were used for the energy required to change the target nucleus (A,Z) to its isobar $(A,Z-1)$ and to obtain neutron binding energies in the nucleus $(A, Z-1)$.[17] The emitted neutrons were assumed to follow an energy distribution given by the function:

$$g(\epsilon)d\epsilon=\text{const}\times\sigma_c\exp[-\epsilon/T(W_n)]d\epsilon,$$

TABLE V. Multiplicity distribution functions $I(\nu)$ obtained by fitting the data to the assumed form (1).

Parameter	Pb	Bi	Sn
a	0.21	0.16	0.27
b	0.23	0.28	0.21
c	0.025	0.024	0.0069

where ϵ is the kinetic energy of the emitted neutron, σ_c is the cross section for neutron capture in the inverse process, W_n is the energy available to the emitted neutron, and $T(W_n)$ is the nuclear "temperature," related to W_n by the expression $T(W_n)=(W_n/a)^{\frac{1}{2}}$. Values for a, which depend on A, are given by Blatt and Weisskopf.[10]

Figure 6 gives the curves $\langle m(\mathcal{E})\rangle$ *vs* $\mathcal{E}$ for the natural isotopic mixture of each element studied.

The abruptness of the threshold θ_1 for single neutron emission is caused by neglecting gamma-ray emission. If this were taken into account, $\langle m(\mathcal{E})\rangle$ would be smaller than the curves indicate at least near the threshold, but this effect is small. The multiple steps near θ_1 for Pb and Sn are caused by the presence of several isotopes with different thresholds.

From these curves, one can find the average nuclear excitation $\langle\mathcal{E}\rangle$ needed to give the average multiplicities observed. By using for $\langle m\rangle$ the values of $\langle\nu\rangle$ given in Table IV and repeated in column 2 of Table VI, one obtains for $\langle\mathcal{E}\rangle$, from Fig. 6, the values in column 3 of Table VI. In column 4 are given values of $\langle\nu_L\rangle$, the

[17] *Nuclear Data*, National Bureau of Standards Circular 499 and Supplment 1 to NBS 499 (1950–1951); J. A. Harvey, Phys. Rev. **81**, 353 (1951); Kinsey, Bartholomew, aud Walker, Phys. Rev. **83**, 519 (1951); H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York, 1947).

average multiplicities calculated on the assumption that the efficiency of the neutron detector for the evaporated neutrons from Pb, Bi, and Sn is 20 percent higher than for RaαBe neutrons (Sec. IV). The values given for $\langle \nu_L \rangle$ are considered to be lower limits for the average neutron multiplicities, and the values of $\langle \mathcal{E}_L \rangle$ are the corresponding excitation energies. The uncertainties given with the values of $\langle \mathcal{E} \rangle$ and $\langle \mathcal{E}_L \rangle$ in Table VI arise from the statistical uncertainties in the average multiplicities.

The curves $\langle m(\mathcal{E}) \rangle$ *vs* $\mathcal{E}$ for Sn, Pb, and Bi are found to be very similar, if the thresholds θ_1 are superposed. This indicates that the results are not very sensitive to the details of the calculation and that the values of $\langle \mathcal{E} \rangle$ are probably reliable, even if the specific assumptions made in the compound nucleus calculation are somewhat in error. It is found, in fact, that the values of $\langle \mathcal{E} \rangle$ obtained by using for $\langle m(\mathcal{E}) \rangle$ *vs* $\mathcal{E}$ a simple step function calculated by assuming that the neutrons are emitted with zero kinetic energy are almost equal to those given in Table VI.

These values of the average excitation energy, which are low compared with the 107-Mev rest energy of the μ^-, indicate that a neutrino, or some other light neutral particle, carries a large part of the meson rest energy out of the nucleus when a μ^- meson is captured. The emission of such a light neutral particle in the case of μ^- capture is also indicated by comparing stars produced by μ^- and π^- mesons stopping in emulsions[18–20] and by comparing neutron multiplicities resulting from the nuclear capture of π^- mesons at rest[21] with those resulting from μ^- capture.

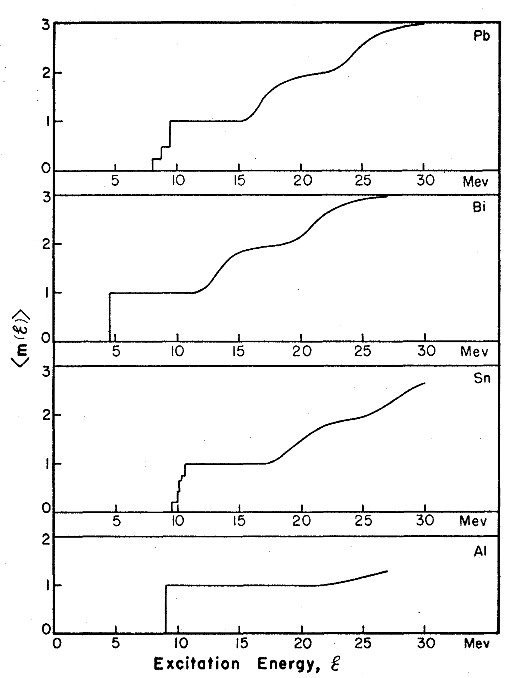

FIG. 6. The average number of evaporated neutrons from Pb, Bi, Sn, and Al as a function of excitation energy, as calculated from compound nucleus theory (see reference 10).

TABLE VI. Experimental average neutron multiplicity $\langle \nu \rangle$ and average excitation energy $\langle \mathcal{E} \rangle$ resulting from the capture of a μ-meson. $\langle \nu_L \rangle$ and $\langle \mathcal{E}_L \rangle$ were obtained by assuming a maximum value for the efficiency of neutron detection. $\langle \nu_{\text{theor}} \rangle$ was calculated from the theory of Tiomno and Wheeler, and Rosenbluth (Sec. VII).

Material	$\langle \nu \rangle$	$\langle \mathcal{E} \rangle$ (Mev)	$\langle \nu_L \rangle$	$\langle \mathcal{E}_L \rangle$ (Mev)	$\langle \nu_{\text{theor}} \rangle$
Pb	2.14±0.13	23±1	1.71±0.10	18±1	0.98
Bi	2.32±0.17	21±1	1.86±0.14	16^{+3}_{-1}	1.43
Sn	1.54±0.12	20±1	1.23±0.10	19±1	0.78
Al	0.95±0.17	$9<\mathcal{E}<25$	—	—	0.76

VII. COMPARISON WITH NUCLEAR THEORY

Tiomno and Wheeler[22] and Rosenbluth[23] have calculated the probability $P(\mathcal{E})d\mathcal{E}$ that a nucleus will receive an excitation energy between $\mathcal{E}$ and $\mathcal{E}+d\mathcal{E}$ in capturing a μ^- meson, on the assumption that the interaction is

$$\mu^- + P \rightarrow N + \nu. \tag{A}$$

If the proton in this interaction were free and at rest, only one neutron of 4.8 Mev could be emitted. The possibility of higher excitation arises from the fact that the nucleons in the nucleus have momentum and also from correlations between nucleons in the nucleus which make higher momentum transfers to the nucleus possible. The theory of the participation of more than one nucleon in the interaction with the μ^- has not been worked out. In the calculations considered here, the nucleons in the nucleus are assumed to constitute a gas of free particles, which follow a Fermi distribution in momentum, and it is assumed that energy and momentum are conserved among the four particles of the interaction (A).

The resulting distribution in excitation energy has a maximum at about 16 Mev and drops quickly to zero at 23 Mev. The average excitation energy expected is 13 Mev.

From this energy distribution function and the compound nucleus theory, one can calculate the average multiplicity $\langle \nu_{\text{theor}} \rangle$ to be expected, and also the expected distribution in n, $f_{\text{theor}}(n)$, of μ^- capture events. The values of $\langle \nu_{\text{theor}} \rangle$ are given in column 6, Table VI, for comparison with the experimental values; the functions $f_{\text{theor}}(n)$ are the dashed curves plotted in Figs. 2 to 5 for comparison with the data. Each of these distribution functions was calculated for a total number of events equal to the experimental value.

[18] E. P. George and J. Evans, Proc. Phys. Soc. (London) **A64**, 193 (1951).
[19] H. Morinaga and W. F. Fry, Phys. Rev. **87**, 182 (1952).
[20] F. L. Adelman and S. B. Jones, Phys. Rev. **75**, 1468 (1949).
[21] V. Tongiorgi and D. A. Edwards, Phys. Rev. **88**, 145 (1952).
[22] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).
[23] M. N. Rosenbluth, Phys. Rev. **75**, 532 (1949).

It can be seen that the theory leads one to expect fewer neutrons than were observed; it predicts too few events of $n=2$ by one order of magnitude and too few events of $n=3$ by at least two orders of magnitude (except for Al). It should be noted that the data indicate that the probability of having $\nu \geq 3$, or $\mathcal{E} \gtrsim 22$ Mev, is about $\frac{1}{4}$ to $\frac{1}{3}$ for Pb, Bi, and Sn, while according to the calculations of Tiomno and Wheeler, and Rosenbluth, $P(\mathcal{E})=0$ for $\mathcal{E}>23$ Mev, and the probability of events with $\nu \geq 3$ is negligible.

The theoretical maximum excitation energy $\mathcal{E}=23$ Mev, which is inconsistent with the experimental results, comes from the assumption that the nucleons in the nucleus constitute a Fermi gas with a maximum energy of 20 Mev and from neglect of the interactions between nucleons. These interactions permit the momentum and energy to be distributed to more than one nucleon in the capture process and thus give rise to higher nuclear excitations.

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Proton-Proton Scattering from 1.8 Mev to 4.2 Mev*

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Measurements of the scattering of protons by protons have been made at six energies in the range from 1.8 Mev to 4.2 Mev. At each energy, the differential cross section was measured as a function of angle from 6° in the laboratory system up to 45° or higher. In the angular range above 20°, the cross sections are determined to within ±0.3 percent. Below 20°, the uncertainties become larger, approaching ±0.6 percent at 6°. Departures from pure S-wave scattering are observed at all energies .These deviations are larger than can be explained by the experimental uncertainties. They closely resemble the effect of a negative P-wave phase shift of approximately −0.1° at 4 Mev and exhibit a reasonable energy dependence.

I. INTRODUCTION

A NUMBER of excellent surveys of experimental results on proton-proton scattering and their theoretical interpretation have appeared in the past few years.[1,2] It is generally concluded that the presence of higher phase shifts has not been established, and the need for improved accuracy in experimental data is strongly indicated.

In the present experiment the scattering of protons by protons is re-examined in the energy range from 1.8 Mev to 4.2 Mev with fractional percent accuracy. Information about the form of the interaction potential is sought through a more accurate determination of the S-wave phase shifts. By a closer inspection of angular distribution and by extending angular distribution measurements to smaller angles, the effects of higher angular momenta are sought.

II. GENERAL FEATURES OF SCATTERING CHAMBER

Details of the chamber design are shown in Figs. 1 and 2. The beam enters the chamber from the large collimating tube at the right, it traverses the chamber and is collected by a Faraday cup at the rear. Hydrogen gas at a pressure of 8-mm Hg fills the entire cylindrical chamber and provides the target. Scattering occurs everywhere along the path of the beam, but only that from a small region at the center of the chamber can reach the detector through the narrow slits of the analyzer. Details of the analyzer slit system can be seen in Fig. 3. The defining slits are mounted on dovetail slides in such a way that any one of three slit sizes may be selected by varying the vertical positions of the slides. Manipulating rods in the top cover of the chamber enable the slits to be changed without interruption of operation.

The analyzer and detector are mounted on a large angle-wheel which can be rotated to any position within ±100° of the forward direction. This wheel is rim supported and rim driven. It was originally the meridian circle of an astronomical telescope and has near its rim an angle scale 50 cm in diameter graduated at 2-minute intervals of arc. On division marks, angles can be set within about 0.001°. As index marks, two fine quartz fibers are used, one in each of the two backward 45° positions. They are mounted on open frames close above the scale and are viewed by telescopes mounted in the lid.

A proportional counter with a thin nickel window

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‡ Now at North American Aviation, Inc., Downey, California.
1 Yovits, Smith, Hull, Bengsten, and Breit, Phys. Rev. **85**, 540 (1952).
2 J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).