Elastic Scattering of Alpha-Particles by Carbon*

RICHARD WILLIAM HILL†
University of Wisconsin, Madison, Wisconsin
(Received January 9, 1953)

The $C^{12}(\alpha,\alpha)C^{12}$ differential scattering cross sections were measured in a gas scattering chamber at $\theta(\text{c.m.})=171.0°$, 147.2°, 125.5°, and 92.0° with alpha-particles accelerated in the Wisconsin electrostatic generator. Cross sections were measured at alpha-particle energies from 0.5 to 4.0 Mev. For zero spin nuclei bombarded by zero spin particles, each partial wave contributing to the cross section vanishes at some angle except that for $l=0$. The P wave vanishes at 90.0°, the D wave at 125.3°, the F wave at 140.8°, and the G wave at 149.5°. Thus, by observing the cross sections near these angles it was possible to determine the J values and parities of the levels. Analyzing the data by means of the Wigner-Eisenbud one-level approximation to determine the widths and resonant energies confirmed the qualitative characterization of the levels. With the O^{16} ground state as the energy zero, the J values, parities, and excitation energies of the two levels observed are $J=1^-$ at 9.58 Mev and $J=2^+$ at 9.835 Mev. The uncertainty in these energies is about 10 kev. The reduced width of the P resonance is approximately 100 percent of the single-particle width, and that of the D resonance is 0.15 percent.

I. INTRODUCTION

THE $C^{12}(\alpha,\alpha)C^{12}$ reaction has been studied previously[1] at three scattering angles, using alpha-particles from RaC′ slowed down by absorbers to energies in the range from 3.9 to 6.9 Mev. Scattering anomalies were found at incident alpha-particle energies of 4.4, 5.0, and 5.5 Mev. Both the energy and angular resolution were too poor to permit unambiguous assignment of J values and parities to the levels.[2]

Adding an alpha-particle to a C^{12} nucleus forms O^{16} with an excitation energy of 7.149 Mev. Thus, bombarding C^{12} with alpha-particles having laboratory energies in the range from 0.5 to 4 Mev will give information about excited states of O^{16} in the range of excitation energies from 7.524 to 10.149 Mev. The known energy levels of O^{16} are given in a recent review article by Hornyak *et al.*[3] In the energy region covered by the experiment to be described, levels at excitation energies of 8.6 and 9.5 Mev were reported previously.[4]

II. EXPERIMENTAL METHODS

The gas scattering chamber and associated equipment used for this experiment are described in the accompanying paper by Cameron. Methane gas was used for the measurements with incident alpha-particles in the energy range from 2.3 to 4.0 Mev and propane was used for the energy range from 0.5 to 2.3 Mev. Cross section measurements were made at scattering angles of 171.0°, 147.2°, 125.5°, and 92.0° in the center-of-mass system.

III. EXPERIMENTAL RESULTS

Two scattering anomalies were observed in the energy region from 2.5 to 4.0 Mev. None was found at energies from 0.5 to 2.5 Mev. Figures 1 and 2 show the differential cross sections in the center-of-mass system as a function of the incident energy of the alpha-particles. Cross sections are given for the four scattering angles in the energy region from 2.5 to 4.0 Mev in Fig. 2 and for the 171.0° scattering angle in the energy region from 0.5 to 2.5 Mev in Fig. 1. The Rutherford scattering cross sections were calculated at 100-kev intervals in the energy range from 0.5 to 2.5 Mev and are indicated on the diagram by heavy dots. The experimental cross sections were calculated using a double charge for the collected alpha-particles over the entire energy range. Below 2 Mev some of the collected alpha-particles are singly charged and some doubly charged. The ratio of He^+ ions to He^{++} ions is a function of the incident alpha-particle energy and is not accurately known between 0.5 Mev and 2 Mev. No attempt has been made to apply corrections. The portion of the curve in Fig. 1 for the energy range from 0.5 to 2.5 Mev is accordingly marked "uncorrected cross section."

As discussed in the preceding paper by Cameron, a partial wave with $l=1$ vanishes only at 90.0°, and a partial wave with $l=2$ vanishes at 125.3°. An inspection of Fig. 2 shows that the broad resonance is present at all angles except 92.0° and that the narrow resonance disappears only at 125.5°. Thus, the J values and parities of the two levels can be assigned as $J=1^-$ for the level at 3.3 Mev and $J=2^+$ for the level at 3.58 Mev.

The largest errors in the absolute cross sections are the statistical uncertainties as determined by the number of particles counted and are less than 3 percent for most of the points taken during this experiment. Uncertainties in the energy loss in the scattering gas introduced uncertainties of about 10 kev in the alpha-particle energies.

IV. ANALYSIS

The J values and parities of the two levels were assigned in the preceding section. The methods used to obtain the level widths and resonant energies will be

* Work supported by the U. S. Atomic Energy Commisions and the Wisconsin Alumni Research Foundation.
† Now at Westinghouse Research Laboratories, East Pittsburgh, Pennsylvania.
[1] A. J. Ferguson and L. R. Walker, Phys. Rev. **58**, 666 (1940).
[2] M. E. Rose, Phys. Rev. **57**, 958 (1940).
[3] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[4] H. W. Fulbright and R. R. Bush, Phys. Rev. **74**, 1323 (1948).

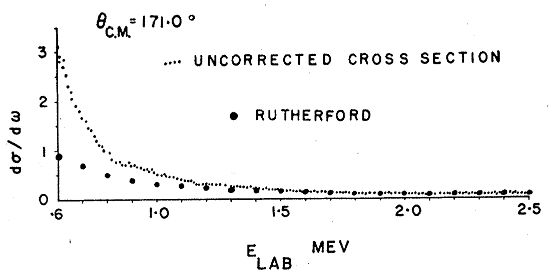

FIG. 1. $C^{12}(\alpha,\alpha)C^{12}$ differential cross section in barns per steradian in the center-of-mass system as a function of the incident energy of the alpha-particles from 0.6 to 2.5 Mev. Cross sections were calculated assuming that the alpha-particles were doubly ionized after passing through the collector cup foil. Rutherford cross sections are shown by heavy dots.

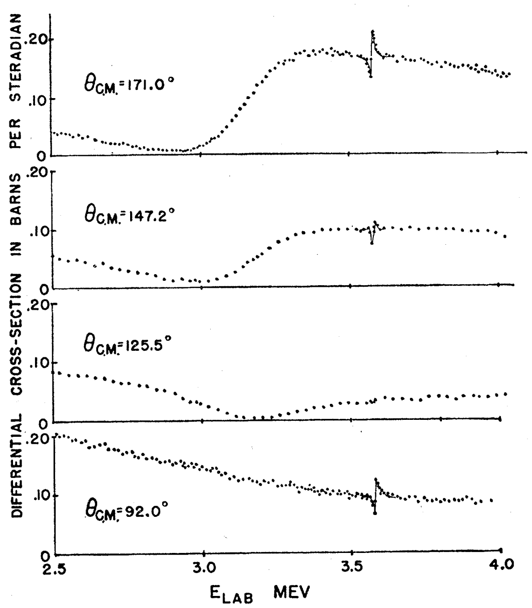

FIG. 2. $C^{12}(\alpha,\alpha)C^{12}$ differential cross sections in the center-of-mass system as a function of the incident energy of the alpha-particles from 2.5 to 4.0 Mev.

described in this section. The partial wave formula and the method of analysis used here are essentially the same as those used by Cameron. Because of the large width of the P resonance the energy dependence of the terms in the cross-section formula was not negligible. Therefore, separate vector diagrams were made for each energy considered, and the phase shift formula was written in terms of the energy independent quantities γ_λ^2 and E_λ. Thus, the P wave phase shift becomes

$$\delta_1=-\tan^{-1}(F_1/G_1)_{r=a}+\tan^{-1}\left(\frac{\gamma_\lambda^2k/A_1^2}{E_\lambda-(\gamma_\lambda^2/a)(g_1+1)-E}\right)_{r=a}$$

where $g_1=d\ln A_1/d\ln\rho$. It may also be expressed as

$$g_1=\Phi_1^*/\Phi_1-\rho^2\Phi_1\theta_1/3F_1^2A_1^2,$$

where $\Phi_1^*/\Phi_1=\rho F_1'/F_1$, or

$$g_1=\Phi_1^*/\Phi_1-\rho G_1/F_1A_1^2,$$

or

$$g_1=-(1+\rho\eta)+\rho(1+\eta^2)^{\frac{1}{2}}(F_1F_0+G_1G_0)/A_1^2.$$

Quantities appearing in these formulas are defined in the preceding article. By taking graphically determined values of δ_1 at two suitably chosen energies for the 147.2° scattering angle and solving the resulting equations, values of γ_λ^2 and E_λ were obtained. These values were used to calculate phase shifts and cross sections for the other energies and angles. The results are shown in Figs 3(a) and 3(b) for the bombarding energy range from 2.5 to 4.0 Mev.

For analysis of the D resonance, which is relatively narrow, the method employed by Cameron in the preceding article was used. The formula for the D wave phase shift was written as

$$\delta_2=-\phi_2+\tan^{-1}\frac{\Gamma_\lambda/2}{E_r-E}.$$

Values of $E_r=3.582$ Mev and $\Gamma_\lambda=0.001$ Mev gave the

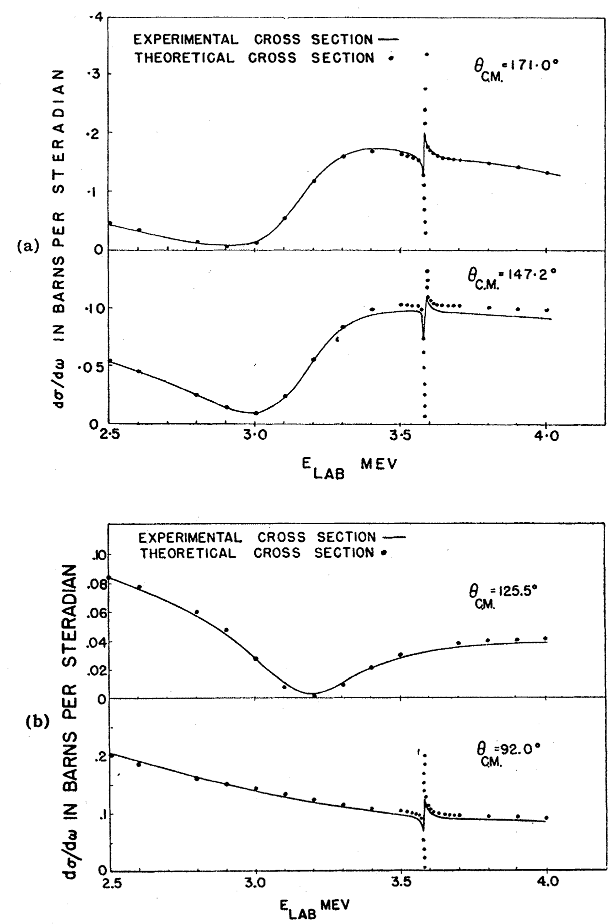

FIG. 3. (a) and (b), $C^{12}(\alpha,\alpha)C^{12}$ theoretical and experimental differential cross sections in the center-of-mass system as a function of the incident energy of the alpha-particles.

best fit to the experimental phase shift curve. The cross section measurements for the D resonance were made with a target thickness of about 15 kev at 171.0° and 3 kev at 92.0°. Because the resonance was not completely resolved, an uncertainty of about ± 0.5 kev should be assigned to the value for the experimental width. The value for the resonant energy has an uncertainty of approximately ± 10 kev caused by the uncertainty in the stopping power of the gas target.

The parameters determined for the two resonances are given in Table I. Values of E_r and Γ_λ are given in the laboratory system of units; all other quantities are given in the center-of-mass system.

The potential phase shifts depend on the value chosen for the interaction parameter (a). This may be a slowly varying function of energy because of cumulative effects of tails of distant resonances and cannot be uniquely determined.[5] However, choosing a value of (a) close to the nuclear radius should be a good approximation. A value of $a = 1.4\times 10^{-13}(A^{\frac{1}{3}}+4^{\frac{1}{3}}) = 5.43\times 10^{-13}$ cm was used for this experiment. The departure of the calculated points from the experimental curves shown in Figs. 3(a) and 3(b) becomes appreciable above 3.5 Mev and amounts to about ten percent for the scattering angles of 147.2° and 92.0°. Increasing the value of the interaction parameter (a) made the fit worse at all angles. Decreasing the value of (a) improved the fit at 147.2° but gave poor results at the other angles. The effect of higher energy levels was considered, but no single level was found that would remove the discrepancy.

This discrepancy may be caused by the effects of a combination of higher energy levels. Ignoring the tails of higher levels assumes that the nonresonant phase shifts are caused solely by potential scattering. This assumption is probably a poor one since levels are known to exist just above 4-Mev bombarding energy.

V. DISCUSSION OF RESULTS

The known energy levels of O^{16} are given in the diagram of Fig. 4. The parameters of the levels that are indicated by heavy lines at 9.84 and 9.58 Mev were determined in this experiment.

The level at 8.6-Mev excitation energy was observed by Fulbright and Bush[4] by the study of the inelastic scattering of protons from O^{16}. Evidence for this level

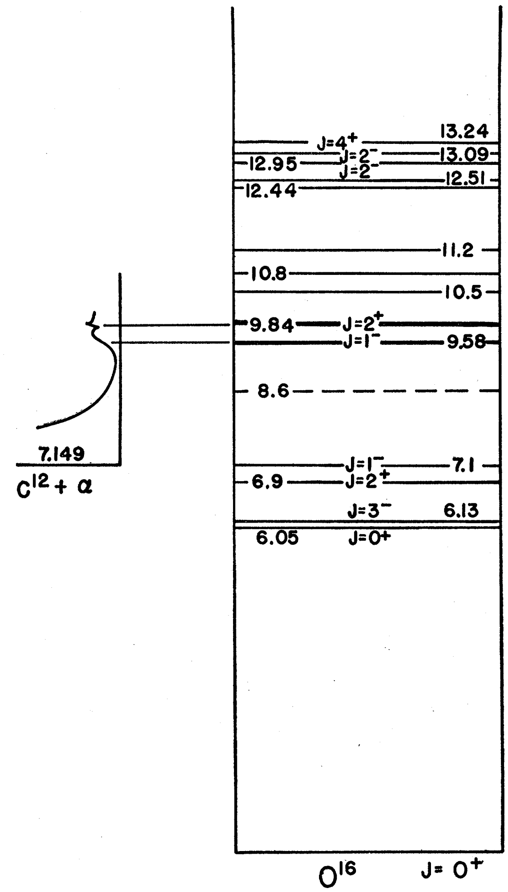

FIG. 4. Known energy levels of O^{16} with the two levels observed in the present experiment indicated by heavy lines.

was not considered conclusive. It would have been missed in the present experiment if its experimental width were much less than 1 kev or if its parity were odd and angular momentum even or vice versa.

The reduced width of the $J=1^-$ level at 9.58 Mev is approximately equal to the single particle width $\hbar^2/\mu a$. For single particle excitation the total wave function χ_λ of the compound state is the product of the wave function of the residual nucleus and of a function of the distance of the extra particle from the residual nucleus. When the single particle picture applies, the reduced width[5] is given approximately by

$$\gamma_\lambda^2 \simeq \hbar^2/\mu a.$$

If many particles are excited, then the reduced width may be expressed by

$$\gamma_\lambda^2 = (3\hbar^2/2\mu a)\sum_n C_{\lambda s n}^2,$$

where the $C_{\lambda s n}$ give the strengths of the individual wave functions entering into the total wave function. The sum of the $C_{\lambda s n}^2$ will be much less than one because normalization demands that the sum of $C_{\lambda t n}^2$ over n and t equal one and the terms with $t=s$ constitute only

TABLE I. Values of the parameters determined for the two levels. Quantities are given in the center-of-mass system of units unless otherwise noted.

Level assignment	$J=1^-$	$J=2^+$
E_r (Mev) lab.	3.24	3.582
Γ_λ (Mev) lab.	0.86	0.001
Excitation energy (Mev)	9.58	9.835
γ_λ^2 (Mev-cm)	3.3×10^{-13}	5.90×10^{-16}
$(\gamma_\lambda^2/3\hbar^2/2\mu a)\times 100$ percent	85 percent	0.15 percent

[5] J. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).

TABLE II. J values, parities and energies of the levels of O^{16} as predicted by the alpha-particle model.

J	P	E
0	+	$\omega_1\hbar$
3	−	$6\hbar^2/A$
2	+	$3\hbar^2/A+\omega_2\hbar$
2	−	$3\hbar^2/A+\omega_2\hbar+2\epsilon_0$
1	−	$\hbar^2/A+\omega_3\hbar+9\hbar^2/8A-\epsilon_1+\epsilon_0$
4	+	$10\hbar^2/A$

a small fraction of all the terms. Thus, for the many particle picture,

$$\gamma_\lambda^2 \ll 3\hbar^2/2\mu a.$$

However, it is theoretically possible to combine wave functions of just a few interacting pairs of particles; and with the proper sort of interference between terms the reduced width may still be of the order of $\hbar^2/\mu a$, even though the single particle picture does not apply. Although the single particle model does imply reduced widths of the order of $\hbar^2/\mu a$, the converse is not necessarily true. Thus, the significance of the broad $J=1^-$ level is that it is probably due to single-particle excitation, and the alpha-particle retains its identity in the compound nucleus.

The energies and order of the levels in O^{16} have been calculated by Dennison[6] by means of the alpha-particle model. He considered four alpha-particles arranged in a regular tetrahedron and derived expressions for the normal frequencies (ω_i) and energies of the system. The J values, parities, and energies of the levels predicted are given in Table II. The quantity A is the moment of inertia; ϵ_i denotes the energy state associated with the tunneling process by which the tetrahedron expressed in a right-handed coordinate system passes over into the left-handed system. Dennison estimates that

$$2\epsilon_0/\omega_8\hbar \approx 3\times 10^{-3}, \quad \text{and} \quad \epsilon_1 \approx 25\epsilon_0.$$

The $J=0^+$, $J=3^-$, and $J=2^+$ levels can be identified with the levels at 6.05, 6.13, and 6.9 Mev, respectively. Thus, the values of $\omega_1\hbar$, $\omega_2\hbar$ and $\hbar^2/A$ can be determined. Because of the restriction on ϵ_0, the separation of the $J=2^+$ and $J=2^-$ levels must be small. By assuming that the D levels are nearly degenerate with a 13-kev separation, Inglis[7] identified the $J=1^-$ level predicted by the alpha-particle model with the $J=1^-$ level observed at 7.1 Mev. Since the $J=1^-$ level observed in the present work at 9.58 Mev has such a large reduced width, we might assume instead that it is the predicted level. Then according to Dennison's alpha-particle model the separation of the $J=2^+$ and $J=2^-$ levels should be about 25 kev. A search for a $J=2^-$ level at an excitation energy of about 6.9 Mev might be profitable.

The energy predicted for the level with $J=4^+$ is 10.2 Mev. If this level exists, it might be observed by extending the energy range of the present experiment.

Finally, I wish to thank all those who contributed to the success of this experiment, specifically: Professor R. G. Herb for his constant encouragement and advice throughout the work; Professor H. T. Richards, Professor J. L. Powell, Professor R. G. Sachs, and Mr. A. I. Galonsky for valuable discussion; Mr. F. J. Eppling and Mr. John Cameron with whom the experiment was performed; and Mr. Ajay Divatia, Mr. Robert H. Davis, Mr. Lee Northcliffe, and Mr. Irving Michael for assistance in taking the data.

[6] David M. Dennison, Phys. Rev. **57**, 454 (1940).

[7] D. R. Inglis, Revs. Modern Phys. (to be published).