Elastic Scattering of Alpha-Particles by Oxygen*

JOHN R. CAMERON†
University of Wisconsin, Madison, Wisconsin
(Received January 9, 1953)

The differential cross sections for elastic scattering of alpha-particles by O^{16} were measured in a gas-scattering chamber using alpha-particles accelerated in an electrostatic generator over the energy region from 0.94 to 4.0 Mev. Cross sections were measured at $\theta(\text{cm})=168.0°$; 140.1°; 124.6°; and 90.0°. Because both particles have zero spin the partial wave analysis is exceptionally simple and fixes unambiguously the angular momentum and parity of each level. The resonances were analyzed using the Wigner-Eisenbud one-level approximation to determine the width and resonant energy of each resonance. Five scattering anomalies were observed which correspond to excited states of Ne^{20} with the following energies, angular momenta, and parities: 6.738 Mev, $J=0^+$; 7.182 Mev, $J=3^-$; 7.218 Mev, $J=0^+$; 7.450 Mev, $J=2^+$; and 7.854 Mev, $J=2^+$. The uncertainty in these energies is about 10 kev, most of which is due to the uncertainty of the energy loss in the gas. None of the levels have reduced widths greater than two percent of the single particle width.

I. INTRODUCTION

THE Ne^{20} nucleus was previously examined in the region of excitation energy above 13 Mev by bombarding F^{19} with protons. References to this work are given in a review article by Hornyak *et al.*[1] Below 10.2-Mev levels from the $F^{19}(d,n)Ne^{20}$ reaction were observed by Bonner[2] using a cloud chamber and by Powell[3] with nuclear emulsions. In both experiments deuterons with an energy of about one Mev were used but resolution was only about 200 kev. The indicated level spacing from these two experiments averaged about one Mev compared to an average level separation of about 100 kev in the region above 13 Mev. Little information is available for the region between 10.2 and 13 Mev.

Another method for obtaining information on Ne^{20} levels is from the study of the elastic scattering of alpha-particles by O^{16}. This method was employed in 1940 by Ferguson and Walker[4] using RaC′ alpha-particles with energies between 3.9 and 6.9 Mev scattered through 157°. They observed levels at incident energies of 5.5 and 6.5 Mev. The wide energy spread of the slowed down alpha-particles resulted in poor resolution.

Analysis of experimental scattering cross sections in terms of partial wave theory is particularly simple for alpha-particles incident on O^{16} nuclei. Both particles have ground states of zero spin[5] and very probably even parity. Thus, as explained in a later section, the J value and parity can be assigned to any observed level from a qualitative examination of the measured cross sections at suitably chosen scattering angles.

In the experiments to be discussed an alpha-particle beam sharply defined in energy and covering the energy region up to 4 Mev was provided by an electrostatic generator. The combined mass of an alpha-particle and an O^{16} nucleus is greater than that of a Ne^{20} nucleus by 4.476 Mev. This value must be added to the center of mass energy of the bombarding alpha-particle to obtain the excitation energy of the Ne^{20} nucleus. Thus the excitation region of the Ne^{20} nucleus available for study in this experiment is between about 5.5 and 7.9 Mev.

II. EXPERIMENTAL ARRANGEMENTS

The experiment was performed with the same differentially pumped gas scattering chamber as was used for recent $C^{12}(p,p)C^{12}$ measurements.[6] A schematic diagram (not to scale) of the experimental set-up is shown in Fig. 1. The proportional counters used by Jackson *et al.* were replaced by a pair that were specifically designed for closed operation, that is, with their

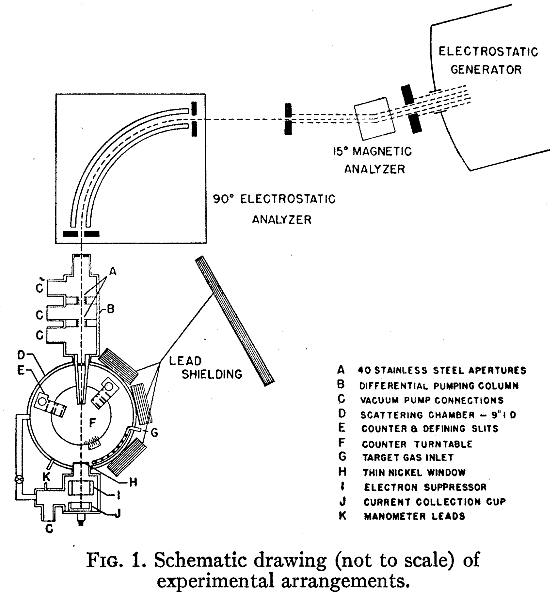

FIG. 1. Schematic drawing (not to scale) of experimental arrangements.

* Supported by the Wisconsin Alumni Research Foundation and the U. S. Atomic Energy Commission.

† U. S. Atomic Energy Commission Predoctoral Fellow. Now at University of Sao Paulo, Sao Paulo, Brazil.

[1] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[2] T. W. Bonner, Proc. Roy. Soc. (London) **174**, 339 (1940).
[3] C. F. Powell, Proc. Roy. Soc. (London) **181**, 344 (1942).
[4] A. J. Ferguson and L. R. Walker, Phys. Rev. **58**, 666 (1940).
[5] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).
[6] Jackson, Galonsky, Eppling, Hill, Goldberg, and Cameron, Phys. Rev. **89**, 365 (1953).

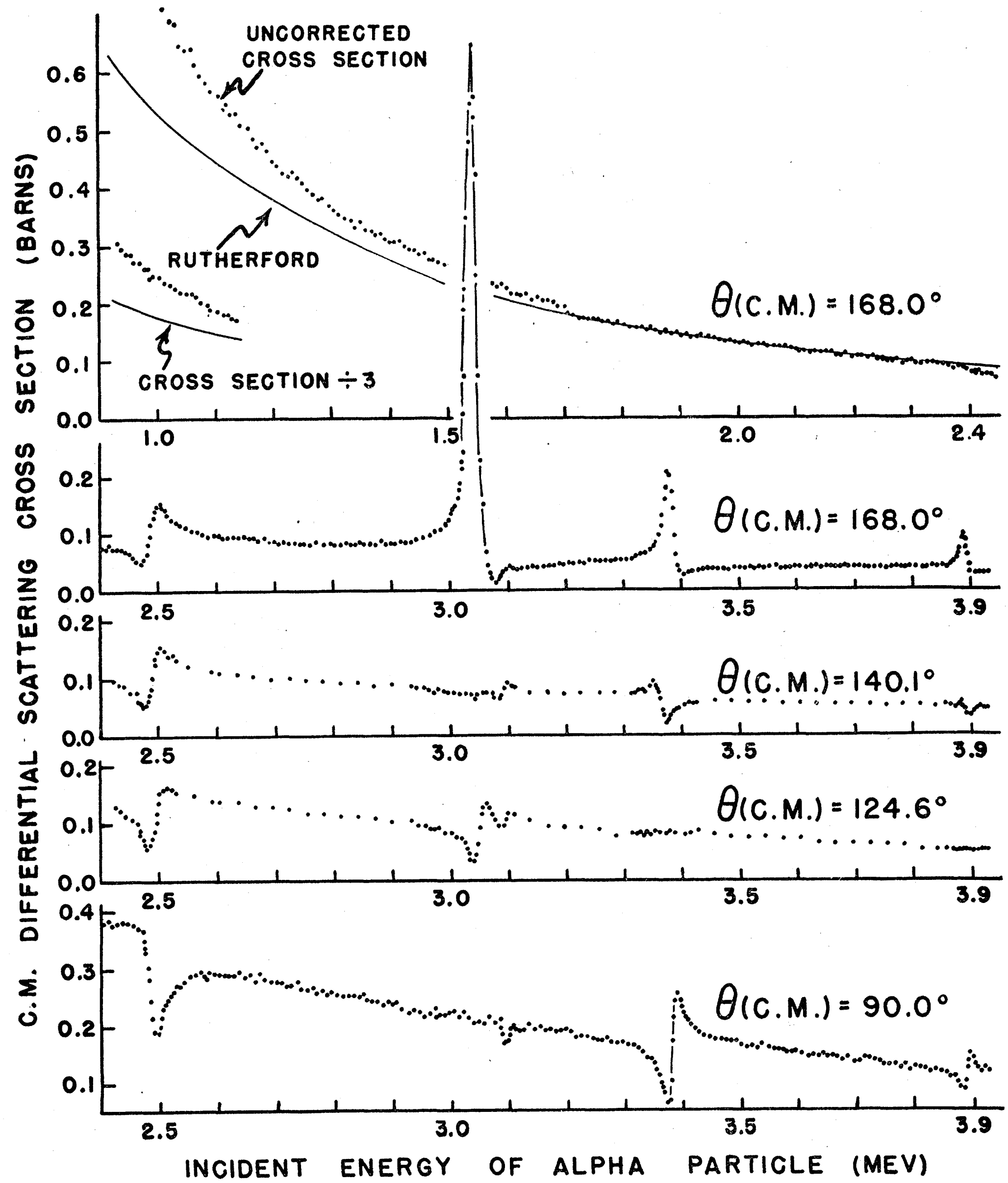

FIG. 2. The $O^{16}(\alpha,\alpha)O^{16}$ experimental cross sections at 168.0° in the region of 0.94 to 2.4 Mev and at 168.0°, 140.1°, 124.6°, and 90.0° in the 2.4- to 4.0-Mev energy range.

own gas supply rather than the target gas in the active volume of the counter.

The target gas supply for this experiment was prepared electrolytically by the Stuart Oxygen Company of San Francisco, California. According to a mass-spectroscopic analysis by the Institute of Gas Technology, Chicago, Illinois, the only important impurities were 0.28 percent H_2O and 0.60 percent H_2. To remove water vapor the gas was passed through a dry ice-acetone cold trap and the pressure contribution of the hydrogen was taken into account in the calculation of the cross sections. Because of the high pressure of the stored gas a ballast tank was maintained at a much lower pressure from which the gas was admitted into the scattering chamber through a long taper needle valve. The storage cylinder had a pressure of about 2000 psig, the ballast tank about 15 psig, and the scattering chamber a pressure of about 5 mm of Hg. This arrangement gave a satisfactorily stable pressure in the scattering chamber.

III. SELECTION OF SCATTERING ANGLES

Considerations leading to the selection of scattering angles were as follows:

(1) Each partial wave includes a Legendre polynomial, $P_l(\cos\theta)$, as a factor, and at 180° all Legendre polynomials have their maximum values.

(2) Since Legendre polynomials of all odd orders are zero at 90° there can be no contribution from partial waves involving $l=1$, 3, 5, etc. at this angle.

(3) $P_2(\cos 125.28°)$ is zero, and therefore the $l=2$ partial wave is zero at 125.28°.

(4) $P_3(\cos 140.77°)$ is zero, and therefore the $l=3$ partial wave is zero at 140.77°.

By taking data at or near the above angles any anomalies that appear in the cross sections due to $l=1$, 2, or 3 partial waves can be identified by their absence at the proper angles. The $l=0$ partial wave is characterized by a uniform intensity at all angles since $P_0(\cos 0)=1$ for all angles. No anomalies which could be associated with l greater than three were found in the present experiment.

The center-of-mass scattering angles used were: 168.0°, 140.1°, 124.6°, and 90.0°. The corresponding laboratory scattering angles are: 164.0°, 128.9°, 111.1°, and 76.0° respectively.

IV. EXPERIMENTAL RESULTS

A. Cross-Section Measurements

Cross sections were measured for alpha-particles with energies between 0.94 and 4.0 Mev. From 0.94 to 2.4 Mev the cross sections decrease monatonically at all angles with no detectable anomalies. Five resonances were found in the 2.4- to 4.0-Mev region. Figure 2 shows the observed cross sections for the 168.0° scattering angles in the lower energy region and for all scattering angles in the upper energy range.

Points were taken at 3-kev intervals over the entire energy range at the 168.0° and 90.0° scattering angles. At the other angles 3-kev steps were taken in the vicinity of each resonance with 30-kev intervals between resonances. Almost 1500 experimental points were taken.

The experimental cross sections were calculated for all energies on the basis of the collected alpha-particles being doubly ionized. Below 2 Mev this assumption is not correct. Since the proportion of He^{++}, He^{+} and neutral atoms is not accurately known between 0.5 Mev and 2 Mev, no attempt has been made to apply corrections. Cross sections in this energy region are probably very close to Rutherford values. This portion of the curve in Fig. 2 is marked "uncorrected cross section."

Data above 1.7 Mev were taken with 0.00002-in. nickel foils on the counter windows. Alpha-particles scattered through 168° had such low energy that some counts were lost due to excessive energy absorption in this foil. It was replaced with 0.00001-in. nickel foil for data taken below 1.7 Mev. The five percent change in cross section at 1.7 Mev at the 168.0° scattering angle is due to improved counting with the thinner foil.

B. Qualitative Assignment of Resonances

From the discussion of the selection of scattering angles and from the assumption of even parity for both nuclei, the following may be concluded:

(1) A $J=0^+$ resonance ($l=0$ partial wave) will appear at all scattering angles and with equal intensity.

(2) A $J=1^-$ resonance ($l=1$ partial wave) will appear at all scattering angles except 90.0°.

(3) A $J=2^+$ resonance ($l=2$ partial wave) will appear at all scattering angles except 124.6°.

(4) A $J=3^-$ resonance ($l=3$ partial wave) will appear at all scattering angles except at 90.0° and 140.1°. From observation of Fig. 3 it is seen that the anomaly at 2.5 Mev is a $J=0^+$ resonance and the 3.045 Mev anomaly is $J=3^-$ resonance. There is another $J=0^+$ resonance at 3.09 Mev and the two resonances at 3.38 Mev and 3.885 Mev are both $J=2^+$ resonances.

V. ANALYSIS OF THE DATA

Angular momentum values and parities of all the observed resonances were determined in the previous section. The other physically interesting parameters of a resonance are its resonant energy and width. Values of these quantities were determined by means of a partial wave analysis to be described. The terminology follows that of previous analyses of this type.[7]

The quantum-mechanical expression for elastic scattering in terms of partial waves for spinless particles on spinless nuclei is given by:

$$\frac{d\sigma}{d\omega}=\frac{1}{k^2}\left|-\frac{1}{2}\eta\csc^2\frac{\theta}{2}\exp\left(i\eta\ln\csc^2\frac{\theta}{2}\right)+\sum(2l+1)P_l(\cos\theta)\sin\delta_l\exp(i\delta_l)\exp(i\alpha_l)\right|^2,$$

where $d\sigma/d\omega$ is the differential scattering cross section, $k=1/\lambda\!\!\!^{-}=\mu v/\hbar$, where μ is the reduced mass of the system, v is the relative velocity, $\eta=ZZ'/\hbar v$ where Z and Z' are the charges of the colliding particles, θ is the scattering angle, $P_l(\cos\theta)$ is the lth Legendre polynomial, δ_l is the phase shift of the partial wave with an orbital angular momentum $l\hbar$,

$$\exp(i\alpha_l)=\prod_{s=1}^{l}\left(\frac{s+i\eta}{s-i\eta}\right)=\exp\left(2\sum_{s=1}^{l}\tan^{-1}\left(\frac{\eta}{s}\right)\right)\text{ for } l>0,$$

and $\exp(i\alpha_0)=1$. All quantities in the above relations are in the center-of-mass system.

Before the analysis can proceed an expression is needed for the δ_l, the only unknowns in the cross-section relation. The presentation up to this point has involved only the theory of partial waves and is not dependent on any other assumptions. The expression to be used for δ_l involves two assumptions: that elastic scattering

[7] R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. **84**, 18 (1951); H. L. Jackson and A. I. Galonsky, Phys. Rev. **84**, 401 (1951); L. J. Koester, Jr., Phys. Rev. **85**, 643 (1952); Kaufmann, Goldberg, Koester, and Mooring, Phys. Rev. **88**, 673 (1952).

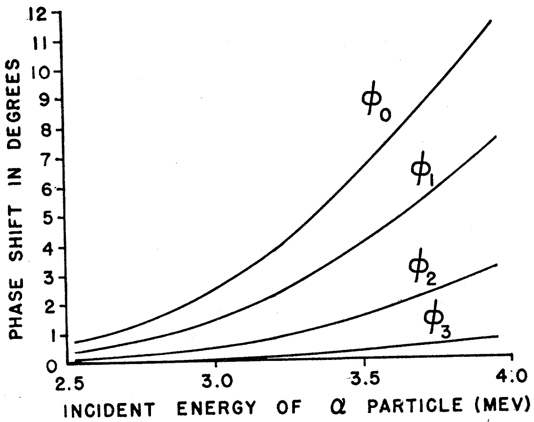

FIG. 3. Potential phase shifts ϕ_l as a function of alpha-particle energy for the $O^{16}(\alpha,\alpha)O^{16}$ reaction using an interaction radius of $a=5.75\times10^{-13}$ cm.

is by far the most probable process taking place, and that any one anomaloy appreciably involves only one level caused by a given partial wave. The first assumption is reasonably well fulfilled since the only other process energetically possible is the capture process which is usually negligible compared to elastic scattering at these energies. The second assumption is well satisfied since the two D resonances and the two S resonances are each separated by 0.5 Mev, which is large compared to their widths. Under these assumptions δ_l can be expressed as:[8]

$$\delta_l=-\tan^{-1}(F_l/G_l)\Big|_{r=a}+\tan^{-1}\frac{\gamma_\lambda^2kA_l^{-2}}{E_\lambda+\Delta_\lambda-E}\Big|_{r=a}.$$

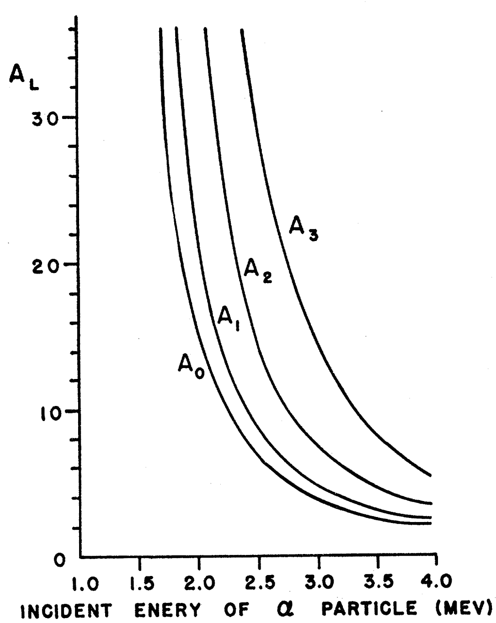

FIG. 4. Values of A as a function of alpha-particle energy using an an interaction radius of $a=5.75\times10^{-13}$ cm.

[8] R. K. Adair, Phys. Rev. 86, 155 (1952).

The first term is usually called the potential scattering phase shift or "hard sphere" phase shift, and the second term is called the resonant phase shift. The potential phase shift depends on the quantities F_l and G_l which are the regular and irregular radial Coulomb wave functions, respectively.[9] These are functions of η and ρ, where $\rho=kr$ and must be evaluated for a value of $r=a$, the interaction radius. The resonant phase shift is a function of the incident energy of the alpha-particle E, the interaction radius a, and two parameters of the level, E_λ and γ_λ^2. The characteristic energy of the level E_λ is closely related to the resonant energy E_r. The reduced width of the level γ_λ^2 is related to the experimental width Γ_λ by the relation:[10] $\gamma_\lambda^2=\Gamma_\lambda A_l^2/2k$, where $A_l^2=F_l^2+G_l^2=$ the reciprocal of the barrier penetrability. The term Δ_λ is defined by

$$\Delta_\lambda=-\frac{\gamma_\lambda^2}{a}\left(\frac{d\ln A_l}{d\ln\rho}+l\right)_{r=a}.$$

The resonant energy E_r of a level is conveniently defined as the energy for which $E_\lambda+\Delta_\lambda-E=0$. The level shift Δ_λ is a slow function of energy and over the range of a narrow resonance can be considered constant, permitting $E_\lambda+\Delta_\lambda$ to be replaced by E_r. Similarly γ_λ^2k/A_l^2 can be considered constant over a narrow resonance and can be replaced by $\Gamma_\lambda/2$, its value at the resonance energy. The approximate expression for the phase shift for a narrow resonance can now be written as:

$$\delta_l=-\phi_l+\tan^{-1}\frac{\Gamma_\lambda}{2(E_r-E)},$$

where $\phi_l=\tan^{-1}(F_l/G_l)|_{r=a}$ is the potential phase shift.

In order to obtain best agreement with the experimental cross sections there is an optimum value for a because its choice determines the magnitude of the potential phase shifts which enter into the expression for the cross section. The value of a chosen for this analysis is $1.40\times10^{-13}(\sqrt[3]{16}+\sqrt[3]{4})$ cm$=5.75\times10^{-13}$ cm.

Because of the assumptions involved in the expression for δ_l the value of a that gives the best fit should be interpreted simply as a parameter of the reaction that is generally of the magnitude of the sum of the nuclear radii of the two interacting particles.[11]

The analysis is quite insensitive to the choice of a at low bombarding energies where the ϕ_l values are all small. When the ϕ_l values become appreciable the value of a has a marked effect. Thus at the lowest resonance a value of a 12 percent larger than the one

[9] The regular Coulomb functions were computed with the aid of *Tables of Coulomb Wave Functions*, Vol. I, National Bureau of Standards, U. S. Government Printing Office. The irregular functions were obtained from tabulated functions furnished by M. Abramowitz, National Bureau of Standards (private communication). See also Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. 23, 147 (1951).

[10] E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947).

[11] T. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).

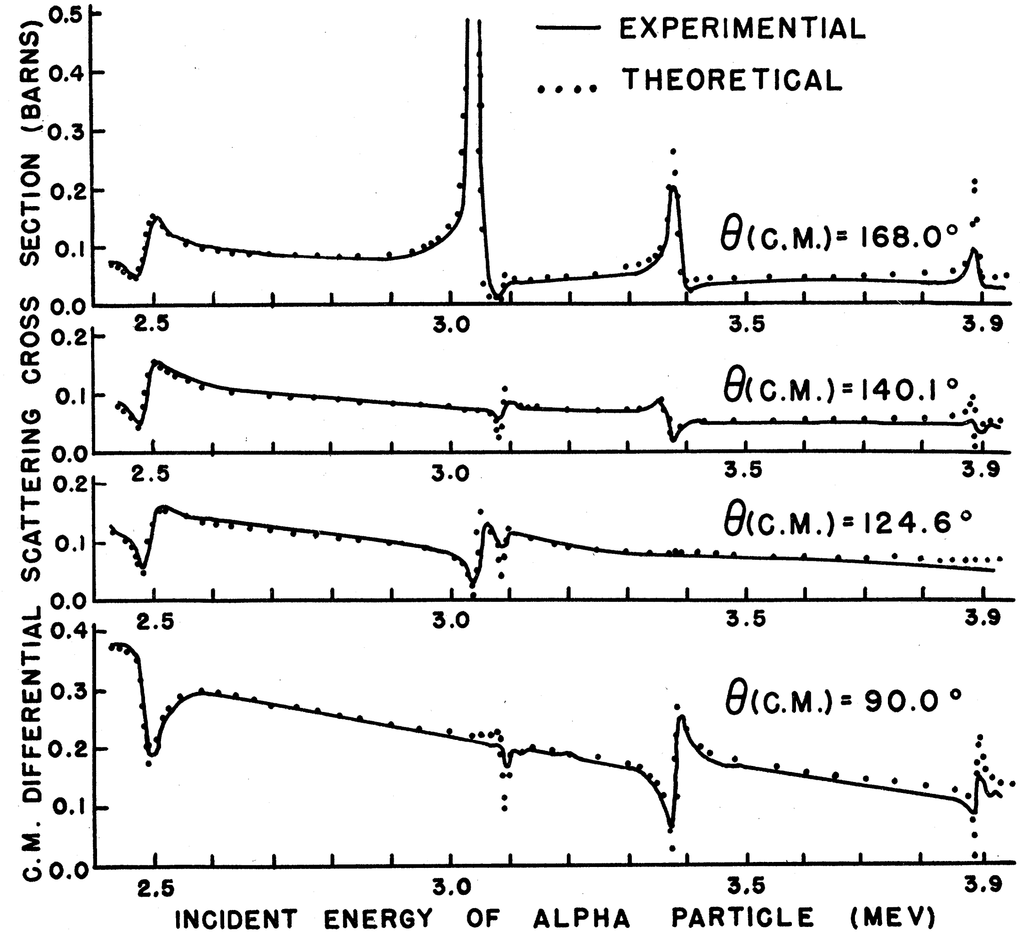

FIG. 5. Comparison of the experimental and calculated $O^{16}(\alpha,\alpha)O^{16}$ cross sections from 2.4 to 4.0 Mev. The dots were calculated using parameters given in Table I.

chosen gave a very slight improvement but at the higher resonances it gave a much poorer fit. Once a value of a has been chosen the functions ϕ_l and A_l can be tabulated and curves made of each *versus* energy. Graphs of these two sets of functions for this reaction using an interaction radius of 5.75×10^{-13} cm are shown in Figs. 3 and 4.

To obtain the parameters Γ_λ and E_r of a resonance it was assumed that the potential scattering phase shifts ϕ_l were correct and from a graphical analysis similar to that used by Jackson and Galonsky,[12] the phase shifts δ_l of the resonant wave were extracted. Then values of Γ_λ and E_r were chosen that gave best agreement with these phase shifts.

The parameters determined for all the resonances are summarized in Table I. Values of E_r and Γ_λ are given in the laboratory system of units; all other quantities are given in the center-of-mass system.

From these parameters the predicted cross sections were obtained by a graphical method and compared with the measured cross sections. This comparison is made in Fig. 5 for all four scattering angles in the energy region 2.4 to 4.0 Mev. The agreement is satisfactory except for increasing deviation in the off-resonance cross sections at energies above 3.5 Mev. This deviation may be due to other resonances above 4.0 Mev which affect the cross sections in this region. Decreasing the interaction radius above 3.5 Mev or assuming a single broad $J=0$ resonance above 4.0 Mev both have the effect of improving the fit at all angles except 90.0°.

TABLE I. Summary of the level parameters.

Level assignment	$J=0^+$	$J=3^-$	$J=0^+$	$J=2^+$	$J=2^+$
E_r (Mev) lab	2.490	3.045	3.090	3.380	3.885
Γ_λ (Mev) lab	0.024	0.010	0.005	0.010	0.003
Excitation energy of Ne^{20} (Mev)	6.738	7.182	7.218	7.450	7.854
γ_λ^2 (Mev-cm) cm$\times10^{13}$	0.75	1.2	0.040	0.16	0.021
Percent single particle width ($\hbar^2/\mu a$)	0.8	1.4	0.044	0.18	0.024

[12] H. L. Jackson and A. I. Galonsky, Phys. Rev. 89, 370 (1953).

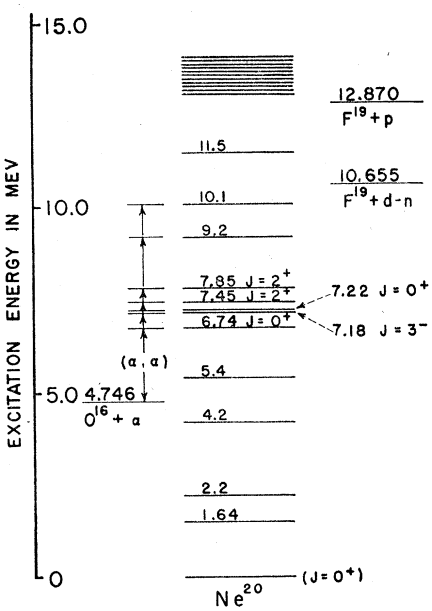

FIG. 6. Energy level diagrams of Ne^{20}.

VI. DISCUSSION OF RESULTS

The primary result of this experiment is the classification of five levels in the Ne^{20} nucleus. The analysis determined the widths and resonant energies of these levels. Other levels in the surveyed region may have been missed because they had even angular momentum and odd parity or vice versa. These levels cannot be formed by this reaction. Levels with isobaric spin $T=1$ would also be forbidden in the resonance scattering of alpha-particles ($T=0$) by $O^{16}(T=0)$. However, the first $T=1$ state of Ne^{20} would correspond to the ground state of F^{20}. The predicted position of the homologous level in Ne^{20} is at 10.5 Mev and hence is beyond the region investigated in the present experiment. An S resonance with an experimental width less than one kev or resonances of higher angular momenta with experimental widths less than 0.5 kev would probably have been missed due to insufficient resolution. Unfortunately at the 168.0° scattering angle where resonances are the most pronounced the target thickness has its greatest value. Thus the resolution is the poorest at this angle.

The failure to detect any resonances below 2.5 Mev may be due to insufficient resolution. The probability of forming a compound nucleus is roughly $1/A_l^2$, which is very small when the available energy is well below the Coulomb barrier (see Fig. 4). At the lowest bombarding energy, which was 0.94 Mev, A_0^2 has a value greater than 100 000 compared to a value of 44 at 2.5 Mev and a value of 13.5 at 3.09 Mev. To detect an S resonance at 0.94-Mev energy would require a reduced width approximately 1000 times the maximum permitted by the Wigner[11] sum rule.

None of the observed resonances have more than two percent of the width of a single-particle level, which is given by $\hbar^2/\mu a$.

Levels observed in this experiment are at 6.74-, 7.18-, 7.22-, 7.45-, and 7.85-Mev excitation energy (Fig. 6). The uncertainty in these energies is about 10 kev, most of which is due to inaccuracies in the determination of the energy loss of the alpha-particle in the gas.

The previously known levels in the region were at 7.1 and 7.8 Mev. Thus the true level density above the observed levels may be much greater than as indicated in the diagram. Additional information can be obtained on the energy level density in this region from the $F^{19}(d,n)Ne^{20}$ reaction. Work on this reaction was started at this laboratory using the photographic emulsion technique, but owing to the large number of levels involved, it is still incomplete.

The author wishes to thank Professor R. G. Herb for his guidance throughout the work; Professor H. T. Richards, Dr. R. K. Adair, Professor J. L. Powell, and Mr. A. I. Galonsky for valuable discussion; Messrs. Richard W. Hill and F. J. Eppling with whom the experiment were performed; Messrs. Ajay Divatia, Robert H. Davis, Lee Northcliffe, and Irving Michael for assistance in taking the data; and Miss Helen MacDuffee for computational assistance.