Effects of Departures from the Single Particle Model on Nuclear Magnetic Moments*

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Departures from the single particle model (SPM) are considered, in which the SPM wave function is mixed with others representing states for which the core no longer has zero angular momentum. It is found that under certain restrictions an experimentally established relation exists between the deviations of odd-N nuclei from the Schmidt lines and those of odd-Z nuclei.

INTRODUCTION

IT was observed long ago that magnetic moments of odd-A nuclei, when divided into two groups according to whether Z or N is odd, show a striking regularity. Schmidt[1] has pointed out that this regularity can be approximately predicted on the basis of a model which assumes that for odd-A nuclei the magnetic moment is due to one single nucleon only, the rest of the nucleus being so coupled as to give a vanishing total magnetic moment.

Recent developments in the theory of nuclear structure have given further support to the original idea of Schmidt. The big pairing energy[2] which tends to couple pairs of equivalent nucleons to a resultant angular momentum zero lends strong support to the assumption that the ground state is formed by coupling the odd nucleon to the spherically symmetric core. The regularities found among the nuclear spins of the ground states also support this interpretation.

The Schmidt lines on which the magnetic moments should lie when plotted against the nuclear spins represent, however, only an approximation to the experimental data. As is well known, nearly all magnetic moments deviate from them, in some cases quite considerably. These deviations, however, show some regularities which suggest that they should be considered more closely. Most important is the fact that, with very few exceptions among the lightest nuclei, all magnetic moments lie between the Schmidt lines. Thus any attempt to explain the deviation should at least agree with this qualitative observation.

Further, on the above Schmidt diagram "forbidden zones" are observed, both for odd-Z and odd-N nuclei, lying more or less midway between the two Schmidt lines and separating the two groups of magnetic moments which correspond to $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$. It should be noted in this connection that the apparent appearance of this forbidden region in the case of odd-N nuclei may very well be due to experimental difficulties. In fact, in this case only small magnetic moments are covered by the forbidden zone (see Fig. 2), and as an odd-N nucleus is usually a comparatively low abundant isotope among many abundant even-even isotopes of the same element, its magnetic moment, if small, can easily escape observation.

Other regularities concern the relative deviation of related nuclei. Thus in the case of nuclei differing by two neutrons or two protons and having the same total and orbital angular momenta—the heavier one almost always lies nearer to the Schmidt line.[3,4] Also the deviations of odd-Z nuclei are bigger than those of odd-N' nuclei if $Z=N'$.

Many attempts have been made to explain the deviations of magnetic moments from the Schmidt lines.[4] The present note is concerned with the extent to which simple departures from the single particle model may explain the observed regularities in the deviations.

THE CORE EFFECTS

Let us assume that there are n nucleons involved in the formation of the ground state under consideration and that each of them has an angular momentum $\mathbf{j}_k$ (we confine ourselves to jj coupling), and magnetic dipole $\boldsymbol{\mu}_k=g_k\mathbf{j}_k$. The total magnetic dipole will then be

$$\boldsymbol{\mu}=\sum \boldsymbol{\mu}_k=\sum g_k\mathbf{j}_k.$$

It is easily verified that $\boldsymbol{\mu}$ satisfies the following commutation relations with $\mathbf{J}=\sum \mathbf{j}_k$:

$$[\mu_x, J_x]=0,\quad [\mu_x, J_y]=i\hbar\mu_z,\quad [\mu_x, J_z]=-i\hbar\mu_y,\quad \text{etc.,}$$

and therefore the average value of $\boldsymbol{\mu}$ in the state $(j_1, j_2,\cdots, j_n, J, M)$ characterized by the eigenvalues of $j_1^2, j_2^2,\cdots, j_n^2, J^2$ and J_z is

$$\langle\boldsymbol{\mu}\rangle=\langle\mathbf{J}\rangle\frac{1}{J(J+1)}\sum g_k\langle(\mathbf{j}_k\cdot\mathbf{J})\rangle,$$

or, in terms of the total g factor,

$$g=\frac{1}{J(J+1)}\sum g_k\ \langle(\mathbf{j}_k\cdot\mathbf{J})\rangle. \tag{1}$$

This is the well-known Landé formula, which in the

* This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
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TABLE I. A comparison of deviations of similar odd-Z and odd-N nuclei from the Schmidt limits.

Odd-Z nucleus	μ	Δg_P	Odd nucleon state	Odd-N nucleus	μ	Δg_N	$\Delta g_P:\Delta g_N$
$_5B_6^{11}$	2.6886	1.104	$p_{3/2}$	$_4Be_5^9$	−1.1776	0.7359	1.50
$_{13}Al_{14}^{27}$	3.639	1.153	$d_{5/2}$	$_{12}Mg_{13}^{25}$	−0.96	0.95	1.21
$_{17}Cl_{18}^{35}$	0.8210	0.697	$d_{3/2}$	$_{16}S_{17}^{33}$	0.64292	0.5052	1.38
$_{17}Cl_{20}^{37}$	0.683	0.559	$d_{3/2}$	$_{16}S_{17}^{33}$	0.64292	0.5052	1.11
$_{37}Rb_{48}^{85}$	1.349	0.487	$f_{5/2}$	$_{30}Zn_{37}^{67}$	0.90	0.46	1.05
$_{49}In_{64}^{113}$	5.460	1.333	$g_{9/2}$	$_{38}Sr_{49}^{87}$	−1.10	0.81	1.64
$_{49}In_{66}^{115}$	5.475	1.318	$g_{9/2}$	$_{38}Sr_{49}^{87}$	−1.10	0.81	1.62
$_{51}Sb_{70}^{121}$	3.70	1.09	$d_{5/2}$	$_{40}Zr_{51}^{91}$	−1.10	0.81	1.35

case $n=2$ takes the form

$$g=\tfrac{1}{2}(g_1+g_2)+(g_1-g_2)\frac{j_1(j_1+1)-j_2(j_2+1)}{2J(J+1)}. \quad (1a)$$

One also sees from this formula that if the g factors of all the nucleons involved are equal to each other, they are also equal to the g factor of the combined angular momentum. In particular, the g factor of any state of equivalent nucleons is equal to that of the correspond-single particle state.

The evaluation of the scalar products $(\mathbf{j}_k\cdot\mathbf{J})$ in (1) for $n>2$ is generally impossible without further assumptions on the way in which the angular momenta are coupled. In a final form of the theory such "assumptions", or "coupling schemes" as they are called, should result from the type and form of the forces between the nucleons, but with the present state of the theory of nuclear forces this can hardly be expected to yield reasonable results.

The experimental data of nuclear spectroscopy seem to indicate that one should first couple among themselves the angular momenta of the protons and the neutrons separately, and the two resulting angular momenta should then be coupled to give the total nuclear spin. This suggests that in introducing departures from the SPM, we do so first for the odd group of particles in the nucleus, leaving the even group in its state of spherical symmetry. Thus the g_k's in Eq. (1) will all refer to nucleons of the same charge character. Now the g factor of a single nucleon can be written in the form,

$$g_k=g_l+(g_s-g_l)\epsilon_k/(2_kl+1), \ (\epsilon_k=\pm 1 \text{ for } j_k=l_k\pm\tfrac{1}{2}), \quad (2)$$

where g_l and g_s are the orbital and spin g factors, respectively, and depend only on the charge of the nucleon. (For protons $g_l=1$, $g_s\approx 5.6$; for neutrons $g_l=0$, $g_s\approx -3.8$.) Introducing (2) into (1) we now obtain:

$$g=g_l+(g_s-g_l)\frac{1}{J(J+1)}\sum\frac{\epsilon_k}{2l_k+1}\langle(\mathbf{j}_k\cdot\mathbf{J})\rangle. \quad (3)$$

As the Schmidt value of the g factor, g_0, is given by a formula similar to (2), we see that the deviation from the Schmidt line for the state considered will be

$$\Delta g=g-g_0=(g_s-g_l)\left\{\frac{1}{J(J+1)}\sum\frac{\epsilon_k}{2l_k+1}\times\langle(\mathbf{j}_k\cdot\mathbf{J})\rangle-\frac{\epsilon_0}{2l_0+1}\right\}; \quad (4)$$

($j_0=J$ is the angular momentum of the odd nucleon).

In Eq. (4) we have separated the two factors which influence the magnetic moment of a certain combination of angular momenta, namely, the electromagnetic properties of the nucleons involved and the way in which the configuration is constructed from its nucleons' angular momenta. We might expect that the latter will be the same whether the odd group consists of protons

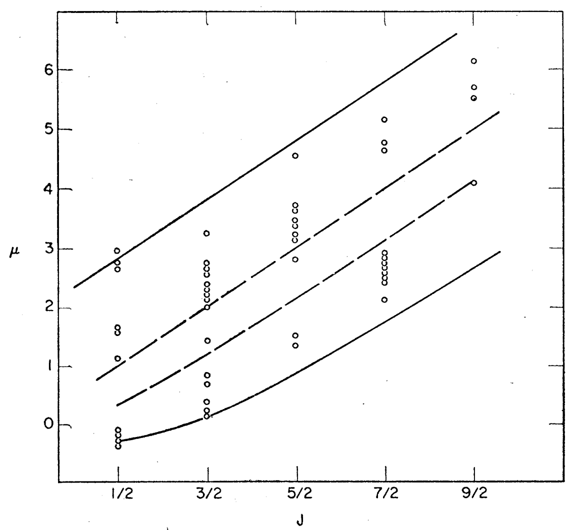

FIG. 1. Magnetic moments (in nuclear magnetons) of odd-Z nuclei plotted against nuclear spin. Full lines are the Schmidt limits; broken lines are limits of the forbidden-zone (taken to be Dirac lines in this case).

or of neutrons (provided, of course, their number and angular momenta are the same). The electromagnetic factor (g_s-g_l) is, however, different. We therefore get for the ratio of the deviations of such similar nuclei from the Schmidt lines the value,

$$|\Delta g_P|/|\Delta g_N|=|g_s{}^P-g_l{}^P|/|g_s{}^N-g_l{}^N|\approx 1.20.‡$$

Thus the bigger deviation of odd-Z nuclei from the Schmidt lines is correctly predicted, as can be seen from Table I.

Another way to check the validity of this approximation is the following: As was mentioned in the introduction, forbidden zones are observed both for odd-Z

‡ *Note added in proof:*—It was pointed out to me by Professor E. Feenberg that this result remains valid for any coupling scheme provided one assumes, as is done here, that the contributions to the angular momentum come only from nucleons of the same charge as that of the odd one.

nuclei as well as for odd-N nuclei; if the above approximation is a good one, the widths of the allowed zones should be in the ratio of 1.20 to one. Figure 1 shows the Schmidt diagram for odd-Z nuclei; the limits of the forbidden zones are taken to be the Dirac lines.[5] Figure 2 shows the Schmidt diagram for odd-N nuclei; the limits of the forbidden zone in this case were obtained from those of the proton case by the above procedure. The agreement is strikingly good.

Equation (4) as it stands does not determine the sign of the deviation. To arrive at this we should make further assumptions on the way in which the real wave function of the ground state is different from that of the SPM. A suggestion may be based on the following consideration.

If the single particle approximation is valid, the bare core of an odd nucleus A should resemble very much the even nucleus A-1. We know that the ground state of an even nucleus has angular momentum zero, positive parity, and that the first excited state nearly always has angular momentum 2, positive parity, indicating that it is very probably an excited state of a configuration of an even number of equivalent nucleons from the core. It is thus natural to assume that the single-particle wave function of the ground state is mixed with the wave function of that state in which the odd nucleon combines with the first excited state of the core. The fact that both the ground state as well as the first excited state of the core have the same parity makes such a mixture possible.

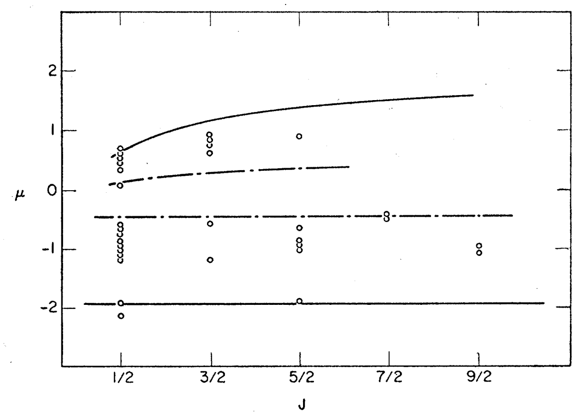

FIG. 2. Magnetic moments of odd-N nuclei plotted against nuclear spin. Full lines are the Schmidt limits; broken lines are limits of the forbidden zone deduced from the corresponding limits in Fig. 1.

Using the Landé formula (1a) for evaluating the g factor of the state in which the core has angular momentum j_c, and the total angular momentum J is equal to that of the odd particle j_o we obtain

$$g=g_o-(g_o-g_c)\frac{j_c(j_c+1)}{j_o(j_o+1)}. \quad (5)$$

[5] F. Bloch, Phys. Rev. **83**, 839 (1951); A. de-Shalit, Helv. Phys. Acta **24**, 296 (1951).

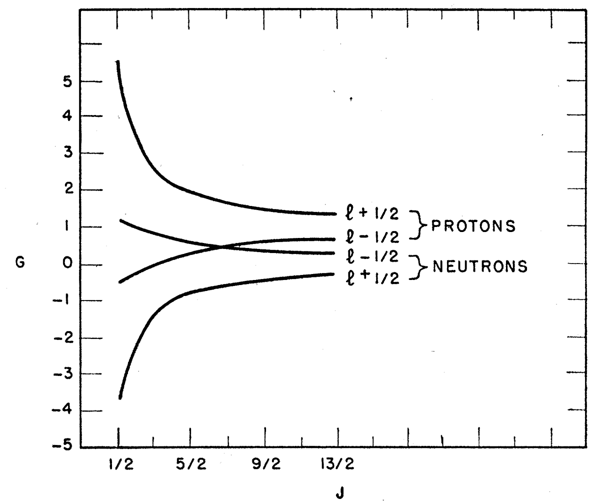

FIG. 3. g factors of single nucleons as a function of total angular momentum.

As the excited state of the core is assumed to be formed by equivalent nucleons, g_c should be set equal to the g factor of one of these nucleons. Figure 3 shows the dependence of the g factor of a single nucleon on its total angular momentum. An inspection of this graph immediately shows that Eq. (5) will give the observed direction of deviation from the Schmidt lines provided the many-particles configurations are formed by combining an $l\pm\frac{1}{2}$ odd nucleon with excited states of either $l'\mp\frac{1}{2}$ pairs or with excited $l''\pm\frac{1}{2}$ pairs where $l''>l$. (By $l\pm\frac{1}{2}$ particle we mean a particle in a state of total angular momentum j and parity $(-1)^{j\mp\frac{1}{2}}$, etc.)

It is evident that if one wants to explain the deviations of magnetic moments from the Schmidt lines by means of mixing single-particle states with many-particle states, some sort of a selection rule should be invented to take account of the fact that the experimental points always lie between the Schmidt lines, and the above restriction represents one possibility. It is interesting to note, however, that this rule seems to be quite reasonable when we inspect the detailed structure of the shells as proposed by Mayer[2] and by Jensen.[6] In fact one may expect that the main contribution to the deviation would come from the "last" pair of nucleons added to the core. Very often this last pair is formed in a state of high angular momentum ($g_{9/2}$, $h_{11/2}$, $i_{13/2}$, etc.) making the situation in favor of the second alternative of the above rule. One also finds that in most of the cases $l\pm\frac{1}{2}$ states have $l'\mp\frac{1}{2}$ states as their neighbors, in which case the first alternative of the rule applies.

DISCUSSION

We have completely ignored the contribution of exchange currents to the observed magnetic moments. This is certainly unjustified.[4] It is, however, still

[6] Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949).

questionable how important their contribution is. On the one hand, the fact that the proton forbidden zone has as its limits the Dirac lines strongly supports the use of a quenching mechanism to explain the deviations from the Schmidt lines,[7] but the occurrence of a forbidden zone for the neutrons raises the question whether this coincidence is not accidental.

It was once suggested that the direction of deviation of the magnetic moments be explained by a mixture of an $l\pm\frac{1}{2}$ state with an $l'\mp\frac{1}{2}$ one. This interpretation has been rejected because of the difference in parity of these two states. The present note somewhat revives these ideas, the parity difficulty being overcome by involving pairs in the admixture. In this respect it is similar to the somewhat less explicit work of Davidson.[8] Although some arguments were given in favor of our rule for the formation of the many-particle configuration stated above, this rule should still be considered as somewhat arbitrary.

The case of nuclei with nuclear spin $\frac{1}{2}$ deserves special mention. Only such a state of the core can affect its magnetic moment which has a total angular momentum 1. States of that angular momentum are rather rare among the known spectra of even-even nuclei and probably need a comparatively higher energy to be excited. There is, however, no indication of a better agreement with the Schmidt limits for spin $\frac{1}{2}$ nuclei. The question is thus left open.

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A Search for Penetrating Showers from Hydrogen at Sea Level Using a Cloud Chamber*

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The extent to which multiple production of π-mesons takes place in local sea-level penetrating showers was studied with a counter controlled cloud chamber in a magnetic field. The rates at which penetrating showers occur under carbon, aluminum, and lead were determined and a search was made for them under liquid hydrogen. In a total operating time of 626 hours with an average thickness of 2.28 g/cm^2 of liquid hydrogen above the chamber, no penetrating showers were found which could have originated in the hydrogen. On the basis of the rates at which such showers occur in heavier materials and the number of them formed in the material of the Dewar while operating with hydrogen, one would have expected to detect a minimum of 6 showers from the hydrogen if the cross section for the production of penetrating showers were the geometric area of the nucleus (taken as 6×10^{-26} cm^2 for hydrogen). It can then be concluded that the majority of sea-level local penetrating showers detected below heavy materials by an apparatus of this kind can be attributed mainly to plural production. From momentum measurements in the magnetic field, the minimum value which can be assigned to the momentum of the incident nucleons which causes the average penetrating shower detected with this apparatus was estimated at 6 Bev/c. It follows that the multiple production of charged mesons in a single nucleon-proton collision at about 6 Bev probably does not occur in more than 15 percent of the cases.

The ratios of the rates at which penetrating showers were detected under C, Al, and Pb were proportional to the geometric area of the nuclei within statistical limits.

An event found in the hydrogen which is very similar in appearance to the μ-meson interaction first observed by Braddick and Hensby is discussed. A photograph of a nuclear collision in lead is described in which very little energy is transferred to the lead nucleus although the incident particle has a momentum estimated to be 40 Bev/c.

I. INTRODUCTION

ONE of the more direct ways to study high energy nuclear interactions of fundamental particles is by means of cloud-chamber observations on local penetrating showers. There have been many cloud-chamber studies of these events at various altitudes and issuing from a wide variety of materials.[1–11] At sea level, practically all such showers are believed to be caused by the collision of very high energy nucleons with atomic nuclei.[12] The resulting penetrating showers have a complex character in general which Janossy suggested is due to successive collisions in the same nucleus (so-called plural production of mesons).[13] A large positive excess among the penetrating particles in local sea-level penetrating showers has been established[8,14] which is interpreted to indicate the presence

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