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Mobility of Holes and Electrons in High Electric Fields

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The field dependence of mobility has been determined for electrons and holes in both germanium and silicon. The observed critical field at 298°K beyond which μ varies as $E^{-\frac{1}{2}}$ is 900 volts/cm for n-type germanium, 1400 volts/cm for p-type germanium, 2500 volts/cm for n-type silicon, and 7500 volts/cm for p-type silicon. These values of critical field are between two to four times those calculated on the basis of spherical constant energy surfaces in the Brillouin zone. A saturation drift velocity of $6(10)^6$ cm/sec is observed in germanium which is in good agreement with predictions based on scattering by the optical modes. Data on n-type germanium at 20°K show a range over which impurity scattering decreases and the mobility increases with field until lattice scattering dominates as at the higher temperatures.

I. INTRODUCTION

IN high electric fields the mobility of electrons (or holes) in semiconductors becomes field dependent. The drift velocity v_d and the current density I should then be expressed as

$$v_d = \mu(T, \sigma)E, \tag{1}$$

and

$$I = nev_d = ne\mu(T, E)E = \sigma(T, E)E. \tag{2}$$

This dependence of mobility or conductivity on field for reasonably pure n-type germanium has been reported previously,[1] and a theoretical interpretation has been presented by Shockley.[2] Similar conductivity measurements have since been made on p-type germanium and on both n-type and p-type silicon. These data are being reported on at this time along with additional data on n-type germanium at a temperature sufficiently low so that impurity scattering rather than lattice scattering is dominant at low fields.

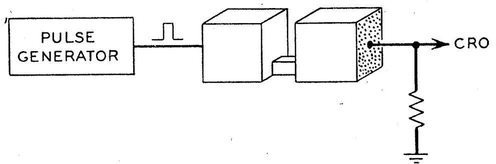

FIG. 1. Schematic diagram of circuit used to measure conductivity.

II. GENERAL CONSIDERATIONS

Several experimental problems confront one when measurements are made at the high electric fields required to observe changes in σ or in the equivalent quantity μ. It is required that high rates of energy be given to the carriers. However, there must be no appreciable heating of the specimen, and there must be no change in the number of carriers. Heating of the specimen will change μ through its dependence on T, and may change n under some circumstances. Injection of minority carriers can also increase the number of carriers. Heating and injection have been minimized (1) by the choice of a semiconductor whose carriers normally have a high mobility, (2) by the choice of one having relatively few carriers, (3) by the use of electrical pulses of short duration, and (4) by special fabrication of the samples.

The rate at which energy is transferred to an electron is μeE^2. When the mobility of carriers is high, it thus becomes possible for these carriers to receive energy at correspondingly high rates. Single-crystal germanium of moderately high resistivity (2–10 ohm-cm) is particularly well suited for high field studies. Haynes has determined the low field mobility of electrons and holes in germanium at 300°K to be 3600 and 1700 cm²/volt sec, respectively.[3] At this temperature a nonlinear drift velocity range is reached with fields of the order of 10^3 volts per cm and has been studied up to over 10^4 volts per cm. These fields are considerably below that predicted by Zener[4,5] at which there is a field-induced generation of electron-hole pairs and a very rapid increase in the number of carriers with field. This Zener field, of the order of 10^5 volts per cm, dictates the upper limit of field in these studies.

The experimental difficulties arising from heating of the crystal have been overcome by using samples of relatively high resistivity and by operating with pulsed electric fields. Germanium of 10 ohm-cm resistivity has roughly one conducting electron per 10^8 atoms. When these electrons become "heated" by virtue of the energy imparted by high but short pulsed fields, the lattice temperature is practically unaffected. Pulse durations of 1 microsecond at moderate fields and 0.1 microsecond at the higher fields prove sufficiently short. The maximum energy delivered to such a sample during a single pulse raises its temperature 1°K at most. However, since the pulses occur repetitively, it is important that the repetition rate be sufficiently low to avoid any accumulative heating of the crystal. The thermal time constant of the samples was less than 10^{-2} sec; a repetition rate of the order of 100 pulses per second

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[1] E. J. Ryder and W. Shockley, Phys. Rev. **81**, 139 (1951).
[2] W. Shockley, Bell System Tech. J., **30**, 990 (1951).
[3] J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835 (1951).
[4] C. Zener, Proc. Roy. Soc. (London) **145**, 523 (1934).
[5] McAfee, Ryder, Shockley, and Sparks, Phys. Rev. **83**, 650 (1951).

was then permissible at the highest fields. This was confirmed experimentally by varying the repetition rate from 50 to 200 pulses per sec. The change of conductivity over this pulse range was negligible.

The second experimental difficulty is associated with changes in the number of carriers. In *n*-type material holes can be injected from the positive electrode. Modulation caused by this hole current was suppressed by fabricating the samples so that the electrodes were "electrically remote" with respect to that portion of the crystal being studied. By special cutting and punching techniques[6] most of the midsection of a crystal was removed leaving a short, slender filament integral with two relatively massive ends, as shown in Fig. 1. Broad ohmic electrodes were then attached to the two ends. When current flows through the sample, the field at the ends is small compared with that along the filament. As a result, holes which are injected from one electrode during a pulse will drift relatively slowly toward the filament—the region which is being studied. However, observation of the pulse current along the filament is completed before these holes reach the filament and modulate its conductivity. These injected holes, therefore, do not manifest themselves.

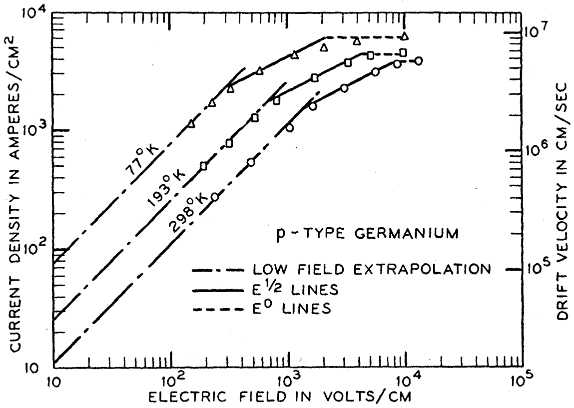

FIG. 2. Current density in *p*-type germanium as a function of electric field.

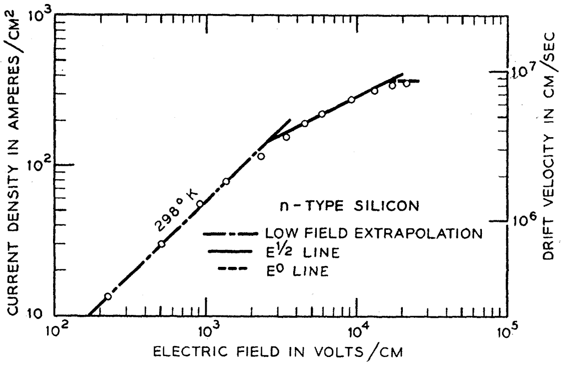

FIG. 3. Current density in *n*-type silicon as a function of electric field.

One additional precaution must be taken to keep the carrier density constant. Apparently, holes can also be generated along the surface of the filament if it is not treated properly. Suitable etching of this surface is necessary to minimize generation of these additional carriers. It appears that etching is effective for two reasons: first, it removes that region of the surface which has been strained by the cutting and punching processes; and second, it reduces surface irregularities which would have abnormally high local fields.

It is easy to determine whether the carrier density actually remains constant with electric field. Any change of carrier density resulting from hole modulation manifests itself as a change of current during a single pulse and is readily observable, so that cases in which such changes occur may be excluded from consideration.

The foregoing remarks about hole injection in *n*-type material apply also to electron injection in *p*-type material. In fact, electron injection can be a more serious problem because of the relatively higher mobility of electrons.

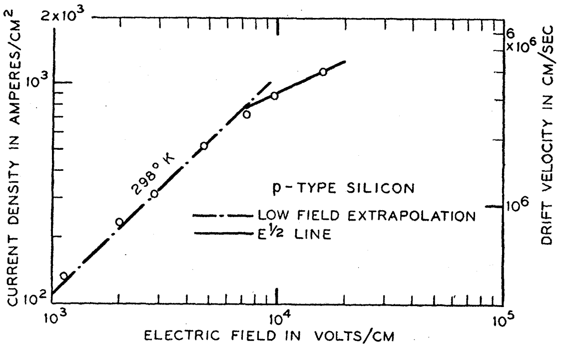

FIG. 4. Current density in *p*-type silicon as a function of electric field.

III. EXPERIMENTAL RESULTS

A schematic diagram of the circuit used to measure conductivity is shown in Fig. 1. The current through a sample was determined from the pulse voltage observed across the small series resistance. The voltage delivered by the pulse generator was measured independently. This voltage was corrected to give that which was actually across the filament of the sample. From the geometry of the filament one then obtained both the current density and the electric field.

Measurements of conductivity were made of a *p*-type germanium filament at temperatures of 77°K, 193°K, and 298°K and of *n*-type and *p*-type silicon filaments at 298°K. These data, shown plotted in Figs. 2, 3 and 4, are consistent with similar data for *n*-type germanium which have been reported previously[2] and which are reproduced in Fig. 5 for convenience of comparison. Each figure has an additional ordinate scale of

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[6] W. L. Bond, Phys. Rev. 78, 646 (1950).

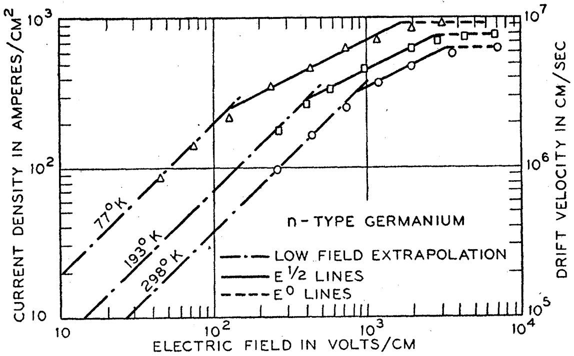

FIG. 5. Current density in n-type germanium as a function of electric field.

drift velocity reckoned from the low field drift mobility of holes and electrons at room temperature, as determined by Haynes and by Prince.[7] These scales of drift velocity for both types of germanium apply equally well to the low temperature data since the carrier density in these two samples of germanium is substantially constant throughout the temperature range investigated.

Three distinct ranges are evident in Figs. 2, 3, and 5. First, there is a range over which the conductivity or mobility is constant and Ohm's law holds. The pulse data in this range fit within experimental error the broken lines of constant conductivity which are extrapolations of dc data taken at extremely low fields. However, beyond a critical drift velocity of roughly 3×10^6 cm per sec there is a departure from Ohm's law, and there is a second range over which the drift velocity varies as $E^{\frac{1}{2}}$ and the mobility is proportional to $E^{-\frac{1}{2}}$. This nonlinear range continues with increased field until there is a third saturation range where the drift velocity is substantially independent of field and the mobility varies as E^{-1}. Figure 4 does not show a saturation range for p-type silicon; it was not possible to reach this range at room temperature because of the relatively low mobility at low fields of 370 cm²/volt/sec for this material.[7]

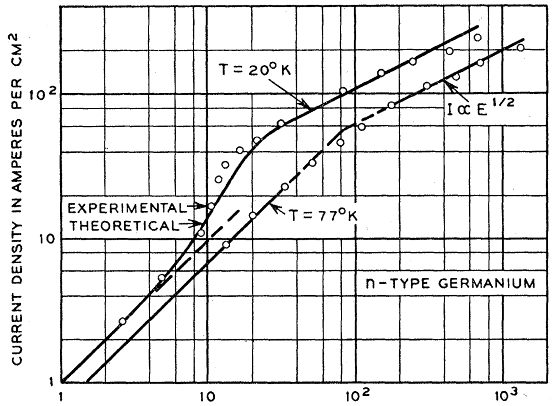

FIG. 6. Current density in n-type germanium as a function of electric field.

Additional conductivity measurements were made on one sample of n-type germanium of reasonably high purity at 20°K. These data are plotted in Fig. 6 along with data taken at 77°K. At 20°K scattering of electrons in this sample for low fields is due mainly to ionized impurity centers rather than to lattice vibrations. Beyond a field of about 10 volts per cm the electron "temperature" rises, and there is a rapid decrease of impurity scattering and a corresponding increase of conductivity or mobility. This decrease of impurity scattering continues with increased field until lattice scattering becomes dominant whereupon the current density (and the drift velocity) vary as $E^{\frac{1}{2}}$ as at the higher temperatures. Heating of the sample prevented extension of these data to include the saturation range. Also shown in Fig. 6 is a theoretical curve determined by Conwell which fits the experimental data at 20°K quite well.[8]

IV. DISCUSSION

TABLE I. Comparison of experimental and calculated values of critical drift velocity and critical field at 298°K.

Material	Experimental v_{dc}	Calculated v_{dc}	Experimental E_c	Calculated E_c
n-type Ge	$3.2(10)^6$	$0.8(10)^6$	900	230
p-type Ge	$2.4(10)^6$	$0.8(10)^6$	1400	480
n-type Si	$3.3(10)^6$	$1.4(10)^6$	2500	1070
p-type Si	$2.8(10)^6$	$1.4(10)^6$	7500	3750

In brief, the interpretation of these data is as follows: When the electric field is low the electrons remain in thermal equilibrium with the lattice and their drift velocity is given by

$$v_d=\mu_0 E, \tag{3}$$

where μ_0 is the low field mobility. However, as the field is increased the electrons gain energy and their "temperature" increases. Their collision frequency increases and their drift mobility decreases. If the rate at which electrons acquire energy from the field is equated to the rate at which they deliver energy to the lattice, it is found that there is a range of field over which the drift velocity varies as $E^{\frac{1}{2}}$. If it is assumed that constant energy surfaces in the Brillouin zone are spherical, the drift velocity in this range is given by[2]

$$v_d=1.23(c\mu_0 E)^{\frac{1}{2}}, \tag{4}$$

where c is the velocity of longitudinal acoustical waves. This velocity has been determined to be $5.4(10)^5$ cm/sec in germanium[9] and $9.2(10)^5$ cm/sec in silicon.[10]

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[7] M. B. Prince (private communication).

[8] E. M. Conwell (following paper) Phys. Rev. **90**, 769 (1953).

[9] Bond, Mason, McSkimin, Olsen, and Teal, Phys. Rev. **78**, 176 (1950).

[10] McSkimin, Bond, Buehler, and Teal, Phys. Rev. **83**, 1080 (1951).

If a critical drift velocity v_{dc} is defined as that velocity at which the Ohm's law and the $E^{\frac{1}{2}}$ regions intersect when extrapolated, we obtain from Eqs. (3) and (4) a critical drift velocity given by

$$v_{dc}=(1.23)^2c=1.51c, \tag{5}$$

and a corresponding critical field given by

$$E_c=1.51c/\mu_0. \tag{6}$$

Table I shows a comparison of the experimental and calculated values of critical drift velocity and critical field for silicon and germanium at 298°K. The observed values are from two to four times those calculated from Eqs. (5) and (6). Evidently then, electrons and holes are able to deliver their energy to the lattice up to higher fields than simple theory would predict. Shockley has proposed that the constant energy surfaces in the Brillouin zone are nonspherical and deeply re-entrant.[2] This concept serves also to explain the magnetoresistance increase of some sevenfold found by Suhl.[11]

In the saturation range at the higher fields the electrons are "heated" sufficiently so that they can transfer energy to optical modes of lattice vibration. These modes are much more effective than the acoustical modes in absorbing energy from the electrons. Theory predicts that the drift velocity is independent of the field in this range and is of the order of $6(10)^6$ cm per sec.[2] This value is in very good agreement with that shown in Figs. 2 and 5 for p-type and n-type germanium at room temperature and is in fair agreement with the observed value of $8(10)^6$ shown in Fig. 3 for n-type silicon.

The author is indebted to many of his associates for their assistance in this work and, in particular, to W. Shockley for his encouragement and advice.

Note added in proof:—In Fig. 2 the low field mobility of holes varies as $T^{-1.5}$ for the particular sample of germanium which was used. F. Morin and M. Prince (private communication) find a similar dependence for somewhat impure material and a $T^{-2.3}$ dependence for purer material.

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[11] H. Suhl, Phys. Rev. 78, 646 (1950).

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High Field Mobility in Germanium with Impurity Scattering Dominant*

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Experimental measurements show a variation of mobility with electric field intensity of electrons in n type germanium which differs at 20°K from that observed in the same specimen at 77°K and higher temperatures. This difference can be accounted for by scattering by ionized impurities. A crude quantitative treatment is carried out along the lines of Shockley's treatment for the case of lattice scattering. As in that case, the resulting theory fits the data well if the rate of energy loss is taken several times higher than that given by the theory assuming that the surfaces of constant energy are spherical.

INTRODUCTION

USING a pulse technique to apply high voltages to a sample of n type germanium, Ryder[1] has obtained the data shown in Fig. 1 for current density *vs* electric field intensity at 77°K and 20°K. Since the number of carriers and the temperature of the sample are kept constant, Ryder's experiment measures essentially the variation of drift velocity or mobility with electric field intensity at constant lattice temperature.[1a]

The significance of Ryder's data can perhaps be made clearer if they are replotted as mobility *vs* electric field intensity. This plot is shown in Fig. 2. The number of conduction electrons per unit volume was taken from Hall effect data.

The 77° behavior is of the same type as has been observed previously in samples at 77°K and higher temperatures. It has been explained by Shockley in the following way.[2,3] In the samples which show this behavior mobility is determined by interaction with acoustical modes of lattice vibration. This mechanism, at these temperatures, gives rise to a constant mean free path and isotropic scattering. In low fields, the

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[*] Presented at the Washington meeting of the American Physical Society, May 1, 1952. Also presented in preliminary form at the Twelfth Annual Conference on Physical Electronics, Massachusetts Institute of Technology. Figure 1 in the written report of this conference is incorrect, however. The corrected version constitutes Fig. 2 of this paper.

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[1] E. J. Ryder (preceding paper), Phys. Rev. 90, 766 (1953).

[1a] *Note added in proof.*—A similar, although steeper, increase of current density with voltage has been observed at 4°K by Sclar, Burstein, Turner, and Davisson, Bull. Am. Phys. Soc. 28, No. 2, 17 (1953) and attributed by them to increase in carrier density through impact ionization of neutral impurities. The current pulses in their experiment had a visible rise time for fields beyond the breakdown field, as would be expected for the mechanism of impact ionization, while the pulses in Ryder's case were flat.

[2] W. Shockley, Bell System Tech. J. 30, 990 (1951).

[3] The problem of electron drift in high fields was treated earlier in essentially this manner by H. Frohlich and F. Seitz, Phys. Rev. 79, 526 (1950), and F. Seitz, Phys. Rev. 76, 1376 (1950). The Seitz paper also contains a review of the literature.