Measurements on this sample were carried out at liquid air temperature (77°K) in order to reduce the saturation current of the reverse junction and to avoid heating effects. (In order to further reduce the effects of heating, pulses of voltage and current were employed.) Thus the mobility should be multiplied by a factor of $(300/77)^{3/2}\sim 7.5$. At this temperature the critical field E_0 has been shown by Ryder's measurements to be approximately 300 volts/cm (rather than 1400 as assumed for Fig. 2). Thus a factor of $(300/1400)^{\frac{1}{2}}$ comes into the determination of J_f. The area of the sample was 0.0445 cm². When all these factors are taken into account, we obtain

$$J_f=7.5\times 10^{-1} \quad \text{at} \quad V_f=40 \text{ volts.} \tag{4.2}$$

Using the values of V_f and J_f of (4.1) and (4.2), we have replotted Fig. 3 as V_a against I instead of the normalized values. This curve appears as the solid line in Fig. 6. The circles in Fig. 6 represent the experimental data. As can be seen the agreement is remarkable. No adjustable constants were used in obtaining this fit, except that the measured rather than calculated value for the punch through voltage was used. As pointed out above the agreement between these two values is better than the precision with which ρ_f is known from external evidence. It should be pointed out that a space-charge penetration experiment of this kind could be used to measure ρ_f.

Thanks are due W. G. Pfann who supplied the germanium, E. Snyder who fabricated the samples and aided in taking the data, and R. A. Logan who supplied the pulser. The author wishes especially to thank R. C. Prim and W. Shockley for helpful discussion and suggestions.

Note added in proof.—In this paper the temperature variation of hole mobility was taken as $T^{-3/2}$. That is, the field required for a given drift velocity was assumed to vary as $T^{3/2}$ for both low and high fields. This is in agreement with the data of E. J. Ryder for samples of about 1 ohm-cm resistivity. Since this paper was written, however, experiments of M. B. Prince[9] and F. J. Morin (private communication) have suggested that the temperature variation of the low field hole mobility may in fact be $T^{-2.3}$ for pure germanium. In the present experiment we are dealing with holes well into the high field range. It may be for such "hot" holes that the field required for a given drift velocity does vary as $T^{3/2}$. In any case, it should be pointed out that the quantitative agreement of theory and experiment shown in Fig. 6 depends on the accuracy of the $T^{3/2}$ variation assumed.

[9] M. B. Prince, Bull. Am. Phys. Soc. **28**, No. 2 (1953) Abstract C6.

PHYSICAL REVIEW VOLUME 90, NUMBER 5 JUNE 1, 1953

Spin Orbit Coupling and the Mesonic Lamb Shift

R. CHISHOLM AND B. TOUSCHEK
Department of Natural Philosophy, University of Glasgow, Glasgow, Scotland
(Received December 10, 1952)

It is shown that the self-energy corrections for a nucleon moving in a scalar potential well lead to a strong spin orbit coupling for pseudoscalar mesons. The effect is, however, opposite in sign to that required by the nuclear shell model.

I.

THE nuclear shell model recently proposed by Haxel, Jensen, and Suess,[1] and Mayer[2] suggests—according to the latter—the existence of a strong spin orbit coupling. In the applications of the shell model this coupling can in many cases be treated as a one-particle property: a single proton or neutron moving in the field of the core of the other nucleons aligns its spin with respect to its angular momentum in such a way that the state of higher moment is energetically favored.

In attempting to find a theoretical picture for this spin orbit coupling one is faced with three possibilities, each representing a crude simplification. The simplest of these is that the coupling is a one-particle phenomenon. Another alternative, more in keeping with the traditional layout of nuclear theory is the introduction of a spin orbit term into the expression for two-particle forces. Finally it is possible that spin orbit coupling in heavy nuclei is a many-particle property, related perhaps to the nonlinearity of the meson equations.

The second of these possibilities has been discussed by Le Couteur.[3] He has shown that the resultant of the two-particle spin orbit forces between the nucleons of a saturated core and an outside nucleon leads to a one-particle spin orbit force for the outer nucleon. This one-nucleon force can be derived from a spin orbit potential,

$$S_c=-(\boldsymbol{\sigma}\,\mathrm{grad}F\mathbf{p}), \tag{1}$$

[1] Haxel, Jensen, and Suess, Naturwiss. **35**, 376 (1948).
[2] M. G. Mayer, Phys. Rev. **75**, 1969 (1949).
[3] J. Hughes and K. H. Le Couteur, Proc. Phys. Soc. (London) **63A**, 1219 (1950).

in which F is a function of r alone, which can be determined if a particular form of the two-particle spin orbit interaction,

$$M^{(12)} = -(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2,\ \mathbf{r}_1-\mathbf{r}_2,\ \mathbf{p}_1-\mathbf{p}_2), \qquad (2)$$

is given.

The first possibility has been discussed by Dancoff.[4] The universal spin orbit coupling of a single particle in a scalar potential U, the Thomas force,

$$S_T = -\frac{1}{4M^2}(\boldsymbol{\sigma}\ \mathrm{grad} U \mathbf{p}), \qquad (3)$$

is two orders of magnitude too small to explain the most recent value of 2.5 Mev of the He^5 doublet and one order too small for an explanation of the Mayer coupling for heavy nuclei.

Another theory of this kind has been discussed independently by Rosenfeld[5] and Gaus.[6] According to Rosenfeld it is possible to choose the vector- and tensor-constants g and f of vector meson theory in such a way that a nucleon moving in the mesostatic field of the core experiences a spin orbit force proportional to gf. Gaus has shown that the spin orbit coupling thus obtained has the right sign and is of the order of magnitude required by the shell model.

Recent developments in the theory of fields have opened up a further possibility for the interpretation of spin orbit forces as a one-particle effect. The self-field of a particle changes if the particle is subjected to external forces. In the following work we show that in the case of a charge symmetric pseudoscalar meson theory this change in the self-field leads to a spin orbit force of the right magnitude but of the wrong sign. (The magnitude is not sufficient to explain the exceptionally large spin orbit coupling of He^5.) Scalar theory, on the other hand, leads to an effect several times smaller than the Thomas effect but of the right sign.

The case of pseudoscalar mesons with pseudoscalar coupling indicates that it is possible to obtain strong spin orbit forces from the Lamb shift in a potential well, and this recommends the adoption of the one-particle viewpoint rather than the assumption of two-particle spin orbit forces. The latter seem to be very small in pseudoscalar meson theory.[7] Furthermore, if mesons are responsible for the spin orbit effect it is probable that the self-mesons of a nucleon play the predominant part in this phenomenon. Spin orbit coupling is then analogous to the anomalous magnetic moment of a nucleon: it is caused by the mesonic cloud around this nucleon and one could expect it to be proportional to $g^2(M/\mu)^2$ times the Thomas coupling. The factor g^2 measures the probability of dissociation of the nucleon and the mass factor indicates that the effect is due to the meson. Inspection of Eq. (11) shows the absence of the mass factor. However, g appearing in this equation is the "large" g corresponding to a pseudoscalar coupling and M/μ times greater than the "small" g of the pseudovector coupling theory.

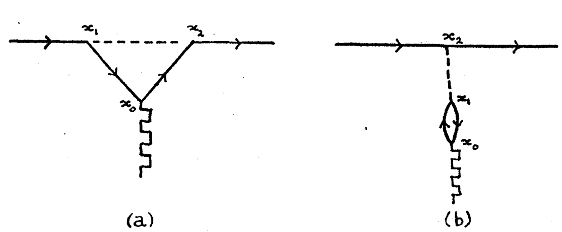

FIG. 1.

II.

Following the notation of Dyson,[8] we have for the second-order correction to the potential energy $U(x_0)$ of a particle situated at x_0,

$$\Delta\int \bar{\psi}(x_0)U(x_0)\psi(x_0)d\tau_0 = -\frac{1}{2}\int d\tau_0\int dx_1\int dx_2 P(H'(x_1)\bar{U}(x_0)H'(x_2)), \qquad (4)$$

where $\int d\tau_0$ is an integration over 3-space, $\bar{U}(x_0) = \bar{\psi}(x_0)U(x_0)\psi(x_0)$ is the density of potential energy at x_0, $\bar{\psi}$ and ψ are the field operators of the nucleon, and U is treated as a scalar quantity. Assuming charge-symmetric meson theory, H' is taken to be of the form

$$H' = g\bar{\psi}\tau_a\phi_a\psi;\quad H' = ig\bar{\psi}\gamma_5\tau_a\phi_a\psi, \qquad (5)$$

for scalar and pseudoscalar theory, respectively. In the latter case pseudoscalar coupling is singled out to insure the possibility of renormalization. There are only two graphs contributing to the expression (4); *viz.*, Figs. 1(a) and 1(b) where the "meander" represents the external potential. Figure 1(b) does not lead to a spin orbit coupling in either scalar or pseudoscalar theory and, therefore, will not be considered in the following.

Using Dyson's rules, one obtains from Eq. (4)

$$\Delta\int \bar{U}d\tau_0 = \frac{3g^2 i}{(2\pi)^4}\int d\tau_0\int dk\int dp_1\int dp_2 \times e^{ix_0(p_2-p_1+k)}\bar{\psi}(p_1)J(p_1p_2)U(k)\psi(p_2), \qquad (6)$$

where

$$\psi(x) = \int e^{iqx}\psi(q)dq \qquad (7)$$

[4] S. Dancoff and D. Inglis, Phys. Rev. **50**, 784 (1936).
[5] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, New York, 1948), p. 368.
[6] H. Gaus. Z. Naturforsch. **4a**, 721 (1949).
[7] D. B. Beard and H. A. Bethe, Phys. Rev. **83**, 1106 (1951).
[8] F. J. Dyson, Phys. Rev. **75**, 486 and 1736 (1949).

and

$$J(p_1p_2)=\begin{cases} -\int dpS(p_1-p)S(p_2-p)D(p) \\ \int dp\gamma_5 S(p_1-p)S(p_2-p)\gamma_5 D(p), \end{cases} \quad (8)$$

respectively, in the scalar and pseudoscalar case. The Fourier representations of the Feynman functions S and D are

$$S(q)=\frac{i(\gamma q)-M}{q^2+M^2}, \quad D(q)=\frac{1}{q^2+\mu^2}, \quad (9)$$

where μ is the mass of the meson. We are only interested in that part $\bar{J}$ of J which leads to spin orbit coupling. This term arises from the product of the terms $i(\gamma\cdot p_1-p)$ and $i(\gamma\cdot p_2-p)$ in the S functions in Eq. (8). $\bar{J}$ can be evaluated by standard methods. In the nonrelativistic limit ($p_1=p_2=iM$) one obtains

$$\bar{J}(p_1p_2)=\pm\sigma_{\mu\nu}(p_{1\mu}p_{2\nu}-p_{1\nu}p_{2\mu})\frac{\pi^2}{2M^2}(\log\lambda-1), \quad (10)$$

where $\sigma_{\mu\nu}=-\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ and $\lambda=M/\mu$. Terms of order $1/\lambda$ have been neglected. Inserting from Eq. (9) into Eq. (6) one finds that the expectation value of the correction to the potential energy can be derived from a spin orbit potential ΔU:

$$\Delta U=\mp 3(g/4\pi M)^2(\boldsymbol{\sigma}\,\mathrm{grad}U\mathbf{p}), \quad (11)$$

the two signs referring to scalar and pseudoscalar mesons, respectively. It follows that scalar mesons favor inverted doublets and pseudoscalar mesons produce normal doublets.

III.

In scalar meson theory g is of the order 1, and it is seen from Eq. (11) that in this case the correction ΔU represents only a small fraction of the Thomas coupling. The effect is therefore, as far as the present approximation goes, quite negligible.

In pseudoscalar theory, however, g is of the order 10 and the spin orbit coupling (10) is an order of magnitude in excess of the Thomas coupling. This magnitude is sufficient to lead to spin orbit splittings of several Mev for heavy nuclei.

The wrong sign for pseudoscalar mesons is not altered if the scalar potential is replaced by a function representing the time component of a 4-vector. This replacement changes the sign of the Thomas coupling. In the present calculation U should then be replaced by $\gamma_4 U$, and the expression (8) becomes (for pseudoscalar mesons)

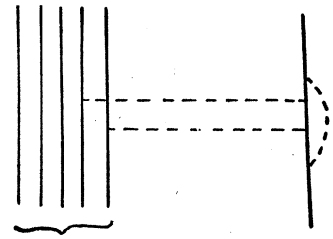

Fig. 2.

$$\bar{J}=\int dp\cdots\gamma_5(\gamma\cdot p_1-p)\gamma_4(\gamma\cdot p_2-p)\gamma_5\cdots.$$

To obtain a nonrelativistic approximation for this expression one has to draw γ_5 through the γ-aggregates $\gamma_i\gamma_4\gamma_\kappa$ and γ_4 through γ_κ. γ_4 acting on a nonrelativistic wave function can be replaced by unity: $\gamma_5\gamma_i\gamma_4\gamma_\kappa\gamma_5=-\gamma_i\gamma_4\gamma_\kappa\rightarrow\gamma_i\gamma_\kappa$, where the arrow indicates the nonrelativistic limit. The opposite situation obtains in the case of scalar mesons owing to the absence of the γ_5. The mesonic coupling for scalar mesons, therefore, depends on the introduction of γ_4 in the same manner as the Thomas effect.

It can be argued that this lack of sensitivity against changes in the transformation character of the potential makes the pseudoscalar meson results more trustworthy than those obtained for scalar mesons. For the present calculation assumes the persistence of the potential U in the intermediate state, and it is doubtful whether this assumption is valid if the intermediate states draw mainly from the negative energy components of the spinor field. The fact that the introduction of γ_4 into the potential energy operator does not alter the result for pseudoscalar mesons indicates that the main contribution to the spin orbit correction arises from the positive energy intermediate states.

A further weakness of the above argument appears to arise from the inconsistency of using a scalar potential in a pseudoscalar meson theory. This argument can be met only by assuming that the main part of the interaction between the outer nucleon and the core arises from an exchange of an even number of mesons represented for example by Fig. 2. This picture, however, assumes a slow convergence of the perturbation method when applied to meson theory. It is difficult, therefore, to see how the first approximation to the spin orbit effect should be sufficient and it is possible that higher approximations will change its sign. This objection, however, does not invalidate the fact that self-field corrections may lead to spin orbit effects far in excess of the Thomas coupling.

The authors wish to thank Professor J. C. Gunn for numerous discussions.