Space-Charge Limited Hole Current in Germanium

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A situation can arise in semiconductors similar to the space-charge limited emission of electrons in vacuum. The theory of Shockley and Prim for this phenomenon has been extended to the high field case using the approximation that the drift velocity of the carriers is $v=\mu(EE_0)^{\frac{1}{2}}$, where μ is the low field mobility, E the electric field, and E_0 the "critical field." For this approximation the current density analogous to Child's law for a plane parallel diode is

$$J=(2/3)(5/3)^{3/2}K\mu E_0^{1/2}V_a^{3/2}/w^{5/2},$$

where V_a is the potential across a diode of thickness w and K is the dielectric constant in mks units. Good agreement between theory and experiment for hole flow in germanium at liquid air temperature has been obtained, using values of μ and E_0 obtained independently by Ryder.

I. INTRODUCTION

It has been suggested by Shockley[1] that the hole current which flows across a p-n-p junction, when sufficient voltage has been applied to extend the space-charge region completely across the n layer, will, in the limit, be determined by space-charge effects analogous to Child's law in thermionic diodes. (The same arguments would apply, both here and throughout this paper, to electron currents through n-p-n structures.) In order that this region of operation may be reached at comparatively low voltages, it is necessary to make the n layer of material approaching intrinsic resistivity. Such material is currently available, and it has been suggested that it be designated by the symbol ν. Thus the structures we shall treat are designated p-ν-p.

We shall take as a model for a p-ν-p, the parallel plane structure pictured in Fig. 1(a). We shall regard the p layers as being very heavily doped and the junctions between the ν and p layers as being stepwise discontinuities in the excess of donors over acceptors, (N_d-N_a), as shown in Fig. 1(b). It is possible by the alloy process to make structures which closely approximate this form.

As voltage is applied across the p-ν-p, one of the junctions will be biased in the forward direction and the other in reverse. Accordingly a space charge barrier will appear at the reverse junction.[2] This barrier will extend deeply into the ν material but only slightly into the relatively more heavily doped p layer. If the thickness of the ν-layer is w, the voltage V_f required to push the barrier completely across the region is given by an appropriate solution to the one-dimensional Poisson equation with constant charge density ρ_f. This voltage is referred to as the "punch-through" voltage and is given by

$$V_f=\rho_f w^2/2K, \tag{1.1}$$

where $K=\kappa\epsilon_0$ is the dielectric constant of the ν region in mks units. The fixed charge density ρ_f is simply

$$\rho_f=q(N_d-N_a), \tag{1.2}$$

where (N_d-N_a) is the excess donor concentration in the ν material. We shall neglect the effects of diffusion[1] and obtain the hole current density J flowing across the p-ν-p by solving the set of equations

$$-K(d^2V/dx^2)=K(dE/dx)=\rho_f+\rho, \tag{1.3}$$

$$\rho=J/v(E). \tag{1.4}$$

Here $v(E)$ is the drift velocity of holes in the ν material. For low fields $v(E)=\mu E$, where μ is the mobility, a constant. For higher fields $v(E)$ is a more complicated function which has been reported for electrons in germanium by Ryder and Shockley[3] and analyzed by Shockley.[4] The author is indebted to E. J. Ryder for the previously unpublished data for holes in germanium shown in Fig. 2.

II. LIMITING CURRENT FOR HIGH APPLIED VOLTAGES

When the applied voltage has been increased beyond V_f sufficiently, enough hole current will flow so that the density ρ of the moving charge will be large compared with the fixed charge density ρ_f. Under these limiting circumstances (1.3) and (1.4) combine to give

$$dE/dx=J/Kv(E). \tag{2.1}$$

We shall solve this equation, assuming for $v(E)$ the following relationship:

$$v(E)=\mu E \quad \text{for} \quad E\leq E_0, \tag{2.2a}$$

$$v(E)=\mu(E_0E)^{1/2} \quad \text{for} \quad E\geq E_0. \tag{2.2b}$$

Let $x=x_0$ be the point at which $E=E_0$, $v=v_0$ and take $V=E=0$ at $x=0$; then, for $x\leq x_0$,

$$E=(2Jx/\mu K)^{1/2}, \tag{2.3a}$$

$$V=-(8Jx^3/9\mu K)^{1/2}, \tag{2.3b}$$

[1] W. Shockley, Proc. Inst. Radio Engrs. **40**, 1289 (1950); and W. Shockley and R. C. Prim, Phys. Rev. **90**, 753 (1953).
[2] See W. Shockley, Bell System Tech. J. **28**, 435 (1949).
[3] E. J. Ryder and W. Shockley, Phys. Rev. **81**, 139 (1951).
[4] W. Shockley, Bell System Tech. J. **30**, 990 (1951).

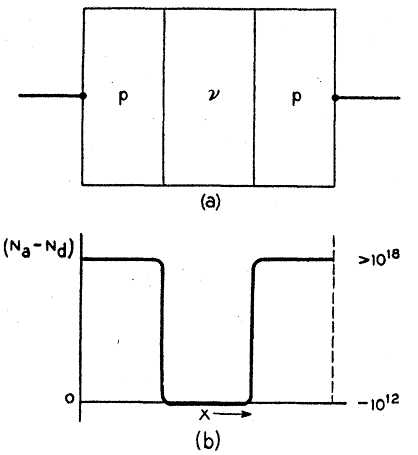

FIG. 1(a). Schematic diagram of parallel plane p-ν-p diode; (b) Excess acceptor concentration *vs* distance in the p-ν-p structure.

and, for $x \geq x_0$,

$$E^{3/2} = (J/2K\mu E_0^{1/2})(3x+x_0). \tag{2.4}$$

Note that (2.4) has been adjusted to give E_0 at $x=x_0$. We accordingly obtain

$$dV/dx = -(J/2K\mu E_0^{1/2})^{2/3}(3x+x_0)^{2/3} \quad \text{for} \quad x\geq x_0. \tag{2.5}$$

Integrating this equation, adjusting the integration constant to give $V=V_0$ at $x=x_0$, and substituting for x_0 its value from (2.3b), we obtain

$$V = -(5J)^{-1}[(2K\mu E_0^{1/2})^{-2/3}(3Jx + \mu KE_0^2/2)^{5/3} - K\mu E_0^3/3] \quad \text{for} \quad x\geq x_0. \tag{2.6}$$

For a typical case of interest we might have $x=3\times10^{-2}$ cm $E_0=1400$ v/cm; $\mu=1700$ cm^2/volt-sec; $K=16\times8.8\times10^{-14}$ f/cm; $J=1$ amp/cm^2. Under such circumstances, the terms in E_0^2 and E_0^3 are negligible and (2.6) reduces to

$$V = -(3/5)(3/2)^{2/3}(J/K\mu E_0^{1/2})^{2/3}x^{5/3}. \tag{2.7}$$

This result is the same as would have been obtained if $v(E)=\mu(E_0E)^{1/2}$ had been assumed throughout the width w of the center layer. Introducing the applied voltage $V_a \equiv -V(w)$, we obtain

$$\begin{aligned} J &= (2/3)(5/3)^{3/2}K\mu E_0^{1/2}V_a^{3/2}/w^{5/2} \\ &= 1.43K\mu E_0^{1/2}V_a^{3/2}/w^{5/2}. \end{aligned} \tag{2.8}$$

Compare this with the formula if $v(E)=\mu E$ had been assumed.

$$J = 9\mu KV_a^2/8w^3 = 1.125K\mu\langle E\rangle V_a^{3/2}/w^{5/2}, \tag{2.9}$$

where $\langle E\rangle$ is the average field V_a/w.

It is sometimes of interest to consider maximum electric field as a parameter rather than the thickness w. The formula corresponding to Eq. (2.8) is

$$J = (0.403K\mu E_0^{1/2}E^{5/2})/V_a. \tag{2.10}$$

The current density given by Eqs. (2.8) or (2.10) will then be the limiting form for high applied voltage to the p-ν-p. In Fig. 3 these equations have been plotted on a log-log scale with x or E as parameter. For the purposes of these curves E_0 has been taken as 1400 volts/cm, the hole mobility as 1700 cm^2/sec volt, and the dielectric constant as 16. The current at other mobilities or critical fields can be obtained by multiplying the value obtained from the curve by appropriate factors. The transit time T of a hole moving across the ν-region may also be of some interest,

$$T = \int_0^w \frac{dx}{v} = \int_0^{E(w)} \frac{K(dE/dx)dx}{J} = \frac{K}{J}E(w)$$

$$= \frac{(3/2)(3/5)^{1/2}w^{3/2}}{E_0^{1/2}\mu V_a^{1/2}}. \tag{2.11}$$

III. CURRENT FOR INTERMEDIATE APPLIED VOLTAGES

When the voltage applied to the p-ν-p is small, the current which flows is simply the saturation current of the reverse junction. As the voltage is raised the space-charge region penetrates more and more deeply into the ν-layer until at the voltage V_f of Eq. (1.1) it reaches entirely across. In Fig. 4 there is shown schematically the potential energy for holes within the ν-region for several applied voltages below the punch-through voltage V_f. As can be seen, for zero applied voltage there is a slight potential rise within the ν-region, which is just that necessary to prevent the flow of hole current from the p regions into the ν-region. As the potential of the right-hand p layer is made more and more negative, the space-charge region of width Y, (shown in Fig. 4 for the case $V=-0.2V_f$) penetrates more and more deeply. Equation (1.1) applies throughout the region Y. The current which flows is the saturation current of this reverse biased

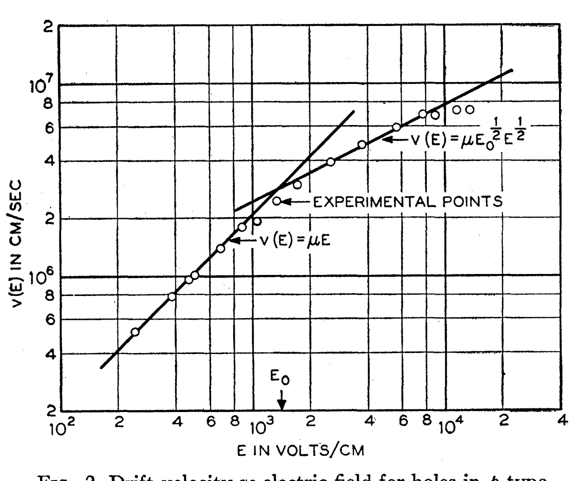

FIG. 2. Drift velocity *vs* electric field for holes in p type germanium at 300°K (from the data of E. J. Ryder).

junction.[5] When, finally, the voltage V_f has been reached, there remains only a slight potential rise or hump at the left side p-ν junction, and as further increases of voltage are made above V_f this hump is pulled down, allowing holes to pour over, leading to a very rapid increase in current when the voltage is raised beyond V_f.

In this intermediate range then, when the current density is not high enough so that the approximation of neglecting the fixed space charge ρ_f is justified, we must use Eqs. (1.3) and (1.4). The result (2.7) of the previous section indicates that the use of the relationship

$$v(E)=\mu(E_0E)^{1/2} \tag{3.1}$$

over the entire range of E should yield a useful approximation. Combining this result with Eqs. (1.3) and (1.4) gives

$$KdE/dx=\rho_f+J/\mu(E_0E)^{1/2}. \tag{3.2}$$

We shall find this equation easy to integrate if we introduce a variable s defined as follows:

$$s=\int_0^x dx/E;\quad ds=dx/E. \tag{3.3}$$

Combining (3.2) and (3.3), we obtain

$$ds/dE=(K\alpha/\rho_f E^{1/2})(1+\alpha E^{1/2})^{-1}, \tag{3.4}$$

where

$$\alpha=\mu E_0^{1/2}\rho_f/J.$$

If we integrate (3.4) and adjust the integration constant so that $s=0\rightarrow E=0$, we obtain

$$s=(2K/\rho_f)\log(1+\alpha E^{1/2}).$$

Solving this equation for E gives

$$E=(J^2/\mu^2\rho_f^2E_0)(e^{\rho_f s/2K}-1)^2=-dV/dx$$
$$=(1/E)(dV/ds); \tag{3.5}$$

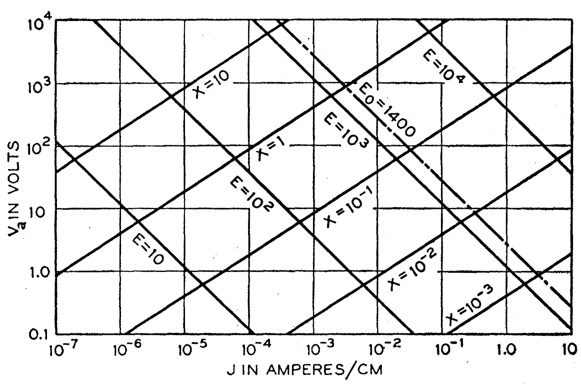

FIG. 3. Space-charge limited drift of "tepid" holes in germanium.

[5] It may be necessary in considering this saturation current to take account of changing lifetime within the space-charge region. The situation is analogous to the saturation current of a p-n-p transistor with floating base.

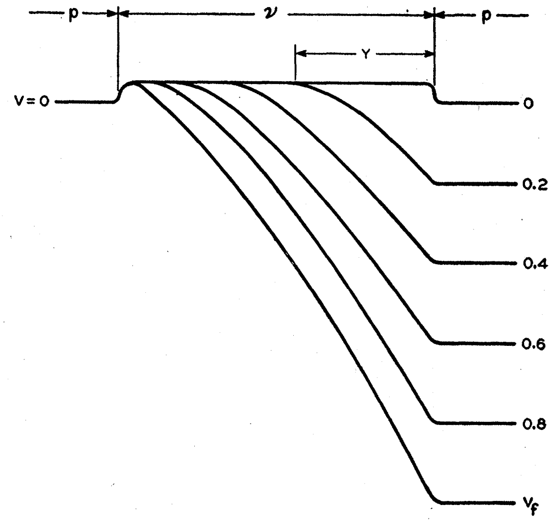

FIG. 4. Schematic plot of potential energy for holes *vs* distance in the p-ν-p diode.

and if (3.5) is integrated and the integration constant chosen so that $s=0$ when $V=0$, we obtain

$$V=-(J^4K/6\mu^4\rho_f^5E_0^2)(3e^{2u}-16e^{3u/2}+36e^u-48e^{u/2}+6u+25), \tag{3.6}$$

where

$$u=\rho_f s/K.$$

The total thickness w, of the ν-layer can be obtained by integrating the following equation:

$$w(s)=\int_0^s Eds. \tag{3.7}$$

Thus

$$w(u)=(J^2K/\mu^2\rho_f^3E_0)(e^u-4e^{u/2}+u+3), \tag{3.8}$$

where u is understood to have its value at $x=w$. If for the case $u\ll 1$ we eliminate u between (3.6) and (3.8), we obtain

$$V^{3/2}=(3/2)(3/5)^{3/2}Jw^{5/2}/\mu KE_0^{1/2}, \tag{3.9}$$

which is the same as expression (2.8) and shows that in the limit, the behavior is the same as if ρ_f were negligible. We find it convenient to introduce the following variables:

$$V_f=-\rho_f w^2/2K,$$
$$J_f=(2/3)(5/3)^{3/2}K\mu E_0^{1/2}V_f^{3/2}/w^{5/2}. \tag{3.10}$$

Here V_f is the voltage necessary to sweep the mobile carriers out of a region of thickness w containing a fixed space charge ρ_f. J_f is the space-charge limited current corresponding to an applied voltage of V_f across a diode of thickness w with no fixed charge and with a mobility law given by (3.1). If we make use of the first of the relations (3.10) and eliminate J between

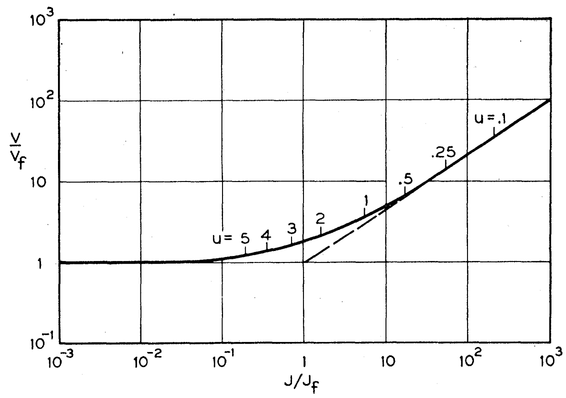

FIG. 5. Theoretical $\ln V$ *vs* $\ln I$ curve for a space-charge limited p-ν-p diode in reduced units.

(3.6) and (3.8), we obtain

$$V=\frac{V_f}{3}\frac{(3e^{2u}-16e^{3u/2}+36e^u-48e^{u/2}+6u+25)}{(e^u-4e^{u/2}+u+3)^2}. \quad (3.11)$$

From (3.8) and the second of the relations (3.10), we obtain

$$J=1.98J_f/(e^u-4e^{u/2}+u+3)^{1/2}. \quad (3.12)$$

It can be seen from an inspection of (3.11) and (3.12) that

$$u\rightarrow\infty \text{ implies } V\rightarrow V_f \text{ and } J\rightarrow 0,$$

while, as previously noted, $u\rightarrow 0$ implies (2.8). This solution therefore gives us the expected rapid increase of current at a voltage V_f and goes into the space-charge limited law (2.8) at high voltages.

A plot of V/V_f *vs* J/J_f has been obtained by numerically eliminating u between (3.11) and (3.12). This result is shown in Fig. 5.

IV. EXPERIMENTAL RESULTS

Structures closely resembling the ideal model of Fig. 1 have been made by a modification of the alloy-diffusion technique.[6] Agreement between theory and experiment has been good for several samples at various temperatures. We shall describe in detail, however, the experimental results for one typical sample.

Sample P-35A was fabricated from n type material in which the excess donor concentration was approximately 10^{12}/cc. This material was purified by the zone process of Pfann.[7] The excess donor concentration was determined by measuring the conductivity at temperatures low enough so that the density of thermally generated carriers was small compared to the excess donor concentration but high enough so that essentially all of the donors were ionized and the electron density equal to (N_d-N_a). The electron density n was then deduced from the conductivity by assuming that $\mu(T)=\mu(300°\text{K})\cdot(300/T)^{3/2}$. In addition to this, measurements of differential capacity *vs* reverse bias were made. The capacity was observed to vary inversely as the square root of the reverse voltage indicating that the impurity densities were discontinuous stepwise as assumed.

It is interesting to note that the presence of *step junctions* makes possible a determination of dielectric constant from a measurement of differential capacity. It is necessary to know the area and thickness of a capacitor in order to obtain κ from the capacity. In the usual graded p-n junction the thickness can be determined independently by probing the space-charge region, but these measurements are difficult to make and rather inaccurate. In the present experiment with an applied bias equal to the punch-through voltage V_f, the capacitor spacing is just the thickness of the specimen and is known to better than 10 percent. The differential capacity at this bias was measured on an impedance bridge at 1 kc (and at 10 kc) and was

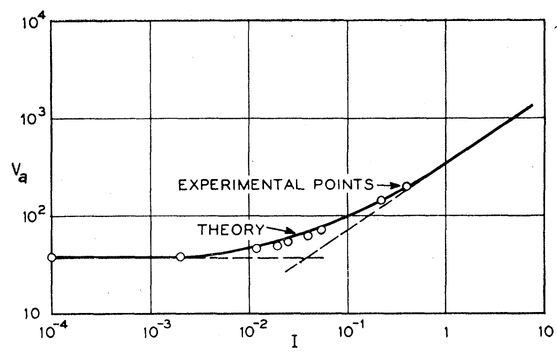

FIG. 6. Comparison of theory and experiment for space-charge limited p-ν-p diode. The observed current I is shown. The theoretical curve is fitted by using the measured area.

consistent with the value of $\kappa=16$ obtained from the optical measurements of Briggs.[8] Furthermore, the differential capacity at lower biases agreed with a spacing w calculated from (1.1) using $\kappa=16$ and the value of ρ_f calculated from the value of (N_d-N_a) obtained from a low temperature conductivity measurement as described above.

The thickness w of the sample was 2.8×10^{-2} cm. Thus the punch-through of the space-charge layer should occur at a voltage of

$$V_f=\frac{\rho_f w^2}{2K}=\frac{1.6\times10^{-19}\times10^{12}\times(2.8\times10^{-2})^2}{2\times16\times8.8\times10^{-14}}=44 \text{ volts.} \quad (4.1)$$

The punch-through for this sample occurred at approximately 40 volts. This agreement is all that can be expected since ρ_f is not known with great precision.

[6] See, for example, R. N. Hall and W. C. Dunlap, Phys. Rev. **80**, 467 (1950); J. S. Saby, Tele. Tech. **10**, 32 (1951); Proc. Inst. Radio Engrs. **40**, 1358 (1952); and Law, Mueller, Pankove, and Armstrong, Proc. Inst. Radio Engrs. **40**, 1352 (1952).

[7] W. G. Pfann and K. M. Olsen, Phys. Rev. **89**, 322 (1953).

[8] H. B. Briggs, Phys. Rev. **77**, 287 (1950).

Measurements on this sample were carried out at liquid air temperature (77°K) in order to reduce the saturation current of the reverse junction and to avoid heating effects. (In order to further reduce the effects of heating, pulses of voltage and current were employed.) Thus the mobility should be multiplied by a factor of $(300/77)^{3/2}\sim 7.5$. At this temperature the critical field E_0 has been shown by Ryder's measurements to be approximately 300 volts/cm (rather than 1400 as assumed for Fig. 2). Thus a factor of $(300/1400)^{\frac{1}{2}}$ comes into the determination of J_f. The area of the sample was 0.0445 cm². When all these factors are taken into account, we obtain

$$J_f=7.5\times 10^{-1} \quad \text{at} \quad V_f=40 \text{ volts}. \tag{4.2}$$

Using the values of V_f and J_f of (4.1) and (4.2), we have replotted Fig. 3 as V_a against I instead of the normalized values. This curve appears as the solid line in Fig. 6. The circles in Fig. 6 represent the experimental data. As can be seen the agreement is remarkable. No adjustable constants were used in obtaining this fit, except that the measured rather than calculated value for the punch through voltage was used. As pointed out above the agreement between these two values is better than the precision with which ρ_f is known from external evidence. It should be pointed out that a space-charge penetration experiment of this kind could be used to measure ρ_f.

Thanks are due W. G. Pfann who supplied the germanium, E. Snyder who fabricated the samples and aided in taking the data, and R. A. Logan who supplied the pulser. The author wishes especially to thank R. C. Prim and W. Shockley for helpful discussion and suggestions.

Note added in proof.—In this paper the temperature variation of hole mobility was taken as $T^{-3/2}$. That is, the field required for a given drift velocity was assumed to vary as $T^{3/2}$ for both low and high fields. This is in agreement with the data of E. J. Ryder for samples of about 1 ohm-cm resistivity. Since this paper was written, however, experiments of M. B. Prince[9] and F. J. Morin (private communication) have suggested that the temperature variation of the low field hole mobility may in fact be $T^{-2.3}$ for pure germanium. In the present experiment we are dealing with holes well into the high field range. It may be for such "hot" holes that the field required for a given drift velocity does vary as $T^{3/2}$. In any case, it should be pointed out that the quantitative agreement of theory and experiment shown in Fig. 6 depends on the accuracy of the $T^{3/2}$ variation assumed.

[9] M. B. Prince, Bull. Am. Phys. Soc. **28**, No. 2 (1953) Abstract C6.

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Spin Orbit Coupling and the Mesonic Lamb Shift

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It is shown that the self-energy corrections for a nucleon moving in a scalar potential well lead to a strong spin orbit coupling for pseudoscalar mesons. The effect is, however, opposite in sign to that required by the nuclear shell model.

I.

THE nuclear shell model recently proposed by Haxel, Jensen, and Suess,[1] and Mayer[2] suggests—according to the latter—the existence of a strong spin orbit coupling. In the applications of the shell model this coupling can in many cases be treated as a one-particle property: a single proton or neutron moving in the field of the core of the other nucleons aligns its spin with respect to its angular momentum in such a way that the state of higher moment is energetically favored.

In attempting to find a theoretical picture for this spin orbit coupling one is faced with three possibilities, each representing a crude simplification. The simplest of these is that the coupling is a one-particle phenomenon. Another alternative, more in keeping with the traditional layout of nuclear theory is the introduction of a spin orbit term into the expression for two-particle forces. Finally it is possible that spin orbit coupling in heavy nuclei is a many-particle property, related perhaps to the nonlinearity of the meson equations.

The second of these possibilities has been discussed by Le Couteur.[3] He has shown that the resultant of the two-particle spin orbit forces between the nucleons of a saturated core and an outside nucleon leads to a one-particle spin orbit force for the outer nucleon. This one-nucleon force can be derived from a spin orbit potential,

$$S_c=-(\boldsymbol{\sigma}\,\mathrm{grad}F\mathbf{p}), \tag{1}$$

[1] Haxel, Jensen, and Suess, Naturwiss. **35**, 376 (1948).
[2] M. G. Mayer, Phys. Rev. **75**, 1969 (1949).
[3] J. Hughes and K. H. Le Couteur, Proc. Phys. Soc. (London) **63A**, 1219 (1950).