Radiation by Electrons in Large Orbits*

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By finding the points in the synchrotron magnetic cycle at which the time rate of change of electron orbit radius vanishes, the total electron energy loss per orbit revolution is measured. The results, for electron energies from 205 Mev to 280 Mev, are in excellent agreement with classical electromagnetic theory. The measurements have been extended to 318 Mev with reduced precision by observing orbit shrink rates after the radiofrequency accelerating voltage is removed. No coherent energy losses and no significant energy losses other than radiation have been observed.

I. INTRODUCTION

AN electron moving in a large orbit radiates energy at a rate

$$P=\tfrac{2}{3}(e^2/\rho)\omega_0(E/mc^2)^4 \quad (1)$$

according to classical electromagnetic theory.[1,2] Here P is the instantaneous power in ergs/sec, e the electronic charge in esu, ρ the instantaneous radius of curvature in cm, ω_0 the angular velocity, and E and mc^2 are the electron's total energy and rest energy, respectively. For an electron moving with angular velocity ω_0 in an orbit of constant radius this power can be conveniently expressed as electron volts per revolution:

$$P=716B^4\rho^3 \text{ ev/rev}, \quad (2)$$

where B is measured in webers/m^2 and ρ in meters. If B is measured in gauss and ρ in cm the right side of Eq. (2) must be divided by 10^{22}.

The frequency distribution of this radiation has been calculated by Schwinger.[3] The energy is radiated at high harmonics of the revolution frequency and much of it lies in the optical range for electron energies of a few hundred Mev. The existence of this optical radiation is well known[4] to experimenters with large synchrotrons and betatrons. In the Cornell synchrotron, for example, some of this light is directed on to a photomultiplier whose output signal is used as a tuning indication.

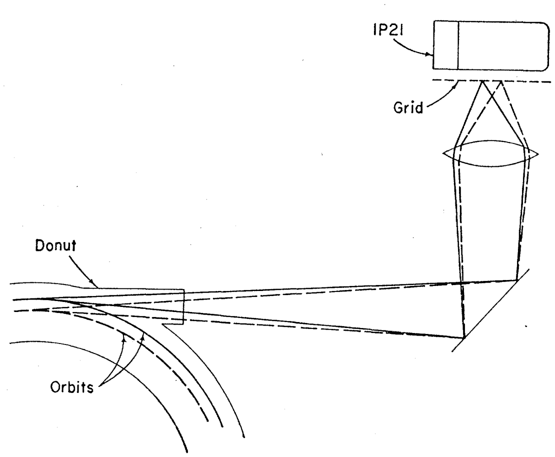

FIG. 1. Light emitted tangentially from electron orbit is brought to a focus on a grid placed over a photomultiplier. When the orbit radius shrinks, the focal spot moves across the grid, modulating the photomultiplier output.

Recently a quantum-mechanical calculation[5] of the radiation frequency distribution showed results at variance with Schwinger's classical distribution and indicated a radiated power markedly below the classical value. Since the fourth power energy dependence of this radiation represents a major barrier in the construction of circular electron accelerators in the Bev range, it is of considerable practical importance to know which result is correct. Measurements previously reported[6] showed that the classical value of total radiated power is the correct one. The work reported here, which is an extension and refinement of that previously reported, shows even better agreement with the classical result. In the meantime the faults of the quantum-mechanical calculation have been pointed out.[7–9]

II. PRINCIPLE OF THE MEASUREMENT

The magnetic field in the Cornell synchrotron varies with time in an approximately sinusoidal fashion at a frequency of 30 cycles per second. The field has a radial dependence given by $B=B_0(\rho_0/\rho)^n$, where the nominal value of n is $\tfrac{2}{3}$. For the total energy of an electron moving with a radius of curvature ρ in a magnetic field B we have $E=300B\rho$, where E is in ev and $B\rho$ in gauss cm. This equation is good to better than one part in 10^3 for a 200-Mev electron. If we allow both B_0 and ρ to vary with time, we have

$$\dot{E}/E=(1-n)\dot{\rho}/\rho+\dot{B}_0/B_0, \quad (3)$$

where the dots indicate differentiation with respect to time. If the magnetic field has a general radial de-

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[1] D. Iwanenko and I. Pomeranchuk, Phys. Rev. **65**, 343 (1944).
[2] L. I. Schiff, Rev. Sci. Instr. **17**, 6 (1946).
[3] J. Schwinger, Phys. Rev. **75**, 1912 (1949); See also L. I. Schiff, Am. J. Phys. **20**, 474 (1952).
[4] Elder, Langmuir, and Pollock, Phys. Rev. **74**, 52 (1948).
[5] G. Parzen, Phys. Rev. **84**, 235 (1951).
[6] D. Corson, Phys. Rev. **86**, 1052 (1952).
[7] Judd, Lepore, Ruderman, and Wolff, Phys. Rev. **86**, 123 (1952).
[8] H. Olsen and H. Wergeland, Phys. Rev. **86**, 123 (1952).
[9] L. I. Schiff, reference 3.

pendence $B=B_0f(\rho)$ the factor $(1-n)$ in Eq. (3) must be replaced by $\{1+[\rho f'(\rho)/f(\rho)]\}$, where the prime indicates differentiation with respect to ρ.

In Eq. (3) $\dot{E}$ is the net rate of change of energy and is the sum of various contributing terms. We have

$$\dot{E}=\dot{E}_{\text{rf}}+\dot{E}_{\beta}+\dot{E}_{\text{rad}}+\dot{E}_0, \quad (4)$$

where $\dot{E}_{\text{rf}}$ is the rate at which energy is supplied by the radiofrequency accelerating field, $\dot{E}_{\beta}$ is the rate at which energy is supplied through the betatron effect by virtue of the changing magnetic flux within the orbit, $\dot{E}_{\text{rad}}$ is the rate at which energy is lost through radiation and $\dot{E}_0$ is the rate at which energy is lost through any other mechanism which may exist, as, for example, through image currents in the conducting walls of the vacuum chamber. If the various electrons in a bunch radiate coherently, the radiation energy loss per electron per revolution will depend on the number of electrons in the bunch.

In all the measurements reported here the electrons are accelerated to an energy E and then the radiofrequency accelerating voltage is removed, so that $\dot{E}_{\text{rf}}=0$. $\dot{E}_{\beta}$ is measurable (see Sec. III C), so that the result of the experiment is a determination of $\dot{E}_{\text{rad}}+\dot{E}_0$. These two terms can be separated by measuring at different electron energies and finding the part of the total energy loss rate which has a fourth power energy dependence. $\dot{E}_0$ turns out to be no more than one or two percent at most of the total energy loss at any of the energies measured here.

It is convenient to express $\dot{E}_{\beta}/E$ as a fraction k of the quantity $\dot{B}_0/B_0$. If k were equal to unity, and neglecting any energy losses, the acceleration would be entirely by betatron action. k is the ratio of flux within the orbit to the flux necessary to satisfy the betatron condition. For the parts of the magnetic cycle where the magnetic shunts, which provide the initial betatron acceleration, have saturated, k is measured to have a constant value of 0.192 ± 0.002.

Expressing $\dot{E}_{\beta}/E$ in this fashion we have

$$(\dot{E}_{\text{rad}}+\dot{E}_0)/E=(1-n)\dot{\rho}/\rho+(1-k)\dot{B}_0/B_0, \quad (5)$$

for the case where $\dot{E}_{\text{rf}}=0$. The most straightforward way to use Eq. (5) to measure $\dot{E}_{\text{rad}}+\dot{E}_0$ is to find the point on the back side of the magnetic cycle, i.e., where the field has passed maximum intensity and is decreasing, where $\dot{\rho}=0$ when the radiofrequency field is removed. At this point in the cycle the electrons are losing energy by radiation and by inverse betatron effect to the magnetic field, but the field is falling at a rate which keeps the orbit radius fixed. If the radiofrequency accelerating field is removed earlier in the cycle the orbit shrinks, if later the orbit expands. If k, $\dot{B}_0$, B_0, and ρ are known, $\dot{E}_{\text{rad}}+\dot{E}_0$ can be calculated. ρ is best known from the frequency $\omega_0/2\pi$ of the radiofrequency accelerating field, so that Eq. (5), in terms of energy loss measured in electron volts per revolution,

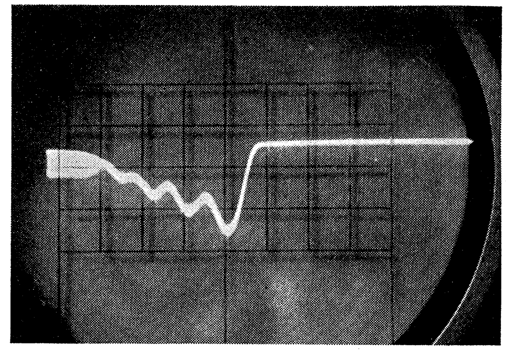


FIG. 2. Modulated signal from photomultiplier. At left, modulation produced by rf pick-up shows where rf is turned off. Light spot moving over grid produces seven half-cycles in photomultiplier output before beam is intercepted by synchrotron target. Average signal level changes because effect of limiting apertures depends on orbit radius.

may be written as

$$\Delta E=\frac{300(1-n)B_0\dot{\rho}}{(\omega_0/2\pi)}+\frac{(1-k)9\times10^{12}\dot{B}_0}{2\pi(\omega_0/2\pi)^2}\ \text{ev/rev.} \quad (6)$$

If the excitation of the magnet is changed and the new $\dot{\rho}=0$ point determined, ΔE can be found for a new electron energy.

III. MEASUREMENTS

A. Determination of $\dot{\rho}=0$ points

The primary method used in finding the $\dot{\rho}=0$ points is illustrated in Fig. 1. As a bunch of electrons moves around the orbit, the narrow cone of light (half-angle $\sim mc^2/E$) sweeps across a lens. The lens brings the light to an approximate point focus on a grid placed over an electron multiplier tube (1P21). As the orbit shrinks, after the accelerating field has been removed, the focal spot moves across the grid, which is alternately transparent and opaque, modulating the multiplier output. As the radiofrequency turn-off time is advanced to later points in the magnetic cycle, a point is reached where the orbit shrinks for a short time and then expands. In this case the modulated multiplier pattern is double—one pattern as the orbit shrinks and a repeat pattern as it expands. The mid-point of this double pattern, corresponding to $\dot{\rho}=0$, can be determined with a probable error of about $\pm0.1°$ of the magnetic cycle. An example of a single modulated output pattern is shown in Fig. 2.

The $\dot{\rho}=0$ point can also be found, as indicated in Fig. 3, by observing the x-rays produced when the electrons strike a target. As long as the accelerating field is applied the orbit radius is constant. If it is turned off near the peak of the magnetic cycle the beam spirals in, as at (a) in Fig. 3, and strikes the synchrotron target at $\rho=\rho_1$ producing x-rays at $t=t_a$. There is a point in the magnetic cycle where the orbit will first

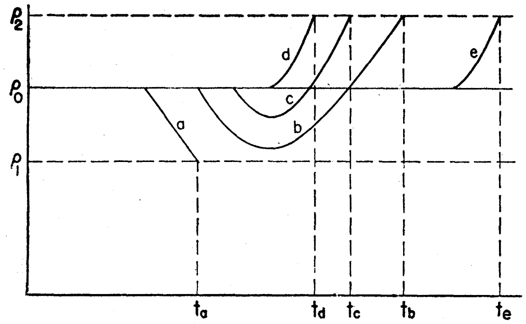


FIG. 3. Illustration of x-ray production time dependence on rf turn-off time. X-rays appear at a minimum time when orbit radius is in equilibrium when the rf is removed.

contract and then expand, as at (b) in Fig. 3. In this case the x-rays appear at $t=t_b$ when the electrons strike the electron injection gun at $\rho=\rho_2$. As the radiofrequency turn-off time is advanced still further, as in (c) Fig. 3, the x-rays appear at $t=t_c$ which is earlier than t_b. There is an earliest time for x-rays to appear, corresponding to the case where the orbit neither shrinks nor expands when the accelerating field is first removed.[10] The radiofrequency turn-off time in this case is the $\dot{\rho}=0$ time.

B. Measurement of B_0 and $\dot{B}_0$

B_0 and $\dot{B}_0$ are measured with the aid of an annular loop of wire fixed in the gap of the synchrotron magnet

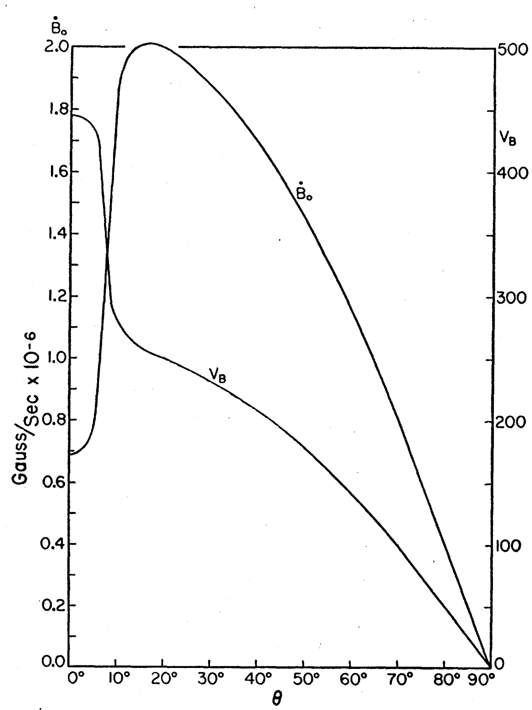


FIG. 4. Wave forms for rate of change of magnet field and for betatron effect in volts/rev.

[10] This phenomenon was called to my attention by Dr. Morton Camac.

and centered on the orbit. This loop is part of the correction coil system regularly employed in the operation of the machine. The output voltage from this loop is proportional to $\dot{B}_0$ in the magnet gap at every instant. The wave form of this voltage, shown in Fig. 4, is precisely determined in the manner indicated below. Since the area of the loop is unknown, $\dot{B}_0$ cannot be determined directly. Instead the wave form is integrated numerically, and the integral set equal to the known peak magnetic field. This determines the scale on the $\dot{B}_0$ *vs* time wave form. The peak magnetic field has been determined in two ways: by measuring the spectrum of electron pairs from the bremsstrahlung x-ray spectrum in a pair spectrometer which has been absolutely calibrated,[11] and by measuring the peak magnetic field at various azimuths around the magnet with a small coil which rotates synchronously with the alternating magnetic field. The rotating coil is calibrated in a dc magnet by comparison with the proton magnetic moment resonance.

In plotting the $\dot{B}_0$ *vs* time wave form it is essential that the time base be strictly linear; otherwise the integration will not establish the proper $\dot{B}_0$ scale. Linearity has been insured here by providing a timing circuit which divides the magnetic cycle into 0.1 degree intervals. This is accomplished by subdividing a 108-kc oscillator frequency by a factor of 3600 to produce 30-cycle pulses, which are held in time coincidence with the 30-cycle "peaking strip" pulses from the magnet by a time discriminator—automatic frequency control circuit. The 108-kc frequency and its various submultiples then provide 10°, 1°, and 0.1° marker pulses for timing events throughout the magnetic cycle. By the use of suitable gates, coincidence circuits, and delays, a continuously variable trigger pulse whose phase is known to ±0.1 degree is available.

TABLE I. Determination of the constant $k=(\dot{E}_\beta B_0/E\dot{B}_0)$ from measurements of the emf V_β induced in a single loop of wire at the orbit radius and from the rate of change of magnetic field at the orbit, at various points in the magnetic cycle.

θ	V_β (volts)	$\dot{B}_0$ (g/sec $\times 10^{-6}$)	k
20.0°	249	2.00	0.195
30.0°	232	1.89	0.193
40.0°	208	1.69	0.192
50.0°	176	1.45	0.190
60.0°	139	1.15	0.188
70.0°	97.5	0.795	0.192
80.0°	51.4	0.412	0.195
			Average = 0.192±0.002

The $\dot{B}_0$ wave form is recorded in comparison with a dc potential difference in a null manner. The ungrounded side of the annular coil is connected to one of the vertical deflection plates of an oscilloscope. The other vertical plate is connected to the variable tap of a precision variable resistance which serves as a po-

[11] J. W. DeWire (unpublished).

tential divider across a battery, one side of which is grounded. The precision timing marker is placed on the horizontal deflection plates. The wave form is then plotted out at closely spaced intervals throughout the magnetic cycle by recording the variable tap position when the timing mark is brought to the no-deflection position on the scope.

Once the $\dot{B}_0$ *vs* time curve is precisely determined, B_0 *vs* time is obtained by integrating backwards (since values near the top of the cycle are the ones of interest) from the known peak magnetic field.

In order to measure the radiation energy loss at different energies it is necessary to vary the magnet excitation and measure the $\dot{\rho}=0$ point at several different magnet currents. The peak magnetic field for each magnet current must be measured, and this is done with the rotating coil in a fixed position in the magnet gap.

C. Measurement of Betatron Voltage

The energy contributed per revolution by the changing flux within the orbit is measured with a single loop of wire in the magnet gap at the orbit radius. The output voltage from this loop is maximum at the beginning of the magnetic cycle, as indicated in Fig. 4, when the magnetic shunts are unsaturated. The peak voltage is measured with a peak reading voltmeter. Amplitudes relative to the peak are measured on an oscilloscope in a null manner similar to that used for the $\dot{B}_0$ measurements.

TABLE II. Determination of the magnetic field radial dependence exponent "n" from measurements of the orbit shrink velocity at symmetrical points early and late relative to the peak of the magnetic cycle.

θ_E	θ_L	$\dot{\rho}_E$ (cm/sec $\times10^{-4}$)	$\dot{\rho}_L$ (cm/sec $\times10^{-4}$)	$\lvert\dot{B}_0\rvert$ (g/sec $\times10^{-6}$)	B_0 (g $\times10^{-4}$)	n
62.1°	117.9°	4.5	0	1.08	0.90	0.57
68.4	111.6	4.3	1.0	0.85	0.94	0.55
78.0	102.0	3.8	1.9	0.49	1.03	0.59
85.5	94.5	3.4	2.7	0.18	1.05	0.60
					Average=	0.58±0.02

The factor k defined in Eq. (5) is given by

$$k=\frac{\dot{E}_\beta}{E}\frac{B_0}{\dot{B}_0}=\frac{6.98\times10^{-13}V_\beta(\omega_0/2\pi)^2}{\dot{B}_0},\qquad (7)$$

where V_β is the emf in volts induced in the loop at a time when the magnetic field is changing at a rate $\dot{B}_0$ measured in gauss per second.

One may get a check on the $\dot{B}_0$ determinations by measuring k at the beginning of the magnetic cycle. Since the initial acceleration is by betatron action, k must be unity. The measured value is 0.998.

D. Measurement of $\dot{\rho}$

The highest energy which can be reached using the $\dot{\rho}=0$ points is about 280 Mev with the synchrotron operating at its normal excitation (peak electron energy of 318 Mev at the orbit for these measurements). To extend the measurements to the peak energy, $\dot{\rho}$ and n in Eq. (6) must be determined. $\dot{\rho}$ is measurable from the modulation frequency of the photomultiplier output, provided the change in orbit radius corresponding to one modulation cycle is known. This has been calibrated by intercepting the electron beam with the synchrotron target and measuring the number of modulation cycles *vs* target position. The $\dot{\rho}$ determination is somewhat uncertain because, as indicated in Fig. 1, the azimuth in the orbit where the observed light is radiated changes as the orbit shrinks, resulting in an apparent slowing up of the orbit motion. Consequently

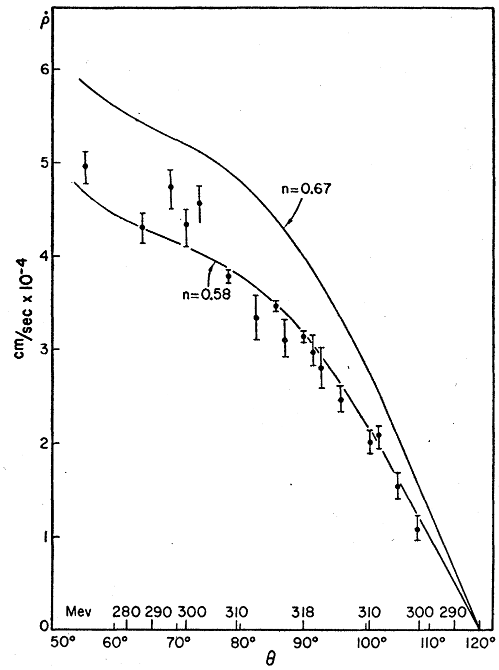


FIG. 5. Measured orbit shrink velocities when rf is turned off at various magnetic cycle angles. Solid curves are calculated on basis of classical radiation for measured and nominal values of radial dependence exponent "n."

TABLE III. Electron energy loss in electron volts/revolution determined from the magnetic cycle angle (θ) corresponding to $\dot{\rho}=0$ at several magnet excitations. B_0, $\dot{B}_0$, and E refer to the $\dot{\rho}=0$ point.

Imag	θ	B_0 (gauss)	$\dot{B}_0$ (g/sec $\times10^{-6}$)	E Mev	ev/rev
2550	118.0°	9320	1.100	280	567
2400	115.4°	9060	0.952	272	491
2300	114.1°	8790	0.870	263	448
2200	112.2°	8600	0.777	258	400
2100	110.6°	8350	0.697	251	359
2000	108.9°	8060	0.610	242	315
1900	106.7°	7790	0.518	234	267
1800	105.4°	7490	0.453	224	234
1700	103.5°	7200	0.382	216	197
1600	101.3°	6880	0.301	206	155

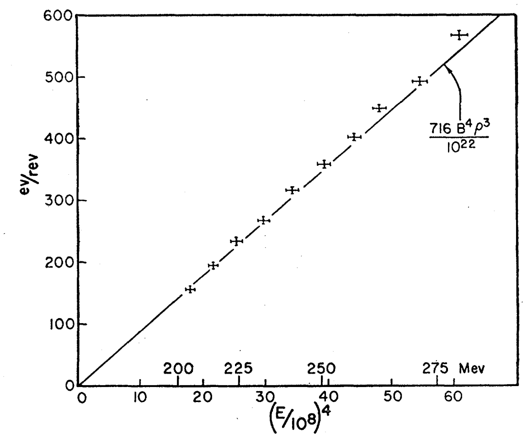


FIG. 6. Measured energy losses in ev/rev *vs* E^4. The straight line is the classical radiation calculation.

only the first few half-cycles of the modulation pattern are useful, even though eight or ten complete cycles have been observed. $\dot{\rho}$ may also be measured by observing x-ray production time *vs* target position. This leads to somewhat uncertain results also, because the electrons are spread out in orbit radius over a few millimeters and it takes several microseconds for the beam to sweep across the target. Consistent results are obtained by measuring times corresponding to the end of the x-ray pulse. Presumably the electrons which arrive at the target last are those which are not executing oscillations about the equilibrium orbit, and consequently are those which give a correct indication of the equilibrium orbit motion.

E. Measurement of "n"

"n" may be determined directly by observing the orbit shrink rates at symmetrical points about the peak of the magnetic cycle. In Eq. (5) one sees that $\dot{E}/E$ must be the same at the two points and $\dot{B}_0/B_0$ is the same in absolute value but is positive at the early point and negative at the late point. Equating $\dot{E}/E$ at the two points we have

$$(1-n)=2(1-k)\frac{|\dot{B}_0|}{B_0}\frac{\rho}{|\dot{\rho}_E-\dot{\rho}_L|}, \quad (8)$$

where $\dot{\rho}_E$ and $\dot{\rho}_L$ are the orbit shrink rates at the symmetrical points early and late relative to the peak of the magnetic cycle.

IV. RESULTS

The results of the measurements to determine "k" from Eq. (7) using the measured synchrotron frequency of 47.3 Mc/sec are shown in Table I.

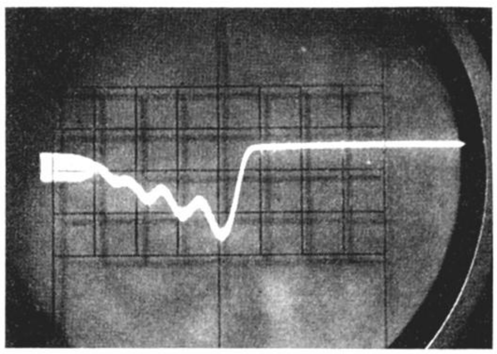
The numbers used to determine "n" are shown in Table II.

Figure 5 shows a plot of orbit shrink velocity *vs* magnetic cycle angle. The solid curve represents the calculated velocity dependence for an "n" of 0.58 and the measured B_0 and $\dot{B}_0$ values, assuming that the only energy loss mechanism is by radiation according to classical theory.

The most precise results are those from the $\dot{\rho}=0$ determinations. These data are tabulated in Table III and plotted in Fig. 6. The straight line in Fig. 6 represents the classical radiation loss. In these measurements, the angle corresponding to $\dot{\rho}=0$ is determined with a probable uncertainty of $\pm 0.1°$. At any given angle, B_0 and $\dot{B}_0$ have an estimated probable uncertainty of 0.5 percent. The synchrotron frequency is known to 0.1 Mc/sec, so that the uncertainty from this source is negligible. $(1-k)$ is known to ± 0.3 percent. It should be noted that the primary method used to find the $\dot{\rho}=0$ points results in a measurement of the corresponding magnetic angle at an orbit radius smaller than the equilibrium synchrotron orbit. One can show that $\theta(\dot{\rho}=0)$ has a slight dependence on orbit radius, depending on "n," but over the range of radii of interest $\Delta\theta$ is no more than $0.1°$. Experimentally, no variation at all can be observed. All results have accordingly been reduced to the synchrotron equilibrium orbit radius. Total energy loss rates are determined with an estimated probable uncertainty of ± 1 percent. No dependence of energy loss per electron per revolution on the number of electrons in the bunch has been observable, i.e., no coherent effects have been detected.

V. CONCLUSIONS

From the plot of Fig. 6 it is evident that nearly all the energy loss is accounted for by the classical radiation loss. The measured points lie slightly above the radiation line, but perhaps not significantly so. The agreement with the classical theory is substantially better than in the previously reported results.[11] The difference lies in the more precise timing during the magnetic cycle in these measurements.



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