

We are indebted to Dr. W. H. Barkas for suggesting this problem to us and for his many helpful discussions. The assistance of Dr. R. S. White, Mr. J. W. McLeod, Mr. George C. McFarland, and the synchrotron crew is gratefully acknowledged.

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⁴ P. G. Hoel, *Introduction to Mathematical Statistics* (John Wiley and Sons, Inc., New York, 1947), p. 66.

The Total Cross Sections of 37-Mev Pions in Hydrogen

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(Received March 26, 1953)

THE total cross sections for the processes $\pi^+ + p$ and $\pi^- + p$ have been measured at a laboratory interaction energy for the pions of 37 ± 6 Mev, using the method previously described.¹

The "slow pions" and proton contamination described in reference 1 have been eliminated from the pion beam by the use of a second focusing magnet, and the pulse-height resolution of the scintillation counter telescope has been improved. These facts enable the analysis of the data to be more nearly independent of the characteristics of the pulse-height circuitry than was previously possible. The second focusing magnet, in conjunction with higher cyclotron operating levels, has increased the incident pion beam intensity to 13 000 π^+ /min and 5000 π^- /min through a 1 in. \times 1 in. \times 5 $\frac{1}{2}$ in. telescope.

A summary of the results is shown in Table I. All numbers have

TABLE I. Measured and calculated cross sections of 37-Mev pions in hydrogen.

	Measured cross section (millibarns)	Calculated total cross sections (millibarns)		
		isotropic	sin ² θ	cos ² θ
π^+	16.0 \pm 1.0	20.8 \pm 1	18.8 \pm 1	26.4 \pm 2
π^-	17.3 \pm 1.4	21.0 \pm 2	19.4 \pm 2	25.3 \pm 3

been corrected for the muon and electron contamination of the beam. The errors indicated are the standard deviations owing to counting statistics or fluctuations from the mean, whichever is larger. Since the π^- cross section includes charge exchange scattering, the value of 5 ± 1.5 mb obtained by Roberts and Tinlot² at approximately this same energy has been used in calculating the total cross sections.

The π^+ and π^- data are each based on 5.6×10^6 incoming pions. The measured cross section is that obtained from the attenuation of the pion beam by the CH₂ and carbon scatterers. It is then corrected for the solid angle subtended by the anticoincidence counter in order to obtain the total cross section. This is done for three assumed angular distributions in the center-of-mass system.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

¹ Barnes, Clark, Perry, and Angell, *Phys. Rev.* **87**, 669 (1952).

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A Binding Energy Calculation on He⁴ with Single-Particle Wave Functions

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(Received March 30, 1953)

A PHENOMENOLOGICAL two-body interaction potential of the form

$$V(r) = V_0 \frac{T_{12}}{3} \left\{ \left[1 - \frac{g}{2} + \frac{g}{2} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \right] \frac{e^{-r/r_c}}{r/r_c} + \gamma S_{12} \frac{e^{-r/r_c}}{r/r_c} \right\},$$

TABLE I. Contributions of the single-particle states to the binding energy of He⁴.

State	Percent	Contribution to binding energy ^a	Parameter value
Ψ_1	75.73	15.72 Mev	$\theta_1 = 0.665$
Ψ_2	21.88		$\theta_2 = 0.88$
Ψ_3	2.39	5.12 Mev	$\theta_3 = 0.58$
		20.84 Mev	

^a The increase in the S state binding energy due to the admixture of Ψ_2 was 4.27 Mev.

where T_{12} determines the exchange character of the forces, has been shown, for suitable choice of the interaction parameters, to give a satisfactory description of low energy scattering processes and the deuteron properties,¹ and also to fit the triton binding energy.^{2,3} It has further been shown by Irving⁴ to give rise to a reasonable value for the binding energy of He⁴. This Letter will show that results for He⁴ comparable to those of Irving may be obtained on the basis of the nuclear shell model when inter-configurational mixing is taken into account.

We use a symmetric interaction [$T_{12} = (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$] rather than one of neutral or Serber type, since, although for the triton and alpha-particle the precise form of the exchange character is immaterial,⁵ for the heavier light nuclei considerable excess binding results with the neutral and Serber interactions. [See work of Kronheimer⁶ on Be⁹, and of the writer on Ca⁴⁰.] A variation method is employed, with wave functions constructed by the methods of Jahn *et al.*,⁷ and the energy matrix elements calculated using the techniques described by Elliott.⁸ The interaction parameter set used is that calculated by Pease and Feshbach⁹ to give 100 percent binding for the triton. Suitably adjusted for the symmetric case this is:

$$V_0 = 46.96 \text{ Mev}, \quad r_t = 1.70 \times 10^{-13} \text{ cm}, \quad g = 1.985, \\ r_t/r_c = 1.44, \quad \gamma = 0.5085.$$

Calculations were first made on the $1s^4$ state arising from the lowest configuration $(1s)^4$, using two types of single-particle radial wave function, *viz.*, oscillator well: $u_{1s}(r) \sim \exp(-\frac{1}{2}r^2/r_c^2\theta^2)$; and "exponential": $u_{1s}(r) \sim \exp(-\frac{1}{2}\gamma r/r_c)$; θ , γ being the respective variation parameters. These calculations showed the oscillator functions to be slightly superior to the exponential functions, but in each case the $(1s)^4$ configuration gave an inadequate description of the He⁴ ground state.

In taking inter-configurational mixing into account, the states $(9s)^2(2p)^2[4]^{11}S$ and $(1s)^2(2p)^2[22]^{15}D$ were considered, together with the $(1s)^4[4]^{11}S$ state. The symmetry characters here label the irreducible representations of the symmetric group S_4 to which the orbital parts of the wave functions belong. A further state which preliminary calculations indicated to be of importance comparable to that of the $(1s)^2(2p)^2[4]^{11}S$ state is the state $(1s)^3(2s)[4]^{11}S$. This state is not however considered here. Writing $1s \equiv s$, the wave functions for the remaining states were found to be:

$$\Psi_1 \equiv \Psi(s^4[4]^{11}S) = \sqrt{\frac{1}{2}} \Phi(s^2[2]^{13}S, s^2[2]^{13}S, {}^{11}S) \\ - \sqrt{\frac{1}{2}} \Phi(s^2[2]^{31}S, s^2[2]^{31}S, {}^{11}S),$$

$$\Psi_2 \equiv \Psi(s^2p^2[4]^{11}S) = \sqrt{\frac{1}{6}} \left[\sqrt{\frac{1}{2}} \Phi(s^2[2]^{13}S, p^2[2]^{13}S, {}^{11}S) \right. \\ \left. - \sqrt{\frac{1}{2}} \Phi(s^2[2]^{31}S, p^2[2]^{31}S, {}^{11}S) \right] \\ + \sqrt{\frac{1}{6}} \left[\sqrt{\frac{1}{2}} \Phi(p^2[2]^{13}S, s^2[2]^{13}S, {}^{11}S) \right. \\ \left. - \sqrt{\frac{1}{2}} \Phi(p^2[2]^{31}S, s^2[2]^{31}S, {}^{11}S) \right] \\ + \sqrt{\frac{2}{3}} \left[\sqrt{\frac{1}{2}} \Phi(sp[2]^{13}S, sp[2]^{13}S, {}^{11}S) \right. \\ \left. - \sqrt{\frac{1}{2}} \Phi(sp[2]^{31}S, sp[2]^{31}S, {}^{11}S) \right],$$

$$\Psi_3 \equiv \Psi(s^2p^2[22]^{15}D) = \sqrt{\frac{1}{6}} \Phi(s^2[2]^{13}S, p^2[2]^{13}D, {}^{15}D) \\ + \sqrt{\frac{1}{6}} \Phi(p^2[2]^{13}D, s^2[2]^{13}S, {}^{15}D) \\ - \sqrt{\frac{1}{6}} \Phi(sp[2]^{13}P, sp[2]^{13}P, {}^{15}D) \\ - \sqrt{\frac{1}{6}} \Phi(sp[11]^{33}P, sp[11]^{33}P, {}^{15}D).$$