expected to be valid. Hence the extent to which the angular distribution can be analyzed in terms of one j value forms a quantitative test of shell model assumptions.

A similar analysis in terms of unique L and S values for the initial and excited nuclear states would test the validity of the $L-S$ coupling scheme.

In this letter we give formulas relating the unique $j-j$ and unique $L-S$ cases to the channel-spin formalism. We give also some explicit γ-distributions of interest on the shell model hypothesis.[4]

The distribution function, Eq. (6) of reference 3, for a unique j may be expanded in terms of Legendre polynomials. Using the same notation for angular momenta as in reference 3 (see also caption of Table I),

$$W(\theta)=1+\Sigma_\nu \eta_\nu(jJ_iJ_e)F_\nu(LJ_fJ_e)P_\nu(\cos\theta),$$

where $\nu=2, 4, \cdots$, the F_ν are tabulated elsewhere,[5] and

$$\eta_\nu=(2J_e+1)^{\frac{1}{2}}(2j+1)(-)^{J_i-J_e-\frac{1}{2}}C_{0\frac{1}{2}-\frac{1}{2}}^{\nu j i}W(J_eJ_ejj;\nu J_i).$$

Comparing this expression with the general channel-spin formula [see Eq. (18), reference 6] we find that the corresponding proportional contribution of channel-spin $s(=J_i\pm\frac{1}{2})$ is given by $|A(s)|^2$, where

$$A(s)\sim(2j+1)(2s+1)^{\frac{1}{2}}W(J_ijsl;J_e\tfrac{1}{2}).$$

In the case of $L-S$ coupling one obtains[4]

$$A(s)\sim(2s+1)^{\frac{1}{2}}W(S_esL_el;L_iJ_e)W(L_iS_is\tfrac{1}{2};J_iS_e),$$

where $(J_iL_iS_i)$ and $(J_eL_eS_e)$, respectively, describe the initial nuclear state and the state after capture.

Thus angular distributions calculated with the channel-spin approach can be used to express the results of unique j and unique $L-S$ assumptions. Unique l is assumed since measurements are to be made in coincidence with outgoing nucleons of a definite l-peak.

Explicit angular distributions for unique j values depend on 5 parameters $(J_iJ_eLJ_f)$, but shell model predictions reduce the number of independent variables in some cases. Thus:

(a) initial nucleus even-even, final even-odd. $J_i=0$, $J_e=j$. If the ground and excited states form a doublet, $J_e=l-\frac{1}{2}$, $J_f=l+\frac{1}{2}$ and $L=1$.

(b) even-odd to even-even. $j=J_i$ if the captured nucleon enters the same orbit as the original "odd" one and forms an excited state by coupling with it. Then $J_e=2$ (or 4), $J_f=0$, (or 2) and $L=2$.

Writing

$$W(\theta)=1+\Sigma_\nu A_\nu P_\nu(\cos\theta),$$

we list in Table I the coefficients A_ν, for $j=3/2$ and $5/2$. ($j=\frac{1}{2}$ gives isotropic distributions always.)

The author is indebted to Dr. J. A. Spiers for advice on this problem.

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The Ratio of Plateau to Minimum Grain Density for Electrons

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RECENT experimental and theoretical work concerning the relativistic increase in ionization of a charged particle moving through matter is discussed elsewhere.[1] The purpose of the present study was to compare the grain density at saturation ionization (plateau) with that at minimum ionization for negative electrons in G5 emulsion.

A single 1 in.×3 in., 200-micron Ilford G5 emulsion was exposed to 300-Mev and 3-Mev electrons. The former were obtained by magnetic analysis of pairs produced from a tantalum converter in the bremsstrahlung beam of the Berkeley synchrotron. The latter were obtained by magnetic analysis of an electron linear accelerator beam. Primary electrons entering the emulsion at ~5 degrees to the emulsion air surface penetrated the emulsion in opposite directions and formed two adjacent bands of electron tracks. Grain counts were recorded in terms of a standard reticule unit, which, under the magnification used (~2500×) was 32 microns in length. For the purposes of comparing grain counts at the two energies, the following criteria were adopted:

(1) A grain was counted as one unit regardless of size.

(2) All grain counts were taken in a layer of developed emulsion between 40 and 10 microns from the emulsion air surface.

(3) Track sections were accepted for grain counting only if their dip angle was in the same sense as would be expected from the exposure set-up.

(4) Consistent with (3), only portions of track were grain counted where the angle of dip in the developed emulsion was between 0 and arc tan 0.059.

The grain density determined on the basis of criterion (1) is essentially proportional to that obtained when clumps are resolved into individual grains for the thin tracks used here. The fourth criterion allows one to neglect the error in track length because of the uncertainty in the shrinkage factor. By grain counting tracks of ~300-Mev electrons as they penetrated the emulsion, the gradient of development in the acceptable layer [criterion (2)] was found to be less than one percent. Because of the large multiple scattering of ~3-Mev electrons, and in light of criterion (4), it was not feasible to grain count successive intervals of these tracks. Instead, the following "field of view" method was adopted: Grain counts per standard unit were taken for all portions of ~3-Mev tracks in a field of view lying within ±22.5 degrees of the mean entrance angle. Tracks of ~300-Mev electrons were first grain counted by taking successive intervals of given tracks (7207 grains). Then the ~3-Mev tracks were grain counted by the "field of view" method. Finally, ~300-Mev tracks were grain counted by the latter method (7197 grains). The means and standard deviations of the two sets of ~300-Mev data agreed within the statistical errors. The error in track length due to multiple scattering over a standard unit is negligible. All counts were taken by one observer whose reproducibility was found to be better than one percent.

Our results are given in Table I. The two sets of ~300-Mev data are combined. The mean energies have been calculated taking into account the energy loss in the acceptable layer due to radiation[2] and ionization.[3] The ionization of 2.8-Mev electrons differs from minimum ionization by less than one percent.[3] From the standard deviations of Table I it is evident that the spreads of the distributions of counted grains are less than those given by Poisson distributions. This result is anticipated since the probability of success in rendering a grain developable is not vanishingly small. Thus, the conditions for a Poisson distribution are not met. The standard errors have been computed from the measured standard deviations in the usual way.[4] Assuming proportionality between grain density and ionization, the ratio in Table I is also that of saturation to minimum ionization.

TABLE I. Grain density measurements.

Mean energy (Mev)	No. of grains counted	Mean grain density with standard error of mean (grains/32μ)	Standard deviation (grains/32μ)	Ratio of mean grain densities with standard error
293	14 404	8.61±0.05	2.13	1.087±0.010
2.8	7 002	7.92±0.06	1.95	

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The Total Cross Sections of 37-Mev Pions in Hydrogen

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THE total cross sections for the processes $\pi^+ + p$ and $\pi^- + p$ have been measured at a laboratory interaction energy for the pions of 37 ± 6 Mev, using the method previously described.[1]

The "slow pions" and proton contamination described in reference 1 have been eliminated from the pion beam by the use of a second focusing magnet, and the pulse-height resolution of the scintillation counter telescope has been improved. These facts enable the analysis of the data to be more nearly independent of the characteristics of the pulse-height circuitry than was previously possible. The second focusing magnet, in conjunction with higher cyclotron operating levels, has increased the incident pion beam intensity to 13 000 π^+/min and 5000 π^-/min through a 1 in.$\times$1in.$\times 5\frac{1}{2}$ in. telescope.

A summary of the results is shown in Table I. All numbers have been corrected for the muon and electron contamination of the beam. The errors indicated are the standard deviations owing to counting statistics or fluctuations from the mean, whichever is larger. Since the π^- cross section includes charge exchange scattering, the value of 5 ± 1.5 mb obtained by Roberts and Tinlot[2] at approximately this same energy has been used in calculating the total cross sections.

TABLE I. Measured and calculated cross sections of 37-Mev pions in hydrogen.

	Measured cross section (millibarns)	Calculated total cross sections (millibarns)		
		isotropic	$\sin^2\theta$	$\cos^2\theta$
π^+	16.0 ± 1.0	20.8 ± 1	18.8 ± 1	26.4 ± 2
π^-	17.3 ± 1.4	21.0 ± 2	19.4 ± 2	25.3 ± 3

The π^+ and π^- data are each based on 5.6×10^6 incoming pions. The measured cross section is that obtained from the attenuation of the pion beam by the CH_2 and carbon scatterers. It is then corrected for the solid angle subtended by the anticoincidence counter in order to obtain the total cross section. This is done for three assumed angular distributions in the center-of-mass system.

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A Binding Energy Calculation on He^4 with Single-Particle Wave Functions

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A PHENOMENOLOGICAL two-body interaction potential of the form

$$V(r) = V_0 \frac{T_{12}}{3}\left\{\left[1 - \frac{g}{2} + \frac{g}{2}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\right]\frac{e^{-r/r_c}}{r/r_c} + \gamma S_{12}\frac{e^{-r/r_t}}{r/r_t}\right\},$$

where T_{12} determines the exchange character of the forces, has been shown, for suitable choice of the interaction parameters, to give a satisfactory description of low energy scattering processes and the deuteron properties,[1] and also to fit the triton binding energy.[2,3] It has further been shown by Irving[4] to give rise to a reasonable value for the binding energy of He^4. This Letter will show that results for He^4 comparable to those of Irving may be obtained on the basis of the nuclear shell model when inter-configurational mixing is taken into account.

TABLE I. Contributions of the single-particle states to the binding energy of He^4.

State	Percent	Contribution to binding energy[a]	Parameter value
Ψ_1	75.73	15.72 Mev	$\theta_1 = 0.665$
Ψ_2	21.88		$\theta_2 = 0.88$
Ψ_3	2.39	5.12 Mev	$\theta_3 = 0.58$
		20.84 Mev	

[a] The increase in the S state binding energy due to the admixture of Ψ_2 was 4.27 Mev.

We use a symmetric interaction [$T_{12} = (\tau_1 \cdot \tau_2)$] rather than one of neutral or Serber type, since, although for the triton and alpha-particle the precise form of the exchange character is immaterial,[5] for the heavier light nuclei considerable excess binding results with the neutral and Serber interactions. [See work of Kronheimer[6] on Be^9, and of the writer on Ca^{40}.] A variation method is employed, with wave functions constructed by the methods of Jahn *et al.*,[7] and the energy matrix elements calculated using the techniques described by Elliott.[8] The interaction parameter set used is that calculated by Pease and Feshbach[2] to give 100 percent binding for the triton. Suitably adjusted for the symmetric case this is:

$$V_0 = 46.96 \text{ Mev}, \quad r_t = 1.70 \times 10^{-13} \text{ cm}, \quad g = 1.985,$$
$$r_t/r_c = 1.44, \quad \gamma = 0.5085.$$

Calculations were first made on the ^{11}S state arising from the lowest configuration $(1s)^4$, using two types of single-particle radial wave function, *viz.*, oscillator well: $u_{1s}(r) \sim \exp(-\frac{1}{2}r^2/r_c^2\theta^2)$: and "exponential": $u_{1s}(r) \sim \exp(-\frac{1}{2}\gamma r/r_c)$; θ, γ being the respective variation parameters. These calculations showed the oscillator functions to be slightly superior to the exponential functions, but in each case the $(1s)^4$ configuration gave an inadequate description of the He^4 ground state.

In taking inter-configurational mixing into account, the states $(9s)^2(2p)^2[4]^{11}S$ and $(1s)^2(2p)^2[22]^{15}D$ were considered, together with the $(1s)^4[4]^{11}S$ state. The symmetry characters here label the irreducible representations of the symmetric group S_4 to which the orbital parts of the wave functions belong. A further state which preliminary calculations indicated to be of importance comparable to that of the $(1s)^2(2p)^2[4]^{11}S$ state is the state $(1s)^3(2s)[4]^{11}S$. This state is not however considered here. Writing $1s \equiv s$, the wave functions for the remaining states were found to be:

$$\Psi_1 \equiv \Psi(s^4[4]^{11}S) = \sqrt{\tfrac{1}{2}}\Phi(s^2[2]^{13}S, s^2[2]^{13}S, {}^{11}S) - \sqrt{\tfrac{1}{2}}\Phi(s^2[2]^{31}S, s^2[2]^{31}S, {}^{11}S),$$

$$\begin{aligned}\Psi_2 \equiv \Psi(s^2p^2[4]^{11}S) = {} & \sqrt{\tfrac{1}{6}}[\sqrt{\tfrac{1}{2}}\Phi(s^2[2]^{13}S, p^2[2]^{13}S, {}^{11}S) - \sqrt{\tfrac{1}{2}}\Phi(s^2[2]^{31}S, p^2[2]^{31}S, {}^{11}S)] \\ & + \sqrt{\tfrac{1}{6}}[\sqrt{\tfrac{1}{2}}\Phi(p^2[2]^{13}S, s^2[2]^{13}S, {}^{11}S) - \sqrt{\tfrac{1}{2}}\Phi(p^2[2]^{31}S, s^2[2]^{31}S, {}^{11}S)] \\ & + \sqrt{\tfrac{2}{3}}[\sqrt{\tfrac{1}{2}}\Phi(sp[2]^{13}S, sp[2]^{13}S, {}^{11}S) - \sqrt{\tfrac{1}{2}}\Phi(sp[2]^{31}S, sp[2]^{31}S, {}^{11}S)],\end{aligned}$$

$$\begin{aligned}\Psi_3 \equiv \Psi(s^2p^2[22]^{15}D) = {} & \sqrt{\tfrac{1}{6}}\Phi(s^2[2]^{13}S, p^2[2]^{13}D, {}^{15}D) \\ & + \sqrt{\tfrac{1}{6}}\Phi(p^2[2]^{13}D, s^2[2]^{13}S, {}^{15}D) \\ & - \sqrt{\tfrac{1}{6}}\Phi(sp[2]^{13}P, sp[2]^{13}P, {}^{15}D) \\ & - \sqrt{\tfrac{1}{2}}\Phi(sp[11]^{33}P, sp[11]^{33}P, {}^{15}D).\end{aligned}$$