Our observations can be explained by a theory in which the particles are produced multiply in the initial interaction.[5] The multiplicity apparently does not seem to increase by a large factor through subsequent interactions inside the target nucleus; this is probably because the shower particles do not interact individually inside the nucleus because of their extreme collimation.[1,6,7] The fluctuations from the average behavior as noted above could be the result of large fluctuations in the angular distribution of the shower particles. A few shower particles of the first or possibly the second generation projected at angles considerably greater than the average could be effective in starting independent cascades inside the nucleus, which could increase the observed multiplicity by a considerable factor.

We have also noted the occurrence of both high and low energy pairs of charged particles produced by neutral radiation, in observing the passage of the high energy showers through the block of stripped emulsions. The most reasonable assumption is that these pairs are electron-positron pairs produced by photons; though our statistics are too low to draw any definite conclusion, the contribution of bremsstrahlung according to Schiff's theory[8] seems to be too low to account for the low energy pairs.

We are indebted to the Aero Medical Field Laboratory, Holloman Air Force Base for their cooperation in obtaining successful balloon flights. We wish to thank Miss Barbara Hull and Miss Katherine Merry for their assistance in scanning the plates.

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Nonuniform Track Shrinkage in Nuclear Emulsions*

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SEVERAL Eastman NTA and NTB nuclear track plates have been exposed to monoenergetic alpha-particles from Po^{210} plated silver foils, and a study has been made of the resulting alpha-tracks as a function of the angle of incidence.

Observation of the tabulated data indicated that the average track length increased as a function of the dip angle θ. Experiments by Rotblat and Tai[1] with lithium-loaded emulsions showed that shrinkage was uniform at all depths of the emulsion. Also the shrinkage factor for alpha-tracks was a function of both the gelatin content and the dip angle. They concluded that the shrinkage factor remained fairly constant with angles of dip up to 25–30°. Roberts,[2] in studying $n-\alpha$ reactions, measured tracks with values of θ up to 40° and found that resolution was considerably improved by limiting θ to $\leq 20°$.

Figure 1 is a plot of the average track range in microns *versus* the angle of dip in the processed emulsions. The empirically determined shrinkage factor 2.7 ± 0.3 was used in these calculations. Both NTA and NTB plates show substantially the same results with the average range of the particles continuing to increase with increasing angles of dip. Extrapolation of the curve to zero yields the range of the alpha particles without shrinkage error. This value, 21.3μ, agrees within experimental error with Steigert's[3] empirical value of 21.2μ as well as his theoretical value of 21.3μ for NTA plates.

Thus, the usual method of correcting for emulsion shrinkage by multiplying the vertical track projection by the shrinkage factor

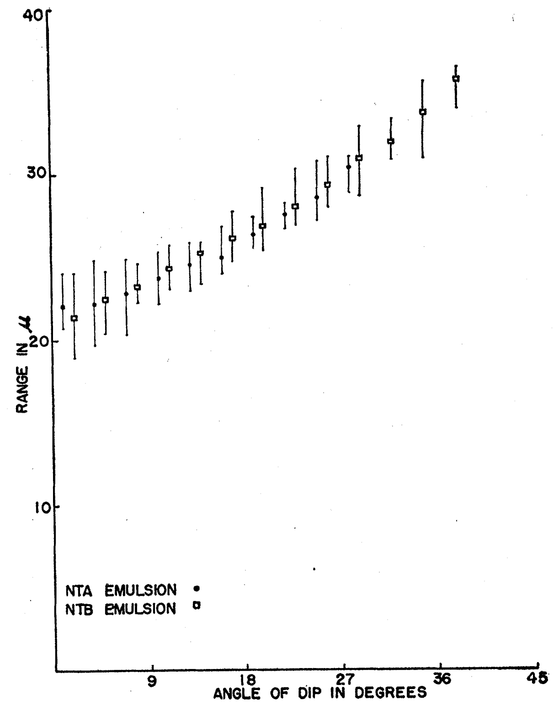

FIG. 1. Average alpha-track range *vs* angle of dip in processed emulsions.

should be restricted to angles of dip less than 6° for heavily ionizing particles if the error of measurement is to be less than 2 percent.

The final criterion for selection was that each alpha track should lie entirely within the depth of focus of the microscope which was approximately 0.5μ, which is equivalent to a variation of less than 2° in the angle of dip. The maximum error introduced by measuring only the horizontal component, with the above limitation, is 0.1μ. This modified procedure resulted in histograms having a maximum dispersion of 5μ and a half-width at half-maximum of 0.5μ.

* Work performed at the Oak Ridge Institute of Nuclear Studies, Oak Ridge, Tennessee.
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Rotational States in Even-Even Nuclei

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IN a recent note,[1] an interpretation of the short-lived $E2$ isomers has been suggested in terms of rotational states of the deformed nucleus. Empirical evidence is rapidly accumulating on the low energy spectra of even-even nuclei;[2–4] the purpose of the present note is to call attention to the extensive support which exists in these data for the above interpretation, and to suggest its usefulness in the analysis of decay schemes.

In the model describing the nucleus in terms of the coupled particle motion and surface oscillations, low-lying rotational states are associated with the large deformations expected in regions with many particles outside of closed shells. In such regions, the rotational spectrum is expected to be given rather accurately by the simple expression[1]

$$E_I=\frac{\hbar^2}{2\mathcal{J}}I(I+1),\quad I=0,2,4,6,\ \text{even parity} \tag{1}$$

where the moment of inertia $\mathcal{I}$ is proportional to the square of the deformation. The relative order of magnitude of the deviations from this formula is rotational energy divided by the potential energy of the deformed surface.[5] Typical among the correction terms is the vibration-rotation interaction, which contributes a term of the form

$$\Delta E_I = -\text{const}\, I^2(I+1)^2, \tag{2}$$

well known from molecular spectra. In the vicinity of closed shells, such correction terms may essentially modify the spectrum, and a more detailed treatment of the coupled particle surface system is required.

The following points summarize some of the relevant evidence on the excited states of even-even nuclei.

I. Energy trends of first excited states.—Figure 1 shows the excitation energies[6] in the region of the rather heavy elements, which is dominated by the doubly closed shell at $Z=82$ and $N=126$. The evidence is consistent with a (2+) assignment for all the states in the figure.[4] The decreasing energy as we move away from closed shells is an immediate consequence of the increasing deformation produced by the extra particles and also observed in the quadrupole moments.[7] The rapid decrease for the first few particles added to closed configurations, which develops into a rather flat minimum, can be understood from the fact that the particle states with large deformative power are the first to be filled, while in the middle of shells the last added particles are less coupled to the deformation. The minimum in the very heavy elements is lower by a factor of two than that in the rare earth region. This is accounted for by the variation with A of the mass parameter B in the moment of inertia,[1] and implies similar deformations in the two regions.

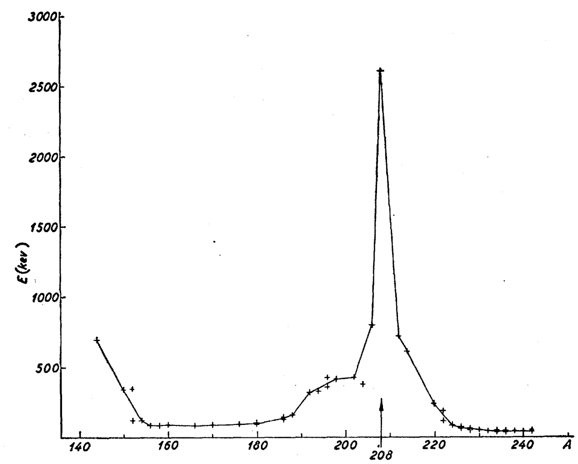

FIG. 1. First excited state in even-even nuclei with $A > 140$. The data are taken from references 3 and 4. The reported 374-kev state in Pb^{204} falls somewhat off the regular curve. [A. H. Wapstra, Physica **18**, 799 (1952).] It may be of a different type from the rest, as is also indicated by its excessively long comparative half-life. (See Wapstra, above, and reference 1.) Some uncertainty exists regarding the mass assignment of this activity. (We are indebted to Dr. P. Stähelin for comments on the Pb^{204} activity.)

II. Lifetimes.—The transition probabilities, more than a hundred times larger than single particle estimates,[1] which are observed for the levels in the rare earth region, furnish direct evidence for the collective nature of the excitation. The interpretation of the states as rotational levels permits one to calculate from the lifetimes the intrinsic quadrupole moments of the deformed surface, which are found to be just of the magnitude of those deduced from spectroscopic data.[1]

III. Higher rotational states.—In regions of large deformations, one expects a number of higher members of the rotational spectrum (1). Table I lists evidence on such higher states for the nuclei of Fig. 1. It is seen that in the regions where Eq. (1) is expected to hold, there is indeed evidence for higher rotational states with the predicted energies. The minor deviations of the energy ratios from the limiting strong coupling values (1) show a systematic behavior which may be attributed to a term of the form (2), but the empirical data are not yet sufficiently accurate to warrant a detailed analysis of the effect.

TABLE I. Higher rotational states in even-even nuclei with $A > 140$. The table lists the energies (in kev) of the (2+) states and of the tentatively suggested (4+) and (6+) states. In many cases, the available empirical evidence for the character of the states in columns three and five is inconclusive and the suggested assignment is based largely on the energy ratio of the observed gamma-rays.

Nucleus	E_2	E_4	E_4:E_2	E_6	E_6:E_2	Ref.
$_{62}Sm^{150}$	337	777	2.3			4
$_{72}Hf^{176}$	89	270	3.0	540	6.1	2
$_{72}Hf^{180}$	93	307	3.3	637	6.9	2
$_{82}Pb^{208}$	2614	3200	1.2			4
$_{88}Ra^{226}$	70	215	3.1	450	6.4	a
$_{90}Th^{228}$	58	187	3.2	371	6.4	b
$_{90}Th^{230}$	50	167	3.3			3
$_{94}Pu^{238}$	43	146	3.4			3
Eq. (1)			3.33		7.00	

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For $_{62}Sm^{150}$, and especially for $_{82}Pb^{208}$, the small E_4:E_2 ratios as well as the large value of E_2 confirm the major deviations from the strong coupling situation expected in the neighborhood of closed shells.

It is characteristic of the rotational spectrum given by (1) that the excitation of a high member is followed by a cascade of $E2$ gamma-transitions with energy values in the ratio 3:7:11:15···, with no cross-overs. Examples of such cascades are given in Fig. 2.

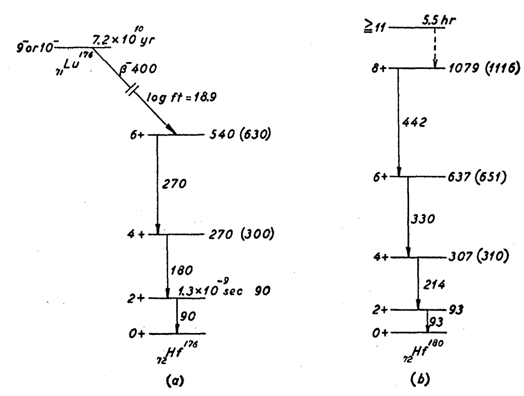

FIG. 2. Suggested decay schemes of $_{71}Lu^{176}$ and $_{72}Hf^{180}$. The gamma-ray energies as well as the β-decay data of Lu^{176} are taken from reference 2. The excitation energies listed in parentheses are obtained from Eq. (1) adjusted to give the energy of the first excited state. All energies are given in kev.

$_{72}Hf^{176}$: The β-decay of $_{71}Lu^{176}$ leads to a 540-kev state[2] of $_{72}Hf^{176}$ which decays in three steps to the ground state [see Fig. 2(a)]. The last transition (89 kev) is known to be of $E2$ type with a strongly enhanced transition probability.[1] The classification of the levels is consistent with the fact that the β-decay of Lu^{176} (measured spin ≥ 7) goes only to the highest (6+) member. The log (ft) value suggests a third-forbidden transition[2] which would lead to the assignment $I=9$ or 10, odd parity, for the Lu^{176} ground state.[8]

$_{72}Hf^{180}$: The decay of the 5.5-hr isomeric state of $_{72}Hf^{180}$ contains four gamma-rays which appear to be in cascade and the last of which (93 kev) has been identified as $E2$.[2] The classification in

Fig. 2(b) assigns an (8+) character to the 1079-kev member of the family and probably implies a spin of at least 11 for the 5.5-hr isomeric state.

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[8] P. F. A. Klinkenberg [Physica **17**, 715 (1951)] has suggested the assignment (10−) for Lu^{176} from an analysis of its magnetic moment.

Recoil Effects in Meson-Nucleon Scattering

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THE asymmetry about 90° of the pion-nucleon scattering cross section shows a considerable even state contribution in addition to a strong P state interaction. We have examined the even state scattering to be expected for pseudoscalar mesons coupled symmetrically to a nucleon with a coupling of the form

$$\sum_{j=1}^{3} g\tau_j \boldsymbol{\sigma}\cdot(\nabla\phi_j - i\mathbf{V}\pi_j). \quad (1)$$

$\mathbf{V}$ is the average velocity of the nucleon before and after meson emission (absorption) so that (1) is Hermitean. The $\boldsymbol{\sigma}\cdot\mathbf{V}$ term[1] is necessary if the interaction is to be invariant under simple Galilean transformations.[2]

To terms in ω/M, the S and D wave phase shifts obtained from (1) are the same for perturbation theory and for the extended source model of Chew.[3] They do not contribute to the noncharge-exchange π^- scattering. The P wave phase shifts, on the other hand, depend critically on the model, but are only slightly affected by recoil effects.

Recoil corrections to the $\boldsymbol{\sigma}\cdot\nabla\phi$ part of the interaction Hamiltonian give the S and D wave phase shifts shown in Table I.

TABLE I. S and D wave recoil phase shifts. Even wave phase shifts in degrees $\times 4\pi/g^2$ obtained from (1). $\eta=k/\mu$ and $\rho=k/\omega$. The superscript is twice the isotopic spin, and the subscript is the total angular momentum.

Phase shift	$S_{1/2}^{(3)}$	$S_{1/2}^{(1)}$	$D_{3/2}^{(3)}$	$D_{3/2}^{(1)}$	$D_{5/2}^{(3)}$	$D_{5/2}^{(1)}$
Recoil from						
$\boldsymbol{\sigma}\cdot\nabla\phi$	$-5.8\eta^3\rho^2$	$2.9\eta^3\rho^2$	$1.2\eta^3\rho^2$	$-0.58\eta^3\rho^2$	$-4.6\eta^3\rho^2$	$2.3\eta^3\rho^2$
Galilean term	$35\eta^3$	$-17\eta^3$	0	0	0	0

The D wave phase shifts, although numerically smaller, give the main contribution (about twice that of the S wave) to the asymmetry of the cross section about 90°. In perturbation theory the calculated $d\sigma(180°)/d\sigma(0°)$ is independent of $g^2/4\pi$ and is in rough agreement with the asymmetry from the best $A+B\cos\theta+C\cos^2\theta$ fit to the experimental π^+ scattering on protons.[4] This is no longer true for Chew's model with $g^2/4\pi\approx 0.2$ to give best agreement with experiment. In this case $d\sigma(180°)/d\sigma(0°)\approx 1.3$ for 78–135 Mev incident π^+ mesons.

Inclusion of the Galilean term $\boldsymbol{\sigma}\cdot\mathbf{V}$ in (1) gives an S wave phase shift opposite in sign to that from the recoil effects of $\boldsymbol{\sigma}\cdot\nabla\phi$, and dominates the even wave scattering as shown in Table I. The resultant asymmetry from both terms is opposite in sign to experiment, when combined with the P wave phase shifts from (1). With Chew's $g^2/4\pi$, the absolute magnitude of the S wave phase shift is about equal to that observed.

Combined with any short-range static repulsion, the fit of experiment and theory[5] for the S wave phase shifts is worse with the inclusion of the Galilean term in (1).

We wish to thank Professor R. Serber for helpful discussion.

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† National Science Foundation Postdoctoral Fellow, on leave from the University of California, Berkeley, California.
[1] The usual $-\gamma_5\pi_j$ of the relativistic interaction becomes $-i\boldsymbol{\sigma}\cdot\mathbf{V}\pi_j$, when taken between positive energy states.
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[3] G. F. Chew, Phys. Rev. **89**, 591 (1953).
[4] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952). Experiment indicates a ratio ~2. Bodansky, Sachs, and Steinberger (to be published) find a best fit ratio ~4 at 59 Mev.
[5] R. E. Marshak, Phys. Rev. **88**, 1208 (1952).

The Transfer Rate of the Liquid Helium II Film near Zero Level Difference

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IT is well known that liquid He II will flow as a film over the walls of a beaker if there is a difference between the level inside the beaker and that of the surrounding bath. The rate of transfer per unit perimeter of the beaker (usually called the critical transfer rate) is practically independent of the level difference, but varies with the temperature.[1,2]

In order to investigate whether or not these transfer rates are due to a critical current density and perhaps to get some information concerning the mechanisms responsible for initiation and limitation of the flow rate, measurements have been made of the flow at very low level differences of the order of several hundredths to several tenths of a millimeter. These were accomplished by suspending an accurately turned Lucite plunger of diameter 7.62 mm inside of, and concentric with, a Pyrex beaker of i.d. 9.60 mm, leaving an annular space approximately 1 mm wide. The plunger could be moved up or down at any desired rate by an electric motor outside the cryostat appropriately geared to a shaft emerging from the top of the cryostat through a vacuum seal. The plunger was hung from a linen thread attached to the shaft.

The observations consisted of measuring the position of the level inside the beaker as a function of time for various rates of downward travel of the plunger. The most interesting results are obtained when the plunger motion is such that the rate at which it displaces liquid is less than the rate at which the beaker would be emptied by flow at the critical transfer rate. Figure 1 shows a

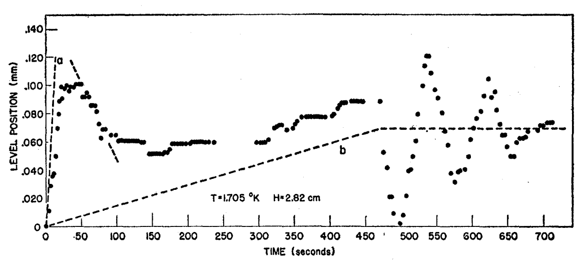

FIG. 1. Position of the level inside the beaker as a function of time.

typical run, made at a temperature of 1.705°K with the rim of the beaker at a height $H=2.82$ cm above the bath. At time $t=0$ sec, the levels inside and outside the beaker were equal, and the plunger was at that instant started down at a rate $x=55.12\times 10^{-5}$ cm/sec. The dashed line a indicates the rate at which the level in