Mev and 90 degrees at 200 Mev; the former gives approximately the correct total cross sections for π^+, π^-, and charge exchange scattering, and roughly the correct angular distribution for π^+ scattering on protons. The actual scattering is of course spin dependent and also includes strong even wave interference so that these results can serve only to illustrate the effects of the multiple scattering.

It is clear from Figs. 1 and 2 that a very marked departure from the impulse approximation results if multiple scattering is included. This is particularly apparent for phase shifts as large as 90 degrees for which the backward elastic scattering is almost entirely eliminated. These results reflect the strong depression of the scattering which sets in when the two scatterers are at distances less than roughly twice the scattering length, the effect when the phase shifts are large being particularly marked for large momentum transfers (large angle scatterings) where the small distance contributions to the cross section are especially important.

It is possible to conclude from these results that the elastic scattering is probably considerably overestimated by the usual impulse approximation and that the angular distribution is also given incorrectly when the phase shifts are large, with strong interference effects reducing the backward scattering. Both of these conclusions are in qualitative agreement with experiment.[1] It is quite possible that the cross sections may be further altered by additional effects such as scatterings which do not conserve energy and the nonadiabatic motion of the nucleons. The correct treatment of these, however, is quite difficult since coupled integral equations are encountered[8] and in addition knowledge of the scattering matrices off the energy shell is required. It is expected that a more detailed treatment of these points will be contained in future papers.

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[4] K. A. Brueckner, Phys. Rev. **89**, 834 (1953).

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[6] Compare, for example, reference 2.

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High Energy Nuclear Interactions in Lead and Light Elements*

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In order to extend previous measurements[1] on nuclear interactions in the energy region 10^{12}–10^{14} ev, an emulsion cloud chamber consisting of a "block" of emulsion 6 in.×4 in.×2 in. placed on top of an emulsion cloud chamber of 25 alternate 3-mm lead plates and 250-micron G-5 emulsion was flown for six hours at White Sands. The "block" of emulsion consisted of 92 G-5 stripped emulsions, 6 in.×4 in.×400μ, which were separated after flight and processed by the NRL technique[2] (emulsions attached to glass backing prior to development).

Figure 1 shows the results for interactions observed starting in the lead. The multiplicity observed on the plate directly below the lead absorber in which the interaction occurred is plotted against the distance back to the point of interaction. The line of the graph which appears to represent the data is calculated assuming an initial production of 20 charged particles and ten neutral π mesons decaying into two photons with a laboratory lifetime less than 3×10^{-12} sec.[3] (For 3×10^{-12} sec, the mean decay distance of 0.9 mm is short compared to the radiation conversion length in lead.) The calculated line seems to fit the data well and accordingly in the majority of showers the initial multiplicity of charged particles would appear to be about 20. The median energy of this group of interactions was about 2×10^{12} ev as estimated from the median angle of the shower particles.

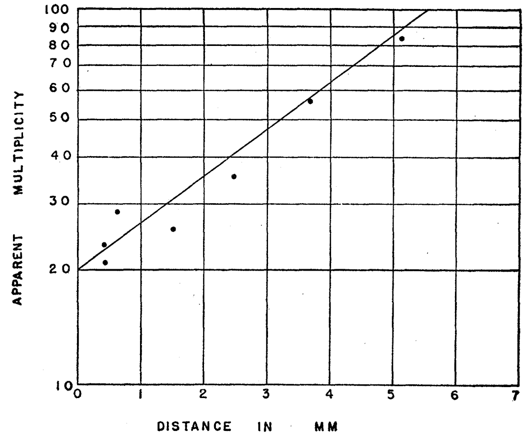

FIG. 1. Apparent multiplicity of showers *vs* distance in lead.

Seven primary interactions were found in emulsion. Table I[9,10] gives the value of N_s, the number of minimum ionizing particles, and N_h, the number of gray plus black prongs, occurring in the interaction. The median energy for this group was $\sim10^{13}$ ev. The multiplicity appears to be independent of N_h with an average value of about 16. If the interaction cross section for primaries of these energies is geometric, approximately half the collisions in emulsion would occur in Ag or Br and the rest in C, N, O, and H. Though the division of interactions on the basis of the number of star prongs is not certain, it seems likely that the cases with $N_h=0$ occur in light elements, and the case with $N_h=15$ must have arisen from an interaction in an Ag or Br nucleus. If the dividing line between the light and heavy elements is taken to be $N_h\approx4$ or 5, then our data are consistent with half the interactions occurring in light elements and half in heavy elements. It is seen that the mean number of shower particles for interactions with $N_h<4$ and $N_h>4$ is 16 for both cases.

If we now utilize previous results[1] and correct for the differences in median energies on the assumption that the multiplicity varies as $E^{\frac{1}{4}}$, we can compare multiplicities of shower particles in various elements for primary energies of $\sim5\times10^{12}$ ev. For lead the multiplicity is $\sim$24, brass$\sim$18, Ag and Br$\sim$14, and light elements$\sim$14. It would appear that the multiplicity in this energy region is almost independent of the target nucleus. There have been cases found in which the multiplicity of shower particles is much higher than those considered above.[4]

TABLE I. High energy stars in emulsion.

No. of evaporation prongs N_h	No. of shower particles N_s
0	10
0	16
2	19
4	12
6	15
15	16
1	15[a]
7	18[b]
Secondary interaction	
1	15

[a] See reference 9.

[b] See reference 10.

Our observations can be explained by a theory in which the particles are produced multiply in the initial interaction.[5] The multiplicity apparently does not seem to increase by a large factor through subsequent interactions inside the target nucleus; this is probably because the shower particles do not interact individually inside the nucleus because of their extreme collimation.[1,6,7] The fluctuations from the average behavior as noted above could be the result of large fluctuations in the angular distribution of the shower particles. A few shower particles of the first or possibly the second generation projected at angles considerably greater than the average could be effective in starting independent cascades inside the nucleus, which could increase the observed multiplicity by a considerable factor.

We have also noted the occurrence of both high and low energy pairs of charged particles produced by neutral radiation, in observing the passage of the high energy showers through the block of stripped emulsions. The most reasonable assumption is that these pairs are electron-positron pairs produced by photons; though our statistics are too low to draw any definite conclusion, the contribution of bremsstrahlung according to Schiff's theory[8] seems to be too low to account for the low energy pairs.

We are indebted to the Aero Medical Field Laboratory, Holloman Air Force Base for their cooperation in obtaining successful balloon flights. We wish to thank Miss Barbara Hull and Miss Katherine Merry for their assistance in scanning the plates.

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Nonuniform Track Shrinkage in Nuclear Emulsions*

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SEVERAL Eastman NTA and NTB nuclear track plates have been exposed to monoenergetic alpha-particles from Po^{210} plated silver foils, and a study has been made of the resulting alpha-tracks as a function of the angle of incidence.

Observation of the tabulated data indicated that the average track length increased as a function of the dip angle θ. Experiments by Rotblat and Tai[1] with lithium-loaded emulsions showed that shrinkage was uniform at all depths of the emulsion. Also the shrinkage factor for alpha-tracks was a function of both the gelatin content and the dip angle. They concluded that the shrinkage factor remained fairly constant with angles of dip up to 25–30°. Roberts,[2] in studying $n-\alpha$ reactions, measured tracks with values of θ up to 40° and found that resolution was considerably improved by limiting θ to $\leq 20°$.

Figure 1 is a plot of the average track range in microns *versus* the angle of dip in the processed emulsions. The empirically determined shrinkage factor 2.7 ± 0.3 was used in these calculations. Both NTA and NTB plates show substantially the same results with the average range of the particles continuing to increase with increasing angles of dip. Extrapolation of the curve to zero yields the range of the alpha particles without shrinkage error. This value, 21.3μ, agrees within experimental error with Steigert's[3] empirical value of 21.2μ as well as his theoretical value of 21.3μ for NTA plates.

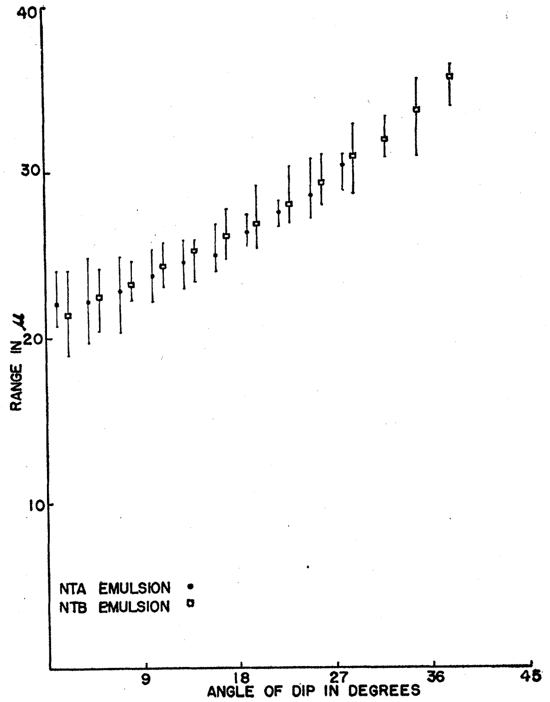

FIG. 1. Average alpha-track range *vs* angle of dip in processed emulsions.

Thus, the usual method of correcting for emulsion shrinkage by multiplying the vertical track projection by the shrinkage factor should be restricted to angles of dip less than 6° for heavily ionizing particles if the error of measurement is to be less than 2 percent.

The final criterion for selection was that each alpha track should lie entirely within the depth of focus of the microscope which was approximately 0.5μ, which is equivalent to a variation of less than 2° in the angle of dip. The maximum error introduced by measuring only the horizontal component, with the above limitation, is 0.1μ. This modified procedure resulted in histograms having a maximum dispersion of 5μ and a half-width at half-maximum of 0.5μ.

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* Work performed at the Oak Ridge Institute of Nuclear Studies, Oak Ridge, Tennessee.

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Rotational States in Even-Even Nuclei

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IN a recent note,[1] an interpretation of the short-lived $E2$ isomers has been suggested in terms of rotational states of the deformed nucleus. Empirical evidence is rapidly accumulating on the low energy spectra of even-even nuclei;[2–4] the purpose of the present note is to call attention to the extensive support which exists in these data for the above interpretation, and to suggest its usefulness in the analysis of decay schemes.

In the model describing the nucleus in terms of the coupled particle motion and surface oscillations, low-lying rotational states are associated with the large deformations expected in regions with many particles outside of closed shells. In such regions, the rotational spectrum is expected to be given rather accurately by the simple expression[1]

$$E_I=\frac{\hbar^2}{2\mathcal{J}}I(I+1),\quad I=0,2,4,6,\ \text{even parity} \qquad (1)$$