inelastic collision cross sections are given, based on an estimate of the effect of multiple scattering. The accuracy of the integrated elastic cross sections shown in the third column is about fifteen percent apart from the effects of multiple scattering. The estimate of the inelastic collision cross section is, therefore, uncertain by at least fifteen percent of the values given in the third column.

The observed angular distributions are in qualitative agreement with results obtained with pile neutrons.[3]

Measurements on other elements are in progress, and final results will be reported after the calculations of the corrections are completed.

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† U. S. Atomic Energy Commission Predoctoral Fellow.

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The Elastic Scattering of Pions in Deuterium*

K. A. BRUECKNER
Indiana University, Bloomington, Indiana
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RECENT experiments by Arase, Goldhaber, and Goldhaber[1] on the elastic scattering of pions in deuterium have shown a considerable discrepancy with the results of a straightforward application[2] of the impulse approximation[3] to the problem. It is the purpose of this note to show that inclusion of the effects of the multiple scattering of the mesons by the two nucleons in the deuteron leads to a considerable improvement of the agreement of experiment with theory. The method used has been developed in a recent paper[4] (to be referred to as I) by the author and there applied to the evaluation of the total cross section. The results obtained can be easily extended to the determination of the elastic cross section.

The approximations made in I were (a) the usual impulse approximation[3] of neglecting both the binding effects in the deuteron on the scattering matrices and the off-the-energy-shell dependence of the scattering matrices; (b) the adiabatic approximation for the treatment of the nucleon motion, i.e., treating the nucleons as very heavy.[5] It was not necessary, however, to make the additional approximation of neglecting the rescattering of the outgoing meson waves from one nucleon by the other nucleon in the deuteron. The coupled algebraic equations obtained could be solved exactly by straightforward methods to give the cross sections. The result for the total cross section for π^+ mesons on deuterons was

$$\sigma_{\text{total}} = 4\pi ƛ \,\text{Im} \int f_R(0)\,|\psi_D(R)|^2 d\mathbf{R}. \quad (1)$$

The elastic cross section is given by[6]

$$\frac{d\sigma_{\text{elastic}}}{d\Omega} = \left| \int f_R(\theta)\,|\psi_D(R)|^2 d\mathbf{R} \right|^2. \quad (2)$$

In both of these results $f_R(\theta)$ is the scattered amplitude into the isotopic spin singlet state with the two nucleons at a separation R. In I, $f_R(\theta)$ was evaluated for P-wave scattering with isotopic spin dependence; for scattering in the isotopic spin state 3/2 state alone, which is a quite good approximation to the experimental results for scattering on free nucleons,[7] $f_R(\theta)$ is given by

$$f_R(\theta) = \frac{2bƛ}{1-b^2f^2}\left\{ \cos\theta\left[e^{i(\mathbf{k}_0-\mathbf{k})\cdot\frac{1}{2}\mathbf{r}} + e^{i(\mathbf{k}_0+\mathbf{k})\cdot\frac{1}{2}\mathbf{r}}\right] + \frac{(\mathbf{k}\cdot\mathbf{R})(\mathbf{k}_0\cdot\mathbf{R})}{R^2} ƛ^2 bg\left[b(f+h)e^{i(\mathbf{k}_0-\mathbf{k})\cdot\frac{1}{2}\mathbf{r}} - (1+b^2fh)e^{i(\mathbf{k}_0+\mathbf{k})\cdot\frac{1}{2}\mathbf{r}}\right]\right\}$$
$$+\text{a term in which } \mathbf{R} \text{ is replaced by } -\mathbf{R}, \quad (3)$$

where $b=e^{i\delta}\sin\delta$, $f=(1/x)(d/x)(e^{ix}/x)$, $g=xdf/dx$, $h=f+g$, and $x=R/ƛ$. The neglect of multiple scattering is equivalent to the retention of terms linear in b only.

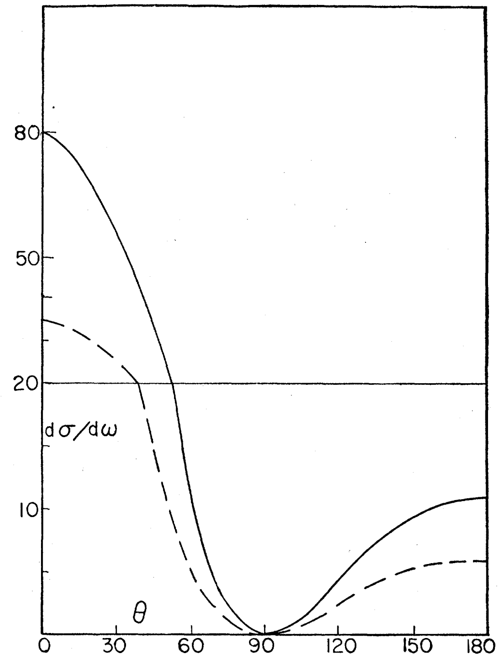

FIG. 1. Elastic P-wave scattering of pions by deuterons at 140 Mev for a phase shift of 45°. The solid curve gives the impulse approximation neglecting multiple scattering; the dashed curve includes the effects of multiple scattering. The curves are in the center-of-mass system for the meson and one nucleon. Note the change in scale at $d\sigma/d\omega = 20$ mb/steradian.

The differential cross sections calculated from Eqs. (2) and (3) are given in Figs. 1 and 2, together with the result obtained if the multiple scattering is neglected. In evaluating the integrals over the deuteron wave function, the Hulthén wave function has been used. The cases given are for phase shifts of 45 degrees at 140

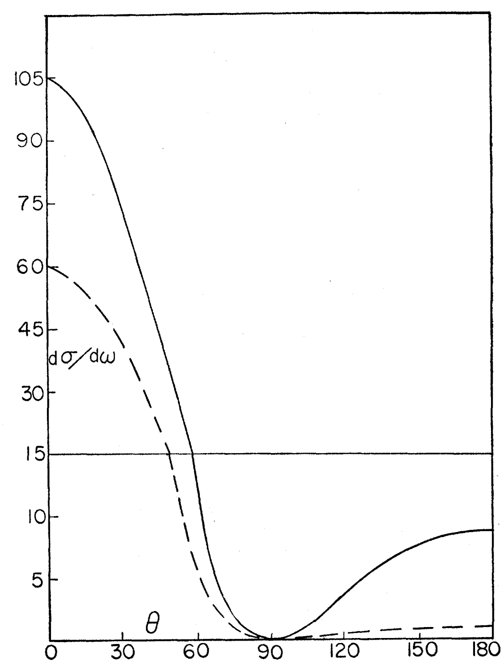

FIG. 2. Elastic scattering at 200 Mev for a phase shift of 90°. The notation is the same as in Fig. 1. Note the change in scale at $d\sigma/d\omega = 15$ mb/steradian.

Mev and 90 degrees at 200 Mev; the former gives approximately the correct total cross sections for π^+, π^-, and charge exchange scattering, and roughly the correct angular distribution for π^+ scattering on protons. The actual scattering is of course spin dependent and also includes strong even wave interference so that these results can serve only to illustrate the effects of the multiple scattering.

It is clear from Figs. 1 and 2 that a very marked departure from the impulse approximation results if multiple scattering is included. This is particularly apparent for phase shifts as large as 90 degrees for which the backward elastic scattering is almost entirely eliminated. These results reflect the strong depression of the scattering which sets in when the two scatterers are at distances less than roughly twice the scattering length, the effect when the phase shifts are large being particularly marked for large momentum transfers (large angle scatterings) where the small distance contributions to the cross section are especially important.

It is possible to conclude from these results that the elastic scattering is probably considerably overestimated by the usual impulse approximation and that the angular distribution is also given incorrectly when the phase shifts are large, with strong interference effects reducing the backward scattering. Both of these conclusions are in qualitative agreement with experiment.[1] It is quite possible that the cross sections may be further altered by additional effects such as scatterings which do not conserve energy and the nonadiabatic motion of the nucleons. The correct treatment of these, however, is quite difficult since coupled integral equations are encountered[8] and in addition knowledge of the scattering matrices off the energy shell is required. It is expected that a more detailed treatment of these points will be contained in future papers.

The author is indebted to Dr. Arase, Dr. G. Goldhaber, and Dr. S. Goldhaber for information on their experimental results in advance of publication and to Dr. T. Green for information on his theoretical interpretation.

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High Energy Nuclear Interactions in Lead and Light Elements*

M. F. Kaplon, D. M. Ritson, and W. D. Walker
University of Rochester, Rochester, New York
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In order to extend previous measurements[1] on nuclear interactions in the energy region 10^{12}–10^{14} ev, an emulsion cloud chamber consisting of a "block" of emulsion 6 in.×4 in.×2 in. placed on top of an emulsion cloud chamber of 25 alternate 3-mm lead plates and 250-micron G-5 emulsion was flown for six hours at White Sands. The "block" of emulsion consisted of 92 G-5 stripped emulsions, 6 in.×4 in.×400μ, which were separated after flight and processed by the NRL technique[2] (emulsions attached to glass backing prior to development).

Figure 1 shows the results for interactions observed starting in the lead. The multiplicity observed on the plate directly below the lead absorber in which the interaction occurred is plotted against the distance back to the point of interaction. The line of the graph which appears to represent the data is calculated assuming an initial production of 20 charged particles and ten neutral π mesons decaying into two photons with a laboratory lifetime less

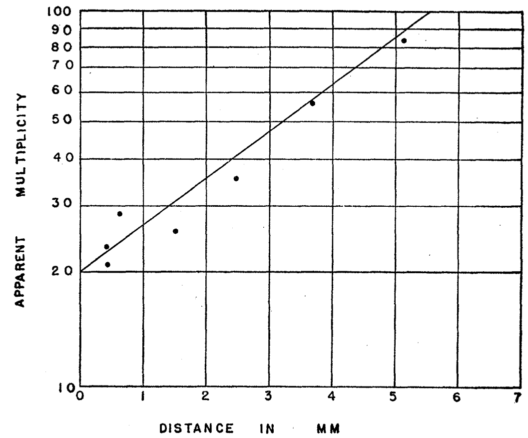

Fig. 1. Apparent multiplicity of showers *vs* distance in lead.

than 3×10^{-12} sec.[3] (For 3×10^{-12} sec, the mean decay distance of 0.9 mm is short compared to the radiation conversion length in lead.) The calculated line seems to fit the data well and accordingly in the majority of showers the initial multiplicity of charged particles would appear to be about 20. The median energy of this group of interactions was about 2×10^{12} ev as estimated from the median angle of the shower particles.

Seven primary interactions were found in emulsion. Table I[9,10] gives the value of N_s, the number of minimum ionizing particles, and N_h, the number of gray plus black prongs, occurring in the interaction. The median energy for this group was $\sim10^{13}$ ev. The multiplicity appears to be independent of N_h with an average value of about 16. If the interaction cross section for primaries of these energies is geometric, approximately half the collisions in emulsion would occur in Ag or Br and the rest in C, N, O, and H. Though the division of interactions on the basis of the number of star prongs is not certain, it seems likely that the cases with $N_h=0$ occur in light elements, and the case with $N_h=15$ must have arisen from an interaction in an Ag or Br nucleus. If the dividing line between the light and heavy elements is taken to be $N_h\approx4$ or 5, then our data are consistent with half the interactions occurring in ight elements and half in heavy elements. It is seen that the mean number of shower particles for interactions with $N_h<4$ and $N_h>4$ is 16 for both cases.

If we now utilize previous results[1] and correct for the differences in median energies on the assumption that the multiplicity varies as $E^{1/4}$, we can compare multiplicities of shower particles in various elements for primary energies of $\sim5\times10^{12}$ ev. For lead the multiplicity is $\sim$24, brass$\sim$18, Ag and Br$\sim$14, and light elements$\sim$14. It would appear that the multiplicity in this energy region is almost independent of the target nucleus. There have been cases found in which the multiplicity of shower particles is much higher than those considered above.[4]

Table I. High energy stars in emulsion.

No. of evaporation prongs N_h	No. of shower particles N_s
0	10
0	16
2	19
4	12
6	15
15	16
1	15[a]
7	18[b]
Secondary interaction	
1	15

[a] See reference 9.
[b] See reference 10.