Positron-Electron Scattering at 1.3 Mev*

HORACE A. HOWE† AND K. R. MACKENZIE
University of California, Los Angeles, California
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Using an incident beam energy of 1.3 Mev, the ratio of electron to positron scattering on atomic electrons has been determined for the case where one-half the incident energy is transferred to the atomic electron. The measured ratio is 1.82 ± 0.11, which agrees with the theoretical ratio of 1.83, calculated from the expressions given by Møller and Bhabha when exchange effects are included. The theoretical ratio, excluding exchange effects, is 1.36.

INTRODUCTION

THE theory of electron-electron scattering, published by Møller[1] in 1932, has been checked experimentally several times in recent years[2] and appears to be correct. However, the theory of positron-electron scattering, published by Bhabha[3] in 1936, has had very little in the way of confirmation. Several experiments using cloud chambers[4] have given indecisive results because of the difficulty of obtaining enough tracks to give good statistics. Recently, using counters, Ashkin and Woodward[5] were able to measure the ratio of positron-electron to electron-electron scattering for 600-kev particles and obtained good agreement with theory. This ratio is easier to measure than the absolute cross section and is the quantity measured in this work.

The Møller formula for electron-electron scattering and the Bhabha formula for positron-electron scattering are reproduced below, where the differential cross-section is given as a function of the fractional energy ϵ imparted to the stationary particle.

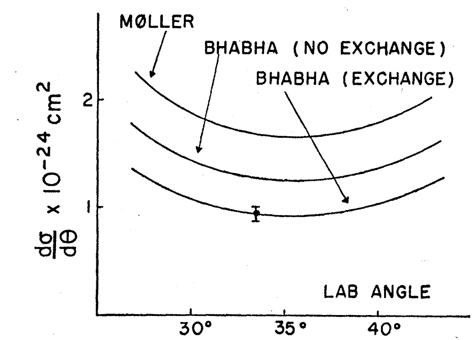

FIG. 1. Theoretical differential scattering cross section in aboratory system as a function of laboratory angle. Experimental point is shown at 33.5°.

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† Now at Reed College, Portland, Oregon.
[1] C. Møller, Ann. phys. **406**, 531 (1932).
[2] L. Page, Phys. Rev. **81**, 1062 (1951); Scott, Hanson, and Lyman, Phys. Rev. **84**, 638 (1951); Groetzinger, Leder, Ribe, and Berger, Phys. Rev. **79**, 454 (1950).
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[4] Ho Zah-Wei, Compt. rend. **226**, 1083 (1948); Von O. Ritter *et al.* Z. Naturforsch. **6a**, 243 (1951); G. R. Hoke, Phys. Rev. **87**, 285 (1952).
[5] A. Ashkin and W. M. Woodward, Phys. Rev. **87**, 236 (1952).

Møller [p. 569, reference 1, with $A=\epsilon$ and $Q=mc^2(\gamma-1)\epsilon$]:

$$\left(\frac{d\sigma}{d\epsilon}\right)_M=\frac{2\pi e^4}{m^2c^4}\frac{\gamma}{(\gamma-1)^2\epsilon^2}G(\gamma,\epsilon),$$

where

$$G(\gamma,\epsilon)=\frac{\gamma}{\gamma+1}\left[1+\frac{\epsilon^2}{(1-\epsilon)^2}-\frac{\epsilon}{(1-\epsilon)}+\left(\frac{\gamma-1}{\gamma}\right)^2\left(\epsilon^2+\frac{\epsilon}{(1-\epsilon)}\right)\right].$$

Bhabha:

$$\left(\frac{d\sigma}{d\epsilon}\right)_B=\frac{2\pi e^4}{m^2c^4}\frac{\gamma}{(\gamma-1)^2\epsilon^2}F(\gamma,\epsilon),$$

where

$$F(\gamma,\epsilon)=\frac{1}{\gamma(\gamma+1)}\Big[\{1+2(\gamma-1)(1-\epsilon)+(\gamma-1)^2(1-\epsilon+\tfrac{1}{2}\epsilon^2)\}+\left(\frac{\gamma-1}{\gamma+1}\right)^2\epsilon^2\times\{3+2(\gamma-1)+(\gamma-1)^2(\tfrac{1}{2}-\epsilon+\epsilon^2)\}-\frac{\gamma-1}{\gamma+1}\epsilon\{3+4(\gamma-1)(1-\epsilon)+(\gamma-1)^2(1-\epsilon^2)\}\Big].$$

$$\gamma=\frac{1}{(1-v^2/c^2)^{\frac{1}{2}}},$$

$$F_0(\gamma,\epsilon)=\frac{1}{\gamma(\gamma+1)}[1+2(\gamma-1)(1-\epsilon)+(\gamma-1)^2(1-\epsilon+\tfrac{1}{2}\epsilon^2)].$$

The quantity $F_0(\gamma,\epsilon)$ is the first term in the expression for $F(\gamma,\epsilon)$ and refers to what Bhabha calls ordinary scattering, or the scattering that would take place if the electron and positron could not annihilate. The second term is the "exchange" term representing the possibility of annihilation and re-creation of a new pair, and the third term is the interference term between the first two. Bhabha plots $F(\gamma,\epsilon)$ and $F_0(\gamma,\epsilon)$ against ϵ for various values of γ. The experimental ratio of positron-electron to electron-electron scattering is to be compared with the ratios F/G and F_0/G.

In order to distinguish between these ratios it is desirable to choose values of γ and ϵ which maximize or minimize the ratio of F/F_0. The maximum value of 2 occurs at $\gamma=\infty$ and $\epsilon=1$ (180° scattering in the center-of-mass system). However, high values of γ and ϵ are rather unfavorable experimentally, since the cross section decreases rapidly as γ approaches ∞ and as ϵ approaches 1, with the added difficulty that the impinging positron is not left with much energy in the laboratory system. At low incident energy, the ratio F_0/F has a broad maximum of about 1.4, for $\gamma=2.4$ (700 kev) and $\epsilon=0.5$. Since coincidence counting of both particles is very desirable to eliminate nuclear scattering and other background, a value of $\epsilon=0.5$, where both particles are scattered through equal angles, is very convenient. The incident energy chosen in this experiment was 1.3 Mev, for which $F_0/F=1.34$. The scattered particles thus have an energy of 650 kev each, which allows discrimination against the 0.51-Mev pulses from annihilation radiation.

Since no distinction is made between positron and electron in a coincidence counting arrangement, the significant ratio is

$$\left\{\frac{1}{\epsilon^2}F(\gamma,\epsilon)+\frac{1}{(1-\epsilon)^2}F(\gamma,(1-\epsilon))\right\}\bigg/\left\{\frac{1}{\epsilon^2}G(\gamma,\epsilon)\right\}.$$

These expressions are in terms of ϵ, but since the experimental arrangement described below selects the particles according to their angle of emission from the scatterer, the cross sections must be expressed as a function of angle (Fig. 1), using the relation $\epsilon=\{(\gamma+1)\sin^2\theta\}/\{2+(\gamma-1)\sin^2\theta\}$. The particles scatter symmetrically at $\epsilon=0.5$, making an angle of 33.5° with the axis, but due to the relativistic distortion of angles, the minimum counting rate for a given solid angle is obtained at approximately 35°.

EXPERIMENTAL EQUIPMENT

The experimental arrangement is shown in Fig. 2. An electron or positron beam of energy up to 2 Mev was produced by a magnetic lens spectrometer 7 feet in length. Such a spectrometer has the advantage that the beam and the source are very far apart. Particles emerging from a source rod 0.08 in. in diameter between the angles of 5° and 10° from the axis were focused into a beam 2 cm in diameter with an angular divergence of 1.5° from the axis. The energy spread was 7 percent as determined from the Cs^{137} line. Helical baffles were used to exclude particles of the wrong sign. The trajectories were determined by means of a light current-carrying wire.

The scattering chamber shown in Fig. 2 was connected to the same vacuum as the spectrometer. The pairs of particles scattered from various foils (mounted on a wheel), were counted in coincidence by high resolution scintillation detectors.

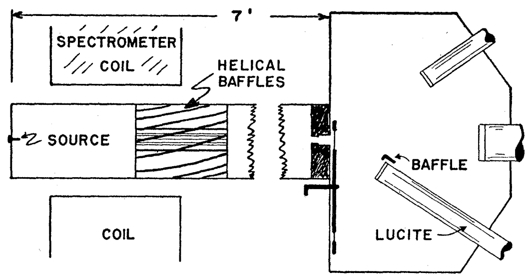

FIG. 2. Spectrometer and scattering chamber.

The electron source was Y^{90}, daughter of 25-year Sr^{90} with an end-point energy of 2.2 Mev. It was prepared by evaporating thirty millicuries of the chloride salt on the end of a glass rod 0.08 in. in diameter. A thin film of polystyrene was then sprayed over the rod to lessen the hazard of contamination. The positron source was Ga^{66} with a 9.4-hr half-life and an end point of 3.9 Mev. It was produced in the UCLA cyclotron by bombarding the end of a 0.08 in. Zn rod with 16-Mev protons. Self-absorption in the Zn made the shape of the gallium spectrum almost indistinguishable from the spectrum of Y^{90} in the region around 1.3 Mev.

Three sodium iodide crystals were used as counters. Although inferior to anthracene for this experiment because of higher sensitivity to gamma-radiation, sodium iodide was used as it happened to be on hand in sufficient quantity. The incident beam was recorded by a crystal 5 cm in diameter and 7 mm thick. The side counters were 2.7 cm in diameter and 4 mm thick. The crystals were mounted on Lucite light pipes of the same diameter, which extended through the vacuum walls and were then cemented to the 5819 photomultiplier tubes. The crystals were first carefully polished, then wiped with a thin coat of mineral oil to prevent absorption of moisture and covered with a very thin Al foil.

One crystal, at 35°, was used as a defining counter and was placed 14.7 cm from the scattering foil, thus defining an angular spread of 10.5° from a point scatterer, and an "effective" angle of 13°, when the size of the source and angular spread of the beam are considered. This angular spread was necessary in order to increase intensity and reduce the effects of multiple scattering. As seen from Fig. 1, the counting rate is very insensitive to angle in this region. The other counter was set at an angle of 33°, and a distance of 6.8 cm from the foil, thus subtending over four times the solid angle. Both solid angles were defined by the edges of the crystals. It was considered undesirable to determine the solid angle with diaphragms, since gamma-ray pairs, produced by positrons striking the diaphragms, might give false coincidences. However a thin baffle between the crystals was considered necessary in order to prevent particles from being scattered by one crystal into the other.

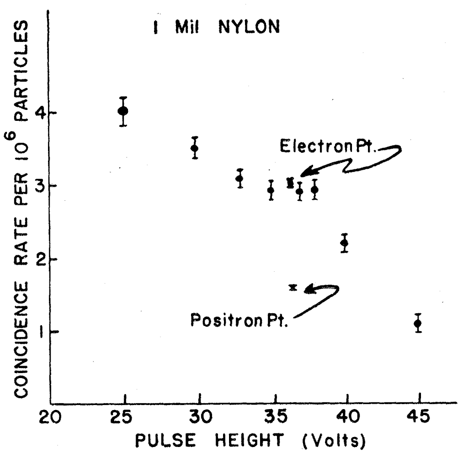

FIG. 3. Number of scattering events recorded per 10^6 incident particles on 1-mil Nylon foil as a function of discriminator bias.

Pulses in each side channel were fed through a cathode follower to a wide band amplifier, then to a discriminator, a coincidence circuit and register. Two scalers and registers recorded the pulses in each side counter. The main beam pulses were recorded after going through a wide band amplifier and a scaler with a resolution time of 0.4 microsecond.

EXPERIMENTAL PROCEDURE

Scattering runs were made with 1-mil and 2-mil Nylon foils. The results after making the necessary corrections are shown in Fig. 3. The runs with 2-mil Nylon foil (Fig. 4) were exploratory, and the statistics were poor, but the data are presented to show that the ratio is not very sensitive to discriminator bias. Most of the counting was done with the 1-mil foil so that the multiple scattering corrections would be smaller.

Theoretically a definite plateau should be observed for coincidences against discriminator bias (varied simultaneously in both channels). Since the counters and the scattering foil are relatively large compared with their separations, the scattered electrons recorded by the defining counter have an energy spread from 0.44 to 0.88 Mev, corresponding to an angular spread of 16.5° as measured from extreme edges of the foil and defining crystal, plus 3° for the spread in the incident beam. Therefore coincidences should first be observed when the bias is lowered to a value corresponding to about 0.65 Mev, since only below this setting will both members of the pair have sufficient height to record a coincidence. As the bias is lowered the coincidence rate should rise until 0.44-Mev particles are admitted, and then it should remain constant, but only on the assumption that there is an exact correspondence between pulse height and energy. The actual distribution in pulse height was measured by setting the spectrometer to produce a monochromatic beam of 0.65-Mev electrons. The half-width of the curve so obtained was 12 percent for the monitor and 9 percent for the defining counter with a tail on the low pulse-height side. This variation in pulse height was presumably due to imperfections in the crystals and light pipe system and possibly to a non-uniform oil film covering the crystals.

Consequently the bias must be reduced essentially to zero in order that all coincidences be recorded and the onset of the theoretical plateau is replaced by a knee in the curve. Many of the coincidences at very low bias are undoubtedly due to scattering events at angles widely different from 35°, in which the high energy electron is recorded by the wide angle monitor and the low energy particle is multiply scattered into the defining counter. To eliminate such events, most of the counting time was used in comparing the positron-electron, electron-electron scattering ratio for a bias which was just over this knee in the curve. This point is also a compromise between maximum discrimination against false coincidences resulting from annihilation quanta, and minimum variation of counting rate due to fluctuations in pulse height caused by changes in either the discriminator bias or high voltage on the photomultiplier tubes.

Since the number of "extra" coincidences at low bias is unknown, it is possible that over 50 percent of the true coincidences were lost by counting near the knee of the curve. This makes an absolute determination of the cross section rather difficult, but does not affect the ratio which depends only on the constancy of the equipment. Great pains were taken to insure this constancy of equipment. In addition to the usual precision regulation of power supplies (less than $\frac{1}{8}$ percent for the photomultipliers), a special circuit "blocked" all registers whenever excessive transients were detected on the ac line.

The runs were made by first warming up the equipment for four or five hours and then making an electron run for a few hours, during which time a positron source was being prepared in the cyclotron. Then the positron source was inserted and counted for some 20 hours. To insure that the equipment had not changed, the electron source was immediately put back and counted again. Counting rates for the electron source were 0.13 coincidence per second with a beam of 44 000 particles per second. The positron rate was 0.013 per second for an initial beam of 8000 particles per second. Several such runs were made until some 1000 positron counts and 5000 electron counts had been recorded.

CORRECTIONS

The counter and foil geometry (Fig. 2) was such that the monitor counter subtends over four times the solid angle seen by the defining counter. With an incident beam of particles $\frac{1}{2}$ cm in diameter, and a thin foil (as originally planned), this geometry was such that the monitor counter would receive a particle for almost every particle that reached the defining counter. However, to obtain sufficient intensity, the beam was widened to 2-cm diameter and the foil thickness increased to 1 mil. The new geometrical factors introduced by the wide beam caused the monitor counter to miss 16 percent of the events that were registered by the defining counter. With a 1-mil Nylon foil, multiple scattering caused another loss estimated to be around 20 percent, based on extrapolation to very thin foils of the curves given by Snyder and Scott.[6] The 16 percent figure is common to both the electron and positron cases, but since the multiple scattering is reported as 10 percent less for positrons than for electrons,[7] a correction of 2 percent was subtracted from the positron counting rate.

As can be seen from Fig. 1, the apparent ratio of electron-electron scattering to positron-electron scattering will be somewhat reduced by the above angular spread. The effect was graphically estimated to be 2 percent; hence the measured ratio was increased by 2 percent.

The positron source produced a large background of annihilation radiation. The coincidence counters were biased so that they were very insensitive to this radiation, but the main beam counter was not so biased, since it was desirable that all the incident positrons and electrons be recorded. A 12 percent correction was necessary for this gamma-background. It was determined by recording the gamma-ray background when the main beam was stopped at the entrance to the lead collimator.

In addition to the above corrections, there were several processes which could give false coincidences. The single counting rates for electrons were 20 counts per second in the defining counter and 28 in the other. Practically all of these counts were stray electrons, scattered primarily from the edges of the lead collimator. This was shown by noting that the single counting rates were not reduced more than the statistical counting error of 1.5 percent when the foil was removed. With a coincidence resolving time of 1 microsecond, the accidental rate was negligible compared with the true coincidence rate of 0.13 count per second. For positrons, the initial rates were about 40 and 50 counts per second, mostly because of annihilation quanta from the source. The order of magnitude of the total number of accidental counts was calculated from a simple integral formula involving the resolving time, the individual rates and the half-life. For the first few hours the relation was integrated numerically because of the presence of the 48- and 68-minute activities resulting from Ga^{64} and Ga^{68}. The correction was approximately 10 percent of the total recorded coincidences.

Another correction must be applied in the case where a positron stops in one counter and one of the annihilation quanta is absorbed in the other. Taking into account the number of positrons striking the counters, the solid angles seen by each counter relative to the other, and the approximate efficiency of the counters for

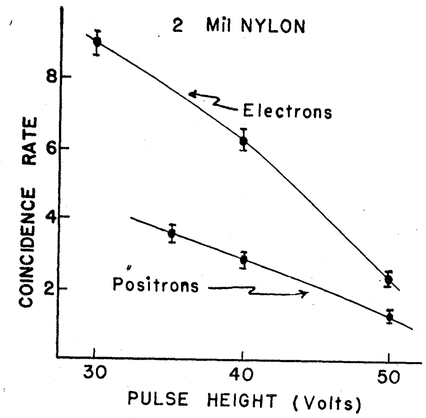

FIG. 4. Number of scattering events recorded per 10^6 incident particles on 2-mil Nylon foil as a function of discriminator bias.

0.51 gamma-rays, the correction was estimated to be around 4 percent.

Since the individual counting rates did not change (within statistical limits) when the foil was removed, it was possible to subtract false coincidences due to both the above causes by making runs with and without the foil in place. The total correction was 15 percent, which was very close to the calculated estimate.

False coincidences, which cannot be subtracted experimentally, can be recorded in the following way: When a positron is absorbed in either the defining or monitor crystal, there is a certain probability that one of the annihilation quanta will be absorbed in the same crystal. For such a gamma-ray the average path length in the crystal, for all positions of the impinging positron, is of the order of 0.4 cm. For 0.51-Mev gamma-rays in NaI, the fractional loss of photons per cm, resulting from the photoelectric effect, is 0.057.[8] For a path length of 0.4 cm, the probability of conversion is then 2.3 percent, giving a total probability of 4.6 percent, since there are two annihilation quanta. Consequently some small pulses, which would normally be rejected by the high bias setting, would have their heights increased by 0.51 Mev and be recorded, whereas the corresponding small pulses in the electron-electron scattering case would not. This effect was estimated by noting the increase in coincidence rate in the electron-electron scattering case when the discriminator bias was lowered an amount corresponding to 0.5 Mev (approximately zero bias) in one side channel at a time. The average increase for such a procedure in both channels was 47 percent. Thus the correction to be subtracted from the positron coincidence rate is 4.6 percent of 47 percent, which is 2.2 percent. There are also Compton electrons which are four times as numerous as the photoelectrons, but have an average energy of only 0.2 Mev. Lowering the bias in one channel by an amount corresponding to 0.2 Mev

[6] H. S. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949).
[7] Groetzinger, Humphrey, and Ribe, Phys. Rev. **85**, 78 (1952).
[8] National Bureau of Standards Report NBS-1003 (unpublished).

increased the rate 20 percent. Hence the amount subtracted is four times 4.6 percent of 20 percent, which is 3.7 percent. When this is added to the above 2.2 percent, the total correction is about 6 percent.

RESULTS

The ratio of electron-electron to positron-electron scattering is shown in Figs. 1 and 3 after all corrections have been applied. The theoretical ratio at 33.5° is 1.83 (with exchange), and 1.36 (no exchange). The experimental ratio is 1.82 ± 0.11. Most of this error is due to uncertainties in the corrections. The Bhabha theory with exchange is thus very definitely favored.

The coincidence rates quoted above are only 0.35 times the rates expected from absolute cross section calculations. When the theoretical rate is reduced by the previously mentioned factors of 16 percent and 20 percent successively, a counting rate for electrons of 0.22 coincidence/sec is obtained. The absolute experimental coincidence rate lies somewhere between 0.27 count/sec, which is the rate just above the noise level near zero bias, and 0.13 count/sec, which is the rate at the knee of the curve. Hence no conclusions can be drawn regarding absolute cross sections.

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The Radiation of a High Energy Electron in a Constant Magnetic Field

MAURICE NEUMAN
Radiation Laboratory, University of California, Berkeley, California

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The quantum-mechanical radiation formula for an electron in a constant magnetic field—rigorous within the approximation of the first power of the fine structure constant—is derived. With it, the quantum effects for energies $E_{\mathrm{Bev}} < 4(10^4/H_{\mathrm{gauss}})$ are analyzed and their importance assessed. The electron and the photon severally are subject to fluctuations which produce effects that are quite significant relative to the difference between the velocity of light and that of the electron, the decisive quantity at high energies. The net result however is that they are inconsequential in the domain investigated and that in its radiative aspects the system behaves—for all practical purposes—classically.

I. INTRODUCTION

A CALCULATION is here presented to show that there are no quantum corrections (in the practical sense) to the radiation of an electron in a constant magnetic field up to energies $E < 4(10^4/H)$ Bev. The magnetic field H in the formula (as elsewhere in this article) is to be expressed in gauss.

A detailed inquiry into a domain, where an elementary application of the correspondence principle renders it plausible that the classical results are valid, seems to require an explanation. It was not undertaken with the thought that this principle could somehow be circumvented. Nor was it deemed necessary or feasible to verify its operation in detail for systems of considerable complexity. In view, however, of persistent reports[1] that significant quantum corrections have been deduced from the theory, a possibility seemingly not inconsistent with the principle, as it has hitherto been applied, suggested itself.

As the electron energy assumes very high values, the difference between its velocity and the velocity of light becomes increasingly important in the description of the frequency spectrum and the angular distribution of the radiation. It is therefore perhaps not entirely fair to gauge the importance of quantum fluctuations by considering their effect on the group velocity of the particle. A relevant quantity in this very high energy domain might also be the difference between the two velocities.

It turned out in the course of the work that elements of competition are present between the quantum uncertainty in the velocity and the difference mentioned, and the result is quite sensitive to the relative magnitude of these two quantities. In the domain investigated, however, the latter quantity is overwhelmingly larger. The possibility that the situation may change at much higher energies, but still well within the range of correspondence of the two theories, cannot be ruled out completely on the basis of the present investigation, but it seems rather unlikely that such effects play any important role in this particular context.

On the practical side, the somewhat remote possibility of such effects had to be weighted against the important consequences their presence would have on future accelerator design. It was concluded that a detailed search for them was not an altogether unreasonable precaution.

In order to exhibit the quantum aspect of the problem

[1] See for example, L. Schiff, Am. J. Phys. **20**, 474 (1952), reference 9.