The Transmission of Electrons through Thin Metallic Foils*

C. H. Chang, C. S. Cook, and H. Primakoff
Physics Department, Washington University, St. Louis, Missouri
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Experimental studies have been made on the relative transmission of positrons and negatrons in the energy range 50–750 kev through aluminum and platinum windows of an end-window-type G-M counter. In qualitative agreement with the theoretical predictions that more scattering takes place in material having higher atomic number, a platinum foil having the same surface density as a corresponding aluminum foil shows lower relative transmission at any given energy even though the low energy cutoffs of the two windows are just about the same. Also in qualitative agreement with theory, a larger percentage of positrons than negatrons are transmitted at any given energy for the same platinum foil. Theoretical transmission curves, with an empirically determined constant, have been developed. These curves are in relatively good agreement with the experimental curves.

I. INTRODUCTION

FROM the point of view of the beta-ray spectroscopist studying nuclear beta- and gamma-ray spectra, a Geiger-Müller counter window introduces experimental distortions in the magnitudes and shapes of low energy spectra. In practice, methods have been devised either for the measurement of the transmission coefficient of the counter window[1–3] in order to correct for this effect, or, attempts have been made to eliminate the counter window entirely.[4] From a more fundamental aspect, however, the problem is actually a form of the general problem of the passage of electrons through matter which has been a subject of much study since the first discovery of cathode rays and which has recently been investigated intensively[5–10] in the range of energies considered in this paper. If attacked from this point of view, the transmission coefficient η of a G-M counter window foil may be considered as consisting of two parts which we shall call η_E and η_I. The quantity η_E is a measure of the amount of elastic scattering of the electrons within the foil. The elastic scattering is important, since some of the electrons do not pass completely through the foil and get into the sensitive region of the G-M counter because they are scattered through too large an angle to enter this region. The second quantity η_I is a measure of the inelastic scattering between the passing electron and the atoms of the foil; such inelastic scattering may lead to the actual stopping of the electron within the foil. The total coefficient is the product of these two parts ($\eta=\eta_E\eta_I$). Actually, of course, these two quantities are not entirely statistically independent one from the other, but handling them as separate entities appears valid in first approximation and leads to reasonably good results.

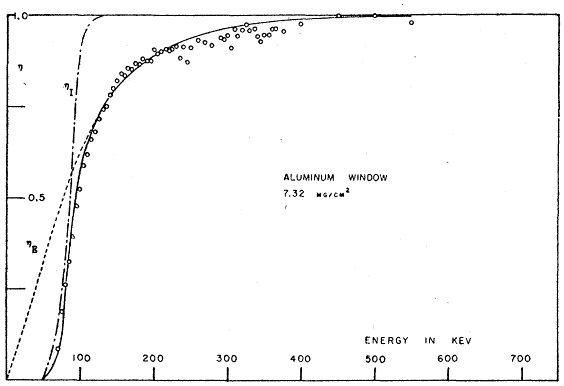

FIG. 1. Relative electron transmission η through a 7.32 mg/cm² aluminum G-M counter window as a function of the electron's incident kinetic energy. In this and subsequent figures the circles represent experimentally determined negatron transmissions, and the plus signs experimentally determined positron transmissions. The continuous line is the theoretical transmission coefficient, $\eta=\eta_E\eta_I$, as determined by the method discussed in the text. To show the relative importance of η_E and η_I at various energies the theoretically determined curves for these two quantities are also shown in this diagram.

II. EXPERIMENTAL RESULTS

The measurements made in the current experiments on the relative transmission of negatrons and positrons through various aluminum and platinum G-M counter windows are indicated in Figs. 1 through 5. These represent seven sets of data, since two of the figures give results for both positrons and negatrons for the same window. The solid lines represent a type of theoretical curve which will be discussed in the next section.

The experimental measurements were performed on the lens spectrometer previously used[3] for this purpose. However, the experimental data presented here were not obtained by means of the acceleration technique[3] but were obtained through a comparison method. Since all aluminum and platinum windows used for the cur-

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3 C. H. Chang and C. S. Cook, Nucleonics **10**, No. 4, 24 (1952).
4 Langer, Motz, and Price, Phys. Rev. **77**, 798 (1950).
5 Groetzinger, Humphrey, and Ribe, Phys. Rev. **85**, 78 (1952).
6 H. J. Lipkin, Phys. Rev. **85**, 517 (1952).
7 H. H. Seliger, Phys. Rev. **88**, 408 (1952).
8 Christian, Dunning, and Martin, Nucleonics **10**, No. 5, 41 (1952).
9 W. Paul and H. Reich, Z. Physik **131**, 326 (1952).
10 E. Hisdal, Phil. Mag. **43**, 790 (1952).

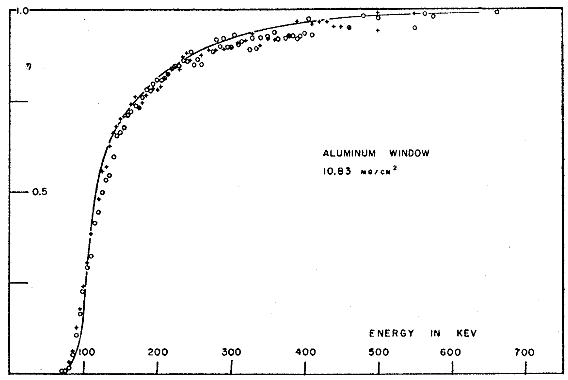

FIG. 2. Relative electron transmission η through a 10.83-mg/cm² aluminum window. See caption for Fig. 1 for explanation.

rent experiment are relatively thick, as G-M counter windows go, a thin Zapon window will transmit, within experimental error, 100 percent of the beta-particles at the energies under consideration. For this reason one can obtain spectra for the negatrons (Ag^{110}) and for the positrons (Cu^{61}) using a thin Zapon window and, by comparison with the spectra obtained using the thicker metallic windows, calculate a relative transmission curve for these thicker windows. This method could be applied quite simply to the Ag^{110} negatron source since its long half-life allowed the same source to be used for all measurements (with appropriate decay corrections). However, the short half-life of the Cu^{61} positron source forced the preparation of a new source for each set of data. The preparation of a Cu^{61} source has been, however, so standardized that it was possible to prepare two or more such sources almost identical one with the other. Corrections were made for small variations in the intensity of the different sources through comparison of the sources with a standard long-lived source under conditions of a standardized geometry.

When it can be applied and when measurements must be made on a number of different sources, the comparison method requires less expenditure of time than does the acceleration technique. However, slight discrepancies between the two techniques still appear in the energy region just above the window cutoff.

III. THE ELASTIC TRANSMISSION COEFFICIENT η_E

Monoenergetic electrons entering a foil may be elastically scattered through any (total) angle, depending upon the number of individual collisions between them and the atoms of foil and upon the angles of scattering in these collisions. The distribution of the electrons upon leaving the foil will be some function $P(\theta)$ of the (total) spatial angle of multiple scattering θ. The form of this function will depend upon the geometrical thickness L_0, the number density of the atoms in the foil N/V, the type of foil material (atomic number Z), the kinetic energy of the incident electrons E, and the polarity of the electron charge (negatrons, $Z'e=-e$, or positrons $Z'e=+e$). The elastic transmission coefficient η_E can then be expressed as

$$\eta_E=\int_0^{\theta_{\max}} P(\theta)d\theta, \tag{1}$$

where $\theta_{\max}$ is the maximum (total) angle through which an electron may be scattered by the G-M counter window foil and still enter the sensitive region of the G-M counter. For ease in calculation we shall normalize $P(\theta)$ in the interval $0\leq\theta<\infty$, even though physically $0\leq\theta<\pi$, since the form of $P(\theta)$ which we shall use is small for $\pi\leq\theta<\infty$.

In the present paper we will not attempt to derive the actual form of $P(\theta)$ from basic theoretical considerations. However, we will show that the crude assumption of a decreasing exponential function of θ for the scattering probability per unit solid angle will lead to results for η_E which can be brought into approximate agreement with the experimental observations. Such a normalized function is

$$P(\theta)d\theta=\alpha^2\exp(-\alpha\theta)\sin\theta d\theta\approx\alpha^2\exp(-\alpha\theta)\theta d\theta, \tag{2}$$

the choice of this function being initially justified on the basis that it is a simple function which at least roughly resembles the spatial distribution associated with the projected "Gaussian" plus "tail" distribution which has been used in most "small angle" multiple scattering theories. From Eqs. (1) and (2) one then obtains

$$\eta_E=1-\{\exp(-\alpha\theta_{\max})\}\{1+\alpha\theta_{\max}\},$$

with α determined via the mean square angle of multiple scattering by

$$\langle\theta^2\rangle=\int_0^{\pi}\theta^2P(\theta)d\theta=6/\alpha^2,$$

$$\alpha=2.45/\langle\theta^2\rangle^{\frac{1}{2}}. \tag{3}$$

There then remains only one arbitrary constant within the equation for η_E, namely, $\theta_{\max}$; for our counter ge-

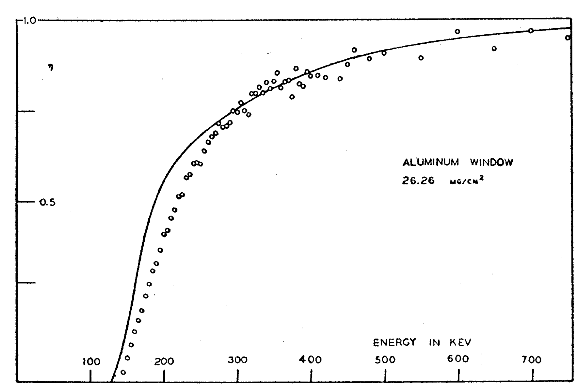

FIG. 3. Relative electron transmission η through a 26.26-mg/cm² aluminum window. See caption for Fig. 1 for explanation.

ometry, the assumption of $\theta_{\max}=1$ radian appears reasonable. Thus, the equation from which η_E has to be determined for each window becomes

$$\eta_E=1-\{\exp[-2.45/\langle\theta^2\rangle^{\frac{1}{2}}]\}\{1+2.45/\langle\theta^2\rangle^{\frac{1}{2}}\}. \quad (4)$$

The mean square angle of multiple scattering can now be found (in the "small angle" approximation) from an expression involving the single scattering probability distribution through an angle ϕ.[11] Using, in addition, in this expression a "spin orbit correction" factor $\gamma(\phi)$,[12] we obtain a formula for the mean square angle of multiple scattering,

$$\langle\theta^2\rangle=\frac{8\pi NL_0}{V}\left(\frac{ZZ'e^2}{pv}\right)^2\int_0^{\pi/2}\frac{\phi^2\gamma(\phi)d\phi}{\{\phi^2+\phi_{\min}{}^2\}^{\frac{3}{2}}}. \quad (5)$$

Here the foil contains NL_0/V atoms per unit area; p and $v=\beta c$ are, respectively, the momentum and velocity of the incident electron;

$$\phi_{\min}=\{1.14mcZ^{\frac{1}{3}}/137p\}\{1.13+3.76(ZZ'/137\beta)^2\}^{\frac{1}{2}},$$

and for small angles,

$$\gamma(\phi)=1-\beta^2\left(\frac{\phi}{2}\right)^2-[\pi ZZ'\beta/137](\phi/2)[1-\phi/2].$$

Actually, for large Z, higher powers of $ZZ'/137$ than the first contribute to $\gamma(\phi)$, but these contributions are relatively unimportant at the rather small ϕ which make the major contribution[13] to $\langle\theta^2\rangle$.

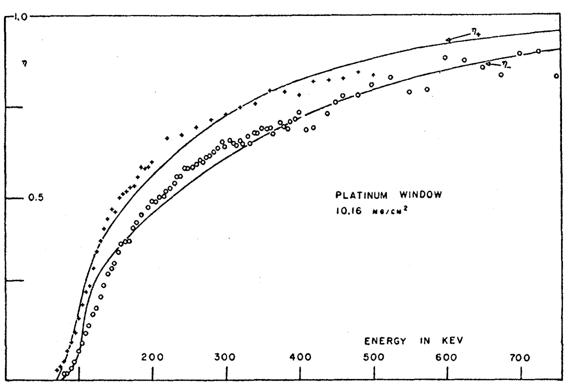

FIG. 4. Relative electron transmission η through a 10.16-mg/cm² platinum window. See caption to Fig. 1 for explanation. The theoretical transmission coefficient designated by the symbol η_+ is the one determined for positrons and that designated by the symbol η_- is the one determined for negatrons.

[11] G. Molière, Z. Naturforsch. **2a**, 133 (1947); **3a**, 78 (1948); S. Olbert, Phys. Rev. **87**, 319 (1952).

[12] W. A. McKinley and H. Feshbach, Phys. Rev. **74**, 1759 (1948); H. Feshbach, Phys. Rev. **88**, 295 (1952).

[13] We neglect the contribution to $\langle\theta^2\rangle$ for values of ϕ between $\pi/2$ and π. In this region both the Rutherford factor in Eq. (5) and the spin orbit correction factor are not accurately given by our very approximate expressions, but we may nevertheless estimate that the net contribution from this region in the integration over the correct single scattering distribution is small compared to the net contribution of the region between $\theta=0$ and $\theta=\pi/2$.

IV. THE INELASTIC TRANSMISSION COEFFICIENT η_I

The inelastic transmission coefficient is determined by the number of electrons which are stopped in the foil. Physically known quantities make it easier to calculate the number of electrons which are stopped rather than the number which penetrate the foil. Therefore it seems appropriate to define a quantity $\eta_S=1-\eta_I$ which represents the probability that an electron is stopped in the foil.

If an electron possessing a fairly high kinetic energy enters a piece of material having semi-infinite extent, it will continue to move until it has been robbed of essentially all its kinetic energy as a consequence of inelastic collisions involving the ionization and excitation of the atoms of the material. The total distance X (effective path length) which the electron travels before stopping will not be the same in all individual cases but will instead be distributed according to a probability function $P_S(X)$ which is in first approximation Gaussian:

$$P_S(X)dX=\exp(-y^2/2\langle y^2\rangle)dy/(2\pi\langle y^2\rangle)^{\frac{1}{2}}.$$

Here $y=X-R$, $\langle X\rangle=R$ is the range of the electron within the material, and $\langle y^2\rangle$, the mean square range straggling, is in first approximation,[14]

$$\langle y^2\rangle=(\langle X-R\rangle^2)=R^2/2\ln(E/I)$$
$$=E^4/32\pi^2(N/V)^2Z^2e^8[\ln(E/I)]^3,$$

where E is the kinetic energy of the incident electron and I is the average ionization energy (we use $13.6Z$ ev for this energy). The actual small difference between the R and the $\langle y^2\rangle$ values for (nonrelativistic) negatrons and for (nonrelativistic) positrons of a given energy is neglected. We then have

$$\begin{aligned}\eta_I&=1-\eta_S=1-\int_0^L P_S(X)dX\\&=1-\{1/(2\pi\langle y^2\rangle)^{\frac{1}{2}}\}\int_{-R}^{L-R}\exp(-y^2/2\langle y^2\rangle)dy\\&=\tfrac{1}{2}-\{1/(2\pi\langle y^2\rangle)^{\frac{1}{2}}\}\left\{\int_0^{L-R}\exp(-y^2/2\langle y^2\rangle)dy\right.\\&\qquad\left.-\int_R^\infty\exp(-y^2/2\langle y^2\rangle)dy\right\}, \quad (7)\end{aligned}$$

where L is the effective path length of an electron

[14] See, for example, H. W. Lewis, Phys. Rev. **85**, 20 (1952). Justification for the use of Lewis' theory of range and of range straggling for a *nonrelativistic* charged particle (negatron or positron) is based upon the fact that η_I affects only the lower energy portion of the transmission curve (see Fig. 1). For higher relativistic electron energies η_I is approximately unity and the transmission curve is determined solely from η_E.

traversing the foil. Using the results of Yang,[15] one has roughly, $L=(1+\langle\theta^2\rangle/4)L_0$; also, the second integral [in the last form of Eq. (7)] can be dropped since $R/(2\langle y^2\rangle)^{\frac{1}{2}}=[\ln(E/I)]^{\frac{1}{2}}\gg 1$, while the first integral can be found from available tables[16] and graphs.[17]

The results of these calculations for η_I, as well as those for η_E and the combined result $\eta=\eta_E\eta_I$, are shown for the 7.32-mg/cm² aluminum window in Fig. 1. Other graphs for the remaining windows show only the final theoretical result $\eta=\eta_E\eta_I$.

V. DISCUSSION

It will be noted that $\langle\theta^2\rangle$ plays a very important role in the present work, since it is instrumental in the determination of both η_E and η_I. Because of this use of the "scattering" approach to the problem of window transmission, it is valid to compare our results with those made through the study of the scattering by various materials of electrons in the energy range considered here (from about 30 kev to a few Mev). Qualitatively the results agree.

Our results show that, in a high Z material—platinum, at a given energy, the transmission coefficient for positrons is greater than for negatrons; in agreement with the observations[5–7] that there is a greater single scattering of negatrons than positrons at high Z. In addition, for a given foil surface density, the transmission coefficient for aluminum is greater than for platinum, in agreement with the observations[7,8] that the amount of single scattering increases with larger atomic number of the scattering material.

For comparison of our distribution function [Eq. (2)] with the measurements of Hisdal[10] on the scattering of 0.5-Mev electrons in an Ilford G5 emulsion, we must transform our distribution for spatial angles into an equivalent form for projected angles. Such a transformation shows that the projected distribution is approximately proportional to $\exp(-\alpha|\Theta|)[|\Theta|+1/\alpha]$ where Θ is the projected scattering angle. Using the data for Ilford G5 emulsions, as given by Voyvodic and Pickup,[18] and the cell length given by Hisdal, to determine α, our distribution is in rough agreement with Hisdal's experimental results.

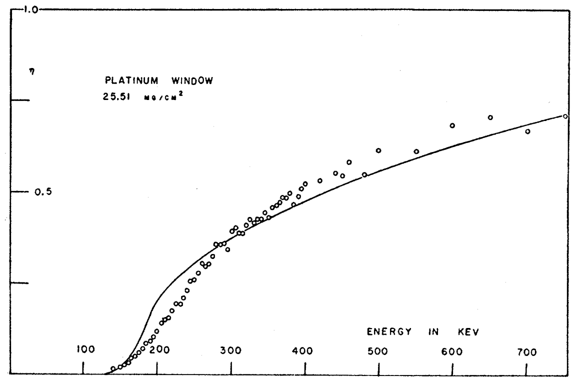

FIG. 5. Relative electron transmission η through a 25.51-mg/cm² platinum window. See caption to Fig. 1 for explanation.

As has already been mentioned, the distribution function for multiple elastic scattering used in the present calculations is of the same general shape in its "small angle" plane projected form as the more commonly used sum of a "Gaussian" plus "tail." Apart from this, possibly the only virtue of our distribution is its simplicity for numerical calculations. Considering this and the number of additional approximations which have been made in the determination of η_E and η_I, the theoretically determined curves for η are in not unsatisfactory agreement with the experimental observations.

These approximations, it will be recalled, are as follows: (a) the omission of any systematic treatment of the inelastic collision energy losses in the treatment of the multiple elastic scattering and the parallel omission of the effect of multiple elastic scattering in the treatment of the stopping probability due to inelastic collision;[19] (b) the use of a crude "small angle" exponential approximation to the "Gaussian" plus "tail" "small angle" multiple scattering distribution in a physical situation where some of the angles of multiple scattering become quite appreciable; and (c) the use of very approximate expressions for the electron range and range straggling. In spite of these perhaps mutually compensating approximations, the agreement between theory and experiment seems to indicate that a future rigorous calculation of the transmission coefficient η should yield results not too different from those developed here.

[15] C. N. Yang, Phys. Rev. **84**, 599 (1951). This formula is, however, applicable only to small angles.

[16] *Table of Probability Functions*, prepared by Federal Works Agency, Works Project Administration, sponsored by National Bureau of Standards, 1941.

[17] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), fourth edition, pp. 23–25.

[18] L. Voyvodic and E. Pickup, Phys. Rev. **85**, 91 (1952).

[19] Except in so far as our use of the approximate relation between L and L_0.