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The Adiabatic Hall Effect in Semiconductors*

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Calculations are carried out to determine the relative difference between the isothermal and adiabatic Hall coefficients of typical semiconductors. The isothermal Hall effect requires a constant temperature in the plane of the electric current flow and Hall field, whereas the adiabatic Hall effect is defined by the alternative requirements that the temperature gradient be zero in the direction of the electric current flow and the heat current be zero in the direction of the Hall field. The relative difference $(R_a-R_i)/R_i$ is studied for the (1) classical impurity semiconductor, (2) degenerate semiconductor, and (3) classical semiconductor at high temperatures. For the thermal and electrical conductivities characteristic of silicon and germanium, the relative differences are found to be of the order one percent or less, thus indicating that it usually is not necessary to distinguish between the isothermal and adiabatic conditions in interpreting measured Hall values. Tabulated values are presented in such form that the relative difference may be readily calculated for any semiconducting sample of known electrical conductivity, thermal conductivity, and variation of carrier density with temperature.

I. INTRODUCTION

The Hall effect[1] is the production of a transverse electric potential difference across a conducting sample as a result of the deflection of current carriers when a magnetic field is applied in a direction perpendicular to the current flow. At equilibrium the electrostatic force due to the Hall field balances the magnetic force on the moving charges composing the current. The Hall coefficient, R, is defined by the relation[2]

$$R=E_y/j_xH_z, \tag{1}$$

where j_x is the longitudinal electric current density, E_y the transverse electric field intensity, and H_z the magnetic field intensity. The transverse electric current density j_y is assumed to be zero. Kinetic theory of conduction indicates that, if the current carriers all have the same sign, R depends upon n, the number of carriers per unit volume, and e, the magnitude of charge on each carrier:

$$R=\pm r/ne, \tag{2}$$

where r is a numerical factor and the sign of R is the sign of the charge on the carriers.

Accurate determination of electron density from measured values of the Hall coefficient requires knowledge of the proportionality factor r, which depends upon (1) the statistical distribution of the electron velocities in the presence of electric and magnetic fields, (2) the dependence of the mean free path upon electron velocity, and (3) the temperature variation and heat flow in the sample.

The "isothermal" Hall effect[3] requires a constant temperature in the plane of the current and Hall field; i.e., the temperature gradients dT/dx and dT/dy must be zero. The absence of a temperature gradient does not insure the absence of the heat current resulting from the electrons if electric or magnetic fields are applied. Thus, the "adiabatic" Hall effect is defined by the alternative requirements that dT/dx and the heat current in the Y direction must be zero. In actual Hall effect measurements, it is usually impracticable to take the precautions necessary to eliminate the transverse temperature gradient; and so there is doubt as to whether the isothermal or the adiabatic conditions apply the more closely to an experimental situation. The object of this paper is to calculate the difference between the isothermal and adiabatic Hall coefficients under varying assumptions as to the nature of the sample; these assumptions correspond particularly to the observed characteristics of silicon, germanium, and tellurium samples.

II. GENERAL EXPRESSION FOR THE ADIABATIC HALL COEFFICIENT

The kinetic theory of conduction,[3–5] based upon the Lorentz solution of the Boltzmann equation, leads to the following set of equations for the electrical and thermal current densities:

$$\begin{aligned}
j_x&=L_{11}E_x+L_{12}H_zE_y+L_{13}dT/dx+L_{14}H_zdT/dy,\\
j_y&=-L_{12}H_zE_x+L_{11}E_y-L_{14}H_zdT/dx+L_{13}dT/dy,\\
w_x&=L_{31}E_x+L_{32}H_zE_y-(L_{33}+\kappa_L)dT/dx\\
&\qquad+L_{34}H_zdT/dy,\\
w_y&=-L_{32}H_zE_x+L_{31}E_y-L_{34}H_zdT/dx\\
&\qquad-(L_{33}+\kappa_L)dT/dy.
\end{aligned} \tag{3}$$

Here w_x and w_y represent thermal current per unit area in the X and Y directions, respectively, and κ_L is the

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[1] E. H. Hall, Am. J. Math. 2, 287 (1879); Phil. Mag. 10, 301 (1880).

[2] Derivations in this paper are carried out in the electromagnetic system of units.

[3] The terms "isothermal Hall effect" and "adiabatic Hall effect" were introduced by R. Gans, Ann. Physik 20, 293 (1906).

[4] H. A. Lorentz, *The Theory of Electrons* (G. E. Stechert and Company, New York, 1923), p. 63 and note 29.

[5] A. Sommerfeld and N. H. Frank, Revs. Modern Phys. 3, 1 (1931).

TABLE I. Coefficients appearing in the electrical and thermal current density equations[a] (current carried by electrons only[b]).

$$L_{11}=-\frac{4\pi e^2}{3}\int_0^\infty v^2l\frac{\partial f_0}{\partial\epsilon}vdv$$

$$L_{12}=\frac{4\pi e^3}{3m}\int_0^\infty vl^2\frac{\partial f_0}{\partial\epsilon}vdv$$

$$L_{13}=-\frac{4\pi e}{3}\int_0^\infty v^2l\frac{\partial f_0}{\partial\epsilon}\left\{\frac{\epsilon}{T}+T\frac{d}{dT}\left(\frac{\zeta}{T}\right)\right\}vdv$$

$$L_{14}=\frac{4\pi e^2}{3m}\int_0^\infty vl^2\frac{\partial f_0}{\partial\epsilon}\left\{\frac{\epsilon}{T}+T\frac{d}{dT}\left(\frac{\zeta}{T}\right)\right\}vdv$$

$$L_{31}=\frac{4\pi me}{6}\int_0^\infty v^4l\frac{\partial f_0}{\partial\epsilon}vdv$$

$$L_{32}=-\frac{4\pi e^2}{6}\int_0^\infty v^3l^2\frac{\partial f_0}{\partial\epsilon}vdv$$

$$L_{33}=-\frac{4\pi m}{6}\int_0^\infty v^4l\frac{\partial f_0}{\partial\epsilon}\left\{\frac{\epsilon}{T}+T\frac{d}{dT}\left(\frac{\zeta}{T}\right)\right\}vdv$$

$$L_{34}=-\frac{4\pi e}{6}\int_0^\infty v^3l^2\frac{\partial f_0}{\partial\epsilon}\left\{\frac{\epsilon}{T}+T\frac{d}{dT}\left(\frac{\zeta}{T}\right)\right\}vdv$$

[a] The symbol f_0 represents the unperturbed electron distribution function, l the mean-free-path, and ϵ the kinetic energy $mv^2/2$, where m is the effective mass. The symbol ζ represents the chemical potential (or Fermi level).

[b] Those coefficients which include an odd power of the electron charge are reversed in sign, and those with an even power of e retain the same sign if holes, rather than electrons, carry the current.

thermal conductivity associated with the lattice vibrations, as distinct from the thermal conductivity associated with kinetic energy transfer by conduction electrons. The coefficients L_{jk} are integrals dependent upon the mean free path, the distribution function, and the chemical potential (Fermi level); these coefficients are given explicitly in Table I.

The total thermal conductivity κ is defined as $-w_x/(dT/dx)$ under the conditions $j_x=dT/dy=H_z=0$. Equations (3) yield the expression

$$\kappa=(L_{13}L_{31}/L_{11})+L_{33}+\kappa_L\equiv\kappa_e+\kappa_L, \tag{4}$$

where κ_e represents the conduction electron contribution to the thermal conductivity.

The isothermal Hall coefficient R_i is found from Eqs. (3) by setting $j_y=dT/dx=dT/dy=0$ and solving for

$$R_i=E_y/j_xH_z=L_{12}(L_{11}^2+L_{12}^2H_z^2)^{-1}. \tag{5}$$

The weak field approximation that terms containing the second or higher power of the magnetic field strength are negligible is used in this paper; hence Eq. (5) reduces to

$$R_i=L_{12}/L_{11}^2. \tag{6}$$

The adiabatic Hall coefficient R_a is also obtained from Eqs. (3), but the auxiliary conditions are $j_y=dT/dx=w_y=0$. Thus one obtains

$$R_a=\frac{L_{12}\{L_{33}+\kappa_L+(L_{13}L_{32}/L_{12})\}}{L_{11}^2\{L_{33}+\kappa_L+(L_{13}L_{31}/L_{11})\}}. \tag{7}$$

Equations (4) and (6) are used to reduce the above to

$$R_a=R_i\left\{1+\frac{L_{13}}{\kappa}\left(\frac{L_{32}}{L_{12}}-\frac{L_{31}}{L_{11}}\right)\right\}. \tag{8}$$

This relation serves as the basis for comparing the adiabatic and isothermal Hall coefficients through evaluation of the ratio R_a/R_i or the relative difference $(R_a-R_i)/R_i$.

The following discussion explicitly applies to n-type or electron conductors, but an exactly parallel treatment applies to p-type or hole conductors; thus the results obtained in the following sections are equally valid for n- and p-type conductors.

III. THE ADIABATIC HALL COEFFICIENT FOR A CLASSICAL IMPURITY SEMICONDUCTOR

At low temperatures conduction in a semiconductor is due to electrons which have been excited from the impurity levels to the conduction band. At a temperature depending upon the sample, nearly all of the impurity atoms are ionized and "exhaustion" occurs. At higher temperatures electrons may be excited from the filled band to the conduction band (intrinsic conduction). The classical impurity region is defined as lying above the temperature at which it is necessary to use quantum statistics but below the temperature at which intrinsic conduction becomes significant. In this range one uses the distribution function

$$f_0=\frac{2m^3}{h^3}\exp\left(\frac{\zeta-\epsilon}{kT}\right), \tag{9}$$

where m denotes effective mass, ζ the chemical potential or Fermi level, and ϵ the electron kinetic energy; the zero of energy is taken as the bottom of the conduction band.

The coefficients L_{jk} of Eqs. (3) all involve integrals of the form

$$\int_0^\infty v^{2\alpha}l^\beta\partial f_0/\partial\epsilon vdv, \tag{10}$$

where 2α and β are integers. Define

$$u=\epsilon/kT; \tag{11}$$

then the integral (10) may be written as

$$-(2kT)^\alpha m^{-(\alpha+1)}\int_0^\infty u^\alpha l^\beta f_0du\equiv(2kT)^\alpha m^{-(\alpha+1)}K_{\alpha,\beta}. \tag{12}$$

Equation (8) combined with Table I and Eq. (12), gives the result

$$R_a=R_i\left\{1+\frac{\sigma T}{\kappa}\frac{k^2}{e^2}F(l,\zeta,T)\right\}, \tag{13}$$

where σ is the electrical conductivity (L_{11}) and where

$$F(l,\zeta,T)=\left\{\frac{K_{2,1}}{K_{1,1}}-\frac{K_{\frac{3}{2},2}}{K_{\frac{1}{2},2}}\right\}\left\{\frac{K_{2,1}}{K_{1,1}}+\frac{T}{k}\frac{d}{dT}\left(\frac{\zeta}{T}\right)\right\}. \tag{14}$$

The Fermi level for a classical semiconductor is given by

$$\zeta = kT \ln\left\{\frac{nh^3}{2(2\pi mkT)^{\frac{3}{2}}}\right\}, \tag{15}$$

and thus

$$\frac{d}{dT}\left(\frac{\zeta}{T}\right) = \frac{k}{n}\frac{dn}{dT} - \frac{3}{2}\frac{k}{T}. \tag{16}$$

In the classical impurity region, scattering of the conduction electrons both by the lattice and by impurity ions may be important. Hence it is of interest to consider the mean free path as given by

$$l = \gamma\epsilon^p. \tag{17}$$

When $p=0$, the mean free path is constant with respect to energy, corresponding to the case when lattice scattering[6] alone is present; if $p=2$, the mean free path is approximately that associated with impurity scattering.[7] One can write

$$K_{\alpha,\beta} = -2m^3h^{-3}e^{\zeta/kT}\{\gamma(kT)^p\}^\beta\Gamma(\alpha+p\beta+1), \tag{18}$$

and thus Eq. (14) reduces to

$$F(l, \zeta, T) = (\tfrac{1}{2}-p)\left(p+\frac{1}{2}+\frac{T}{n}\frac{dn}{dT}\right). \tag{19}$$

For lattice scattering only ($p=0$), Eq. (13) becomes

$$R_a = R_i\left\{1+\frac{\sigma T}{k}\frac{k^2}{e^2}\left(\frac{1}{4}+\frac{1}{2}\frac{T}{n}\frac{dn}{dT}\right)\right\}, \tag{20}$$

and for impurity scattering only ($p=2$), it becomes

$$R_a = R_i\left\{1+\frac{\sigma T}{\kappa}\frac{k^2}{e^2}\left(-\frac{15}{4}-\frac{3}{2}\frac{T}{n}\frac{dn}{dT}\right)\right\}. \tag{21}$$

It is seldom that either of the two limiting cases, completely dominant lattice scattering or completely dominant impurity scattering, exists throughout the impurity temperature range. The effective mean free path l for combined scattering may be taken as

$$1/l = 1/l_L + 1/l_I, \tag{22}$$

where $l_I = \gamma\epsilon^2 = \gamma(kT)^2u^2$ and l_L is independent of ϵ. Then

$$K_{\alpha,\beta} = -\frac{2m^3}{h^3}e^{\zeta/kT}l_L{}^\beta\int_0^\infty\frac{e^{-u}u^{\alpha+2\beta}du}{(u^2+b^2)^\beta}, \tag{23}$$

where

$$b^2 \equiv l_L/\{\gamma(kT)^2\} = 6\rho_I/\rho_L, \tag{24}$$

in which ρ_I and ρ_L are the resistivities due to impurity and lattice scattering, respectively.[8] Define the functions

$$G_k(b) = \int_0^\infty e^{-u}u^k(u^2+b^2)^{-1}du \tag{25}$$

and

$$H_k(b) = \int_0^\infty e^{-u}u^k(u^2+b^2)^{-2}du; \tag{26}$$

then Eq. (14) becomes

$$R_a = R_i\left[1+\frac{\sigma T}{\kappa}\frac{k^2}{e^2}\left\{\frac{G_4(b)}{G_3(b)}-\frac{H_{11/2}(b)}{H_{9/2}(b)}\right\}\right.$$
$$\left.\times\left\{\frac{G_4(b)}{G_3(b)}-\frac{3}{2}+\frac{T}{n}\frac{dn}{dT}\right\}\right]. \tag{27}$$

The required functions $G_k(b)$ and $H_k(b)$ have been evaluated by numerical methods.

Examination of Eqs. (20), (21), and (27) shows that the ratio R_a/R_i is dependent upon temperature, electrical conductivity, thermal conductivity, the temperature rate of change of carrier density, and the nature of the scattering encountered by the carriers. In the following paragraphs R_a/R_i is evaluated under several sets of conditions chosen to correspond to the observed characteristics of semiconducting samples.

A. The Gans Expression

Gans[3] considered only the electronic contribution to the thermal conductivity and took the mean free path as independent of energy. He used $\kappa/\sigma T = \kappa_e/\sigma T = 2k^2/e^2$ and hence found

$$\frac{R_a-R_i}{R_i} = \frac{1}{8}+\frac{1}{4}\frac{T}{n}\frac{dn}{dT}. \tag{28}$$

This indicates an appreciable difference between the isothermal and adiabatic Hall coefficients; the difference is even more pronounced for impurity scattering in which case one obtains

$$\frac{R_a-R_i}{R_i} = -\frac{3}{2}-\frac{3}{5}\frac{T}{n}\frac{dn}{dT}. \tag{29}$$

However, the measured heat conductivity of a semiconductor at ordinary temperature is far greater than that obtained by the assumption of electronic thermal current alone. This fact indicates a large contribution to heat conductivity from the lattice, so that Eqs. (28) and (29) are not valid.

B. Exhaustion Temperature Range: $dn/dT=0$

In many samples the conduction electron density does not change measurably over a portion of the tem-

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[6] A. Sommerfeld and H. Bethe, *Handbuch der Physik* (Julius Springer, Berlin, 1934), Vol. 24, No. 2, p. 558.

[7] E. Conwell and V. F. Weisskopf, Phys. Rev. **77**, 388 (1950).

[8] V. A. Johnson and K. Lark-Horovitz, Phys. Rev. **82**, 977 (1951).

TABLE II. $F(l, \zeta, T)$ as a function of the ratio of impurity to lattice scattering in a classical impurity semiconductor.

$\rho_I/(\rho_I+\rho_L)$	$b^2=6\rho_I/\rho_L$	$F(l, \zeta, T)$
0.0	0	0.2500
0.1	2/3	0.0816
0.2	3/2	−0.0160
0.3	18/7	−0.113
0.4	4	−0.250
0.5	6	−0.375
0.6	9	−0.581
0.7	14	−0.816
0.8	24	−1.15
0.9	54	−1.72
1.0	∞	−3.75

perature range. In this "exhaustion" region the assumption $dn/dT=0$ may be made and the function F simplified accordingly. Equation (20) leads to

$$\frac{R_a-R_i}{R_i}=\frac{1}{4}\frac{\sigma T}{\kappa}\frac{k^2}{e^2}, \tag{30}$$

which is applicable in the exhaustion range if only lattice scattering exists.

Measurements of the heat conductivity of germanium[9,10] give 0.59 watt/cm-°K at room temperature and little deviation from this value over a wide temperature range in the neighborhood of 300°K. Since the thermal conductivity of silicon[11] is about the same as for germanium, the $(R_a-R_i)/R_i$ values are also about the same for equivalent σT values; the thermal conductivity of tellurium[12] at 300°K is about one-tenth that of germanium and hence the fraction $(R_a-R_i)/R_i$ is ten times higher for tellurium than for germanium for a given σT value.

The relative difference between the adiabatic and isothermal Hall coefficients, as calculated from Eq. (30) with $\kappa=0.59$ watt/cm-°K, varies from 2×10^{-8} for $\sigma T=6$°K mho/cm to 3×10^{-3} for $\sigma T=9\times10^5$°K mho/cm. Hence one can conclude that the adiabatic and isothermal Hall coefficients are practically indistinguishable for the type of sample to which Eq. (30) applies.

If impurity scattering dominates, as may occur at the low temperature end of the exhaustion range for a low resistivity sample, Eq. (21) predicts that

$$\frac{R_a-R_i}{R_i}=-\frac{15}{4}\frac{\sigma T}{\kappa}\frac{k^2}{e^2}. \tag{31}$$

This relation is applicable under conditions such that σT is in the range 10^4-10^5 °K mho/cm, and thus Eq. (31) leads to $(R_a-R_i)/R_i$ values in the range -5×10^{-4} to -5×10^{-3}. Again the adiabatic and isothermal values are practically indistinguishable.

If both lattice and impurity scattering are considered, the function F is evaluated numerically in accordance with Eq. (27); the results are given in Table II. If one takes $\kappa=0.59$ watt/°K-cm and $\sigma T=3\times10^5$ °K-mho/cm, the fractional difference $(R_a-R_i)/R_i$ varies from +0.1 percent to −1.4 percent as the proportion of impurity scattering increases from zero to unity.

C. Low Temperature Range: $dn/dT\neq0$

The assumption $dn/dT=0$ is not a good approximation throughout the classical impurity region. An expression for dn/dT may be obtained from the classical equilibrium equation:[13]

$$\frac{n^2}{N-n}=\frac{2(2\pi mkT)^{\frac{3}{2}}}{h^3}\exp\left(-\frac{E_D}{kT}\right), \tag{32}$$

where N is the number of ionizable impurities (donors) per unit volume and E_D the activation energy required to excite an electron from the donor level to the conduction band. From Eq. (32) one obtains

$$\frac{dn}{dT}=\frac{n}{T}\left(\frac{3}{2}+\frac{E_D}{kT}\right)\left(\frac{N-n}{2N-n}\right). \tag{33}$$

TABLE III. Relative difference between adiabatic and isothermal Hall coefficients at 100°K calculated with dn/dT found from dissociation equation.

N cm^{-3}	E_D ev	n cm^{-3}	$\rho_I/(\rho_I+\rho_L)$	$F(l, \zeta, T)$	σ mho/cm	$(R_a-R_i)/R_i$
10^{15}	0.07	6.784×10^{14}	0.1	0.299	1.3	0.49×10^{-6}
			0.2	−0.0514	1.1	-0.072×10^{-6}
			0.3	−0.334	0.93	-0.39×10^{-6}
			0.4	−0.697	0.79	-0.70×10^{-6}
10^{17}	0.03	6.846×10^{16}	0.5	−0.685	66	-57×10^{-6}
			0.6	−1.02	53	-69×10^{-6}
			0.7	−1.39	42	-74×10^{-6}
10^{18}	0.01	6.875×10^{18}	0.8	−1.54	285	-560×10^{-6}
			0.9	−2.23	163	-460×10^{-6}

Table III gives values of $(R_a-R_i)/R_i$ calculated from Eq. (27) by using the dn/dT expression of Eq. (33). Numerical values of N, E_D, ρ_I/ρ_L, and σ have been chosen so as to be representative of germanium sample of varying degrees of purity; the value 0.59 watt/cm-°K has been used for the thermal conductivity. The largest difference, about 0.5 percent, is found in the impure material having $N=10^{18}$ per cm^3. Recent measurements[14,15] of the thermal conductivity of germanium at at low temperatures indicate that at 100°K the value

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[9] K. Lark-Horovitz, National Defense Council Report 14-585, p. 61, Nov., 1945, unpublished.

[10] A. Grieco and H. C. Montgomery, Phys. Rev. **86**, 570 (1952).

[11] Given as 0.84 watt/cm-°K by J. W. Mellor, *Comprehensive Treatise of Inorganic Chemistry* (Longmans, Green and Company, New York, 1925), Vol. VI, p. 152.

[12] W. W. Scanlon, Purdue Progress Report on Semiconductor Research (January, 1948).

[13] R. H. Fowler, Proc. Roy. Soc. (London) **A140**, 505 (1933).

[14] I. Estermann and J. E. Zimmerman, U. S. Office of Naval Research report from Carnegie Institute of Technology, 1950, unpublished.

[15] J. F. Goff, Purdue University, private communication.

of κ may be as high as 3 watt/cm-°K, and thus the $(R_a-R_i)/R_i$ values of Table III may be too high by a factor as large as 5.

IV. THE ADIABATIC HALL COEFFICIENT OF A DEGENERATE SEMICONDUCTOR

Hall effect measurements of germanium and silicon samples show that samples with large impurity content become degenerate at low temperatures.[16–18] In this case the classical distribution function of Eq. (9) is replaced by the Fermi-Dirac distribution function

$$\begin{aligned} f_0 &= \frac{2m^3}{h^3}\left\{1+\exp\left(\frac{\epsilon-\zeta}{kT}\right)\right\}^{-1} \\ &= \frac{2m^3}{h^3}(1+e^{u-\zeta^*})^{-1}, \end{aligned} \tag{34}$$

where $\zeta^*=\zeta/kT$. In a degenerate sample the impurity content is high and the temperature relatively low; thus impurity scattering dominates and one can take $l=l_I=\gamma\epsilon^2=\gamma(kT)^2u^2$. Equation (18) is replaced by

$$\begin{aligned} K_{\alpha,\beta} &= -\frac{2m^3}{h^3}\gamma^\beta(kT)^{2\beta}\int_0^\infty \frac{u^{\alpha+2\beta}e^{u-\zeta^*}du}{(1+e^{u-\zeta^*})^2} \\ &= -\frac{2m^3}{h^3}\gamma^\beta(kT)^{2\beta}(\alpha+2\beta)\int_0^\infty \frac{u^{\alpha+2\beta-1}du}{1+e^{u-\zeta^*}} \\ &= -\frac{2m^3}{h^3}\gamma^\beta(kT)^{2\beta}(\alpha+2\beta)J_{\alpha+2\beta-1}(\zeta^*), \end{aligned} \tag{35}$$

where $J_k(\zeta^*)$ is the Fermi-Dirac integral of kth order. With the change of statistics, $d/dT(\zeta/T)$ is no longer given by Eq. (16); instead

$$\frac{d}{dT}\left(\frac{\zeta}{T}\right)=\left(\frac{2k}{n}\frac{dn}{dT}-\frac{3k}{T}\right)\frac{J_{\frac{1}{2}}(\zeta^*)}{J_{-\frac{1}{2}}(\zeta^*)}. \tag{36}$$

Now the adiabatic Hall coefficient can be written in the form of Eq. (13) with

$$\begin{aligned} F(l,\zeta,T) &= \left\{\frac{4}{3}\frac{J_3(\zeta^*)}{J_2(\zeta^*)}-\frac{11}{9}\frac{J_{9/2}(\zeta^*)}{J_{7/2}(\zeta^*)}\right\} \\ &\times\left\{\frac{4}{3}\frac{J_3(\zeta^*)}{J_2(\zeta^*)}-\left(3+\frac{2T}{n}\frac{dn}{dT}\right)\frac{J_{\frac{1}{2}}(\zeta^*)}{J_{-\frac{1}{2}}(\zeta^*)}\right\}. \end{aligned} \tag{37}$$

The integral $J_k(\zeta^*)$ has been discussed by McDougall and Stoner,[19] who treat the evaluation of the integral in three different ways, depending upon the value of ζ^*:

(1) $\zeta^*\leq 0$:

$$J_k(\zeta^*)=\sum_{r=1}^{\infty}(-1)^{r-1}e^{r\zeta^*}r^{-(k+1)}\Gamma(k+1). \tag{38}$$

(2) $\zeta^*\gg 0$: An asymptotic expansion due to Sommerfeld[20] can be used:

$$J_k(\zeta^*)=(\zeta^*)^{k+1}\{1+\sum_{r=1}^{n}a_{2r}(\zeta^*)^{-2r}\}+\mathcal{R}_{2n}, \tag{39}$$

where

$$a_{2r}=2C_{2r}(k+1)(k)\cdots(k-2r+2),$$

$$C_{2r}=(1-2^{1-2r})\sum_{m=1}^{\infty}m^{-2r},$$

and the remainder $\mathcal{R}_{2n}$ is given an upper limit by the relation

$$\mathcal{R}_{2n}<(2n+2)a_{2n}+2(\zeta^*)^{k-2n-1}.$$

(3) ζ^* a small positive quantity: For ζ^* between 0 and about 4, the error incurred by using the asymptotic expression (39) is prohibitive. In this range $J_k(\zeta^*)$ must be found by numerical methods. The integrals $J_{\frac{1}{2}}(\zeta^*)$, and $J_{-\frac{1}{2}}(\zeta^*)$ have been tabulated by McDougall and Stoner[11] at close intervals in the range $-4\leq\zeta^*\leq 20$. Table IV gives values of $J_k(\zeta^*)$, for $k=2$, 3, 7/2, and 9/2, evaluated by the authors for use in this paper.

Table IV also gives value of $F(l,\zeta,T)$ calculated from Eq. (37) for $-4\leq\zeta^*\leq 20$; dn/dT is taken as zero in this evaluation. As ζ^* assumes large positive values, F approaches zero; this is consistent with the conclusion of Sommerfeld that $R_a=R_i$ for a metal.

Estermann and Zimmerman[14] have measured the thermal conductivity, at low temperatures, of an

TABLE IV. Values of the Fermi-Dirac integral $J_k(\zeta^*)$ and $F(l,\zeta,T)$ for a degenerate semiconductor (impurity scattering only).

ζ^*	$J_2(\zeta^*)$	$J_3(\zeta^*)$	$J_{7/2}(\zeta^*)$	$J_{9/2}(\zeta^*)$	$-F(l,\zeta,T)$
−4.0	0.03656	0.10977	0.21287	0.95830	3.736
−3.5	0.06018	0.18084	0.35078	1.57956	3.729
−3.0	0.09896	0.29780	0.57782	2.60314	3.714
−2.5	0.16252	0.49002	0.95137	4.28883	3.691
−2.0	0.26626	0.80532	1.56496	7.06295	3.656
−1.5	0.43456	1.32088	2.57067	11.6329	3.612
−1.0	0.70510	2.15984	4.21327	19.105	3.560
−0.5	1.1343	3.51520	6.88232	31.347	3.396
0.0	1.803	5.68220	11.184	51.290	3.324
1.0	4.312	14.39	28.83	135.4	2.773
2.0	9.445	34.30	70.76	346.6	2.232
3.0	18.87	75.73	162.6	846.2	1.736
4.0	34.49	154.3	346.7	1947	1.34
5.0	58.12	291.0	687.3	4200	1.04
6.0	91.74	513.0	1273	8500	0.814
7.0	137.36	853.4	2223	16 206	0.647
8.0	196.99	1351	3687	29 279	0.522
9.0	272.61	2051	5847	50 430	0.429
10	366.23	3005	8926	83 281	0.357
11	479.86	4269	13 184	132 532	0.301
12	615.48	5806	18 920	204 150	0.257
13	775.10	7986	26 482	305 550	0.223
14	960.72	10 583	36 257	445 804	0.194
15	1174.3	13 778	48 684	635 840	0.170
16	1418.0	17 659	64 248	888 663	0.151
17	1693.6	22 318	83 488	1 219 590	0.134
18	2003.2	29 854	106 992	1 646 455	0.120
19	2348.8	34 373	135 402	2 189 874	0.108
20	2732.5	41 985	169 419	2 873 483	0.0981

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impure germanium sample having a 25°C resistivity of 0.0021 ohm-cm. They found κ (in watt/cm-°K) equal to 1.9 at 65°K, 1.1 at 15°K, and 0.05 at 3.6°K. If these κ values are used, Eqs. (37) and (13) yield values of $(R_i-R_a)/(\sigma R_i)$, at 65°K, dropping from 9.3×10^{-6} to 0.89×10^{-6} ohm-cm as ζ^* increases from -4 to 10; the corresponding variation at 15°K is from 3.8×10^{-6} to 0.36×10^{-6}, and at 3.6°K the range is from 2.0×10^{-6} to 0.19×10^{-6}. Since σ for degenerate germanium samples is between 100 and 1000 mho/cm in this temperature range, one finds that $(R_i-R_a)/R_i$ is of the order 10^{-4} to 10^{-3} and thus that there is no appreciable difference between the adiabatic and isothermal Hall coefficients of a degenerate germanium sample.

V. THE ADIABATIC HALL COEFFICIENT OF A CLASSICAL SEMICONDUCTOR AT HIGH TEMPERATURES

The high temperature behavior of a semiconductor is characterized by thermal excitation of electrons from the full band to the conduction band. When both electrons and holes take part in conduction, the two currents are additive. It is convenient to retain the formal notation for the electrical and thermal current density equations (3) in applying them to semiconductors displaying both electron and hole conduction. This may be done by defining "total" coefficients $L_{jk}(t)$ which are appropriate algebraic sums of coefficients $L_{jk}(1)$, ascribed to electrons, and $L_{jk}(2)$, due to holes. The $L_{jk}(1)$ and $L_{jk}(2)$ coefficients are the expressions of Table I except that the mass, mean free path, and carrier density carry the subscript 1 (electrons) or 2 (holes). The difference in the sign of charge is correctly taken into account if those coefficients $L_{jk}(2)$, such as $L_{12}(2)$, which include e to an odd power are prefixed by a negative sign. By following this convention one obtains

$$\begin{aligned} L_{11}(t)&=L_{11}(1)+L_{11}(2), & L_{12}(t)&=L_{12}(1)-L_{12}(2),\\ L_{13}(t)&=L_{13}(1)-L_{13}(2), & L_{14}(t)&=L_{14}(1)+L_{14}(2),\\ L_{31}(t)&=L_{31}(1)-L_{31}(2), & L_{32}(t)&=L_{32}(1)+L_{32}(2),\\ L_{33}(t)&=L_{33}(1)+L_{33}(2), & L_{34}(t)&=L_{34}(1)-L_{34}(2). \end{aligned} \tag{40}$$

Since the sign of charge has now been provided for, e denotes only the magnitude of the electron charge, a positive quantity, in all of the $L_{jk}(1, 2)$ integrals.

TABLE V. Coefficients $L_{jk}(t)$ for a high temperature semiconductor.

$$L_{11}(t)=n_1e\mu_1+n_2e\mu_2$$

$$L_{12}(t)=-\frac{3\pi e}{8}(n_1\mu_1^2-n_2\mu_2^2)$$

$$L_{13}(t)=\frac{k}{2}(n_1\mu_1-n_2\mu_2)+kT\left(\mu_1\frac{dn_1}{dT}-\mu_2\frac{dn_2}{dT}\right)$$

$$L_{14}(t)=\frac{3\pi}{8}kT\left(\mu_1^2\frac{dn_1}{dT}-\mu_2^2\frac{dn_2}{dT}\right)$$

$$L_{31}(t)=-2kT(n_1\mu_1-n_2\mu_2)$$

$$L_{32}(t)=\frac{9\pi}{16}kT(n_1\mu_1^2+n_2\mu_2^2)$$

$$L_{33}(t)=\frac{3k^2T}{e}(n_1\mu_1+n_2\mu_2)+2\frac{k^2T^2}{e}\left(\mu_1\frac{dn_1}{dT}+\mu_2\frac{dn_2}{dT}\right)$$

$$L_{34}(t)=\frac{9\pi}{16}\frac{k^2T}{e}\left(n_1\mu_1^2-n_2\mu_2^2+T\mu_1^2\frac{dn_1}{dT}-T\mu_2^2\frac{dn_2}{dT}\right)$$

$$\mu_{1,2}=\frac{2}{3}\frac{(2)^{\frac{1}{2}}}{\pi}el_{1,2}(m_{1,2}kT)^{-\frac{1}{2}}$$

Classical statistics and a mean free path independent of energy are used in evaluating $L_{jk}(t)$. Table V gives these coefficients in terms of the electron and hole mobilities, μ_1 and μ_2, respectively. Equation (8) is valid if each L_{jk} appearing therein is taken as a "total" $L_{jk}(t)$. Thus one gets

$$R_a=R_i\left(1+\frac{T}{\kappa}\frac{k^2}{e^2}AB\right), \tag{41}$$

where

$$A=\mu_2(n_1c-n_2)+2\mu_2T(cdn_1/dT-dn_2/dT),$$

$$B=\frac{n_1c-n_2}{n_1c+n_2}-\frac{3}{4}\frac{n_1c^2+n_2}{n_1c^2-n_2},$$

and $c=\mu_1/\mu_2$. The quantities dn_1/dT and dn_2/dT are estimated from the equilibrium condition:[21]

$$n_1n_2=32h^{-6}(m_1m_2)^{\frac{3}{2}}(\pi kT)^3\exp(-E_G/kT), \tag{42}$$

where E_G is the width of the forbidden energy gap. If $n_1=n_2+N$ and $dN/dT=0$, then

$$\frac{dn_1}{dT}=\frac{dn_2}{dT}=\frac{n_1n_2}{T}\left(3+\frac{E_G}{kT}\right)(N^2+4n_1n_2)^{-\frac{1}{2}}. \tag{43}$$

The relative difference between R_a and R_i is calculated for germanium samples from Eq. (41) and presented in Table VI. The following numerical values were used in these computations:

$E_G=0.74$ ev.,[22] $c=1.5$,[23]
$\mu_1=1.7\times10^7T^{-\frac{3}{2}}$ cm^2/volt-sec,[24]
$\kappa=0.586$ watt/cm-°K.[9,10]

The values of Table VI show again that there is no appreciable difference between R_a and R_i since $(R_a-R_i)/R_i$ is of the order of one percent or less for temperatures as high as 900°K and conduction electron concentrations as low as 10^{15} cm^{-3}.

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VI. SUMMARY

The relative difference between the adiabatic and isothermal Hall coefficients has been calculated for various sets of assumptions chosen to correspond to the observed behavior of semiconducting samples in the different temperature ranges.

A. Classical Impurity Semiconductor in the Exhaustion Range

In this case classical statistics are employed, both impurity and lattice scattering are considered, and the carrier density is taken as independent of the temperature. The quantity F of Eq. (13) takes values between 0.25 and -3.75. When this is combined with a thermal conductivity of 0.6 watt/°K-cm and an electrical conductivity of 1000 mho/cm, $(R_a-R_i)/R_i$ varies between 10^{-3} and 10^{-2} at 300°K.

B. Classical Impurity Semiconductor at Temperatures below the Exhaustion Range

The variation of the carrier density with temperature is calculated from the dissociation equation, and the conductivities used are varied to correspond to different impurity contents in germanium. The resulting $(R_a-R_i)/R_i$ values at 100°K range between 10^{-6} and 10^{-3}, depending upon impurity content and impurity activation energy.

C. Degenerate Impurity Semiconductor

This treatment applies at low temperatures to samples having high impurity content. Fermi-Dirac statistics are employed, impurity scattering is assumed to be dominant, and the carrier density is approximately temperature independent. As ζ^* goes from negative values, through zero, to increasing positive values, F values go from -3.75 toward 0, and hence $(R_a-R_i)/R_i$ approaches zero under these conditions.

TABLE VI. $(R_a-R_i)/R_i$ for a semiconductor at high temperatures.

T(°K)	N(cm^{-3})	n_1(cm^{-3})	n_2(cm^{-2})	$(R_a-R_i)/R_i$
300	1.0×10^{18}	1.0×10^{18}		50×10^{-5}
300	0.1×10^{18}	0.1×10^{18}		5×10^{-5}
300	0.001×10^{18}	0.001×10^{18}		0.05×10^{-5}
600	1.0×10^{18}	1.01×10^{18}	0.014×10^{18}	-4×10^{-5}
600	0.1×10^{18}	0.18×10^{18}	0.077×10^{18}	-60×10^{-5}
600	0.001×10^{18}	0.12×10^{18}	0.12×10^{18}	-180×10^{-5}
900	1.0×10^{18}	3.0×10^{18}	3.0×10^{18}	-5.5×10^{-3}
900	0.1×10^{18}	1.6×10^{18}	1.6×10^{18}	-12×10^{-3}
900	0.001×10^{18}	1.4×10^{18}	1.4×10^{18}	-13×10^{-3}

D. Classical Semiconductor at High Temperatures

The expression for $(R_a-R_i)/R_i$ reflects the fact that both electrons and holes take part in conduction. The magnitude of the relative difference increases strongly with temperature, especially for relatively pure material. However, even at 900°K, $(R_a-R_i)/R_i$ is not over about 10^{-2} for germanium and silicon samples.

Thus one can conclude that the adiabatic and isothermal Hall coefficients are practically indistinguishable as long as the ratio of electrical conductivity to thermal conductivity, σ/κ, does not appreciably differ from the values, characteristic of germanium and silicon, used in the foregoing calculations. For a material such that σ/κ is appreciably larger, the relative difference between R_a and R_i may be readily estimated from the tabulated values of the $F(l, \zeta, T)$ function.

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