Corrected Values of Fowler-Nordheim Field Emission Functions $v(y)$ and $s(y)$

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Corrected values are given for the function $v(y)$ which appears in the exponent of the Fowler-Nordheim equation for field emission from a metal. Values are also given for the function $s(y)$ which relates to the slope of logarithmic plots of the Fowler-Nordheim equation.

NORDHEIM[1] has given an expression for a function $v(y)$, which appeared when he calculated the probability that an electron might pass through a potential barrier composed of a mirror-image potential plus a linear potential that resulted from an applied electric field. This function appears in the exponent of the Fowler-Nordheim equation for electron field emission from a metal. The variable y is a nondimensional function of the applied electric field and the surface work-function.

Burgess and Kroemer have recently independently discovered a mathematical error in Nordheim's analysis. The correct expression for $v(y)$ is:

$$v(y)=2^{-\frac{1}{2}}[1+(1-y^2)^{\frac{1}{2}}]^{\frac{1}{2}} \times\{E(k^2)-y^2K(k^2)/[1+(1-y^2)^{\frac{1}{2}}]\},$$

where

$$E(k^2)=\int_0^{\pi/2}(1-k^2\sin^2\phi)^{\frac{1}{2}}d\phi,$$

$$K(k^2)=\int_0^{\pi/2}(1-k^2\sin^2\phi)^{-\frac{1}{2}}d\phi,$$

$$k^2=2(1-y^2)^{\frac{1}{2}}/[1+(1-y^2)^{\frac{1}{2}}].$$

$K(k^2)$ and $E(k^2)$ are the complete elliptic integrals of the first and second kind. In his paper Nordheim defined k incorrectly.

This error renders incorrect Houston's[2] tables of $v(y)$ and $s(y)$, the latter being defined as: $s(y)=v(y)-\frac{1}{2}ydv/dy$. The function $s(y)$ appears in expressions[2] for the slope of logarithmic plots of the Fowler-Nordheim equation. In Table I are listed recalculated values of $v(y)$ and $s(y)$ correct to four figures.

TABLE I. Values of $v(y)$ and $s(y)$.

y	$v(y)$	$s(y)$
0	1.0000	1.0000
0.05	0.9948	0.9995
0.1	0.9817	0.9981
0.15	0.9622	0.9958
0.2	0.9370	0.9926
0.25	0.9068	0.9885
0.3	0.8718	0.9835
0.35	0.8323	0.9777
0.4	0.7888	0.9711
0.45	0.7413	0.9637
0.5	0.6900	0.9554
0.55	0.6351	0.9464
0.6	0.5768	0.9366
0.65	0.5152	0.9261
0.7	0.4504	0.9149
0.75	0.3825	0.9030
0.8	0.3117	0.8903
0.85	0.2379	0.8770
0.9	0.1613	0.8630
0.95	0.0820	0.8483
1.	0	0.8330

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[1] L. Nordheim, Proc. Roy. Soc. (London) **121**, 626 (1928).
[2] J. M. Houston, Phys. Rev. **88**, 349 (1952).