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The Energy Loss of Protons in Metallic Foils and Mica*

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The rate of loss of energy of 500- to 1300-kev protons in various metals and mica has been determined. Foils of Be, Al, Cu, and Au were prepared by evaporation techniques, and the density of the foil was determined by weighing the amount of metal deposited on a known area. The energy loss rate $\Delta E/\Delta x$ was measured with a standard deviation of <3 percent. The energy loss curve was found to join smoothly to the previously measured curve for lower energy protons ($50 \text{ keV} < E < 350 \text{ keV}$), and to approach the theoretical curves of Bethe and Bloch at the high energy end.

I. INTRODUCTION

THE energy loss of charged particles in matter has been of interest since the early days of nuclear physics. The range of an energetic charged particle has long been used to measure its energy; but the importance of the energy loss phenomenon is due not only to its close connection with the problem of range-energy relations, but also in that it allows an evaluation of our ideas concerning the interactions of charged particles with atoms and electrons.¹

The problem of particle interaction with matter was first experimentally investigated by observing the range and ionization of naturally occurring α -particles in various gases.² To understand better the resultant data, the problem was investigated theoretically on a classical basis by Bohr³ and from a quantum-mechanical standpoint by Bethe⁴ and Bloch.⁵ These equations could be tested by determining the energy loss of α -particles in foils of known thickness.

The first observations on protons were made by utilizing the recoil protons resulting from collisions of α -particles in gases containing an admixture of hydro-

gen.⁶ The range-energy curve for protons over a wider energy range was constructed by relating the proton curve to that for α -particles (which was based on more plentiful and accurate data) by means of formulas, both theoretical² and empirical.⁷

The development of particle accelerators provided a source of protons of variable and accurately known energies, and observations on the range of artificially accelerated protons were made in 1937.⁸ While the experimental results of energy loss and range agree suitably with the theory for high particle energies,¹ the agreement is unsatisfactory at lower energies where the assumptions made in deriving the formulas are increasingly invalid. For the regions in which the theory is not applicable resort must be made to empirically determined data if the energy loss of particles is to be known for these energies. The energy loss in metallic foils for protons with energies in the region 50 to 400 keV has been measured in this laboratory using the Cockcroft-Walton proton accelerator.⁹ Some measurements have been made at higher energies in gold¹⁰ and beryllium.¹¹

In order to extend the range of measurement, the Van de Graaff accelerator of the Institute of Radiobiology and Biophysics was used to measure the energy loss of

* Supported in part by the U. S. Atomic Energy Commission.

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¹ A. E. Taylor, Repts. Progr. Phys. **15**, 49 (1952).

² M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 245 (1937). This article contains an extensive bibliography and review of experimental and theoretical work done prior to 1937.

³ N. Bohr, Phil. Mag. **25**, 10 (1913).

⁴ H. A. Bethe, Ann. Physik **5**, 325 (1930).

⁵ F. Bloch, Ann. Physik **16**, 285 (1935).

⁶ P. M. S. Blackett and D. S. Lees, Proc. Roy. Soc. (London) **134**, 658 (1932).

⁷ W. P. Jesse and J. Sadauskis, Phys. Rev. **75**, 1110 (1949); **78**, 1 (1950).

⁸ Parkinson, Herb, Bellamy, and Hudson, Phys. Rev. **52**, 75 (1937).

⁹ S. D. Warshaw, Phys. Rev. **76**, 1759 (1949).

¹⁰ T. Huus and C. B. Madsen, Phys. Rev. **76**, 323 (1949).

¹¹ C. B. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 648 (1948); **74**, 1782 (1948).

protons with energies between 500 and 1400 kev in several metals and mica.¹²

II. APPARATUS

In order to obtain as uniform foils as possible, evaporation techniques were used in preparing them.¹³ Glass microscope slides were successively washed with detergent, distilled water, hydrogen peroxide, ether, and ethyl alcohol, and then dried. These slides were then dipped into a 5 percent solution of Parlodion (celluloid) in equal parts of ether and alcohol, and dried. The Al, Cu, and Au foils were deposited onto the coated slides by evaporation from a heated tungsten filament in a vacuum of less than 10^{-4} mm Hg. An equal amount of metal was deposited simultaneously onto two previously weighed platinum foils exposed behind a mesh with circular apertures of known area.

The Parlodion coating containing the metal foil was then stripped from the glass and mounted on rectangular frames with an opening of approximately one by one-half centimeter. The remaining Parlodion backing was removed by immersing the frame and foil into the ether-alcohol solvent. The last traces of the backing were removed by placing the frames cooled by dry ice above the gently warmed solvent mixture.

The proton beam was produced by a Van de Graaff electrostatic accelerator.¹⁴ The construction of the machine is similar to that of the previous models.¹⁵ After acceleration and magnetic analysis, the emerging proton beam defined by the Van de Graaff slit system struck a beryllium target button supporting an evapo-

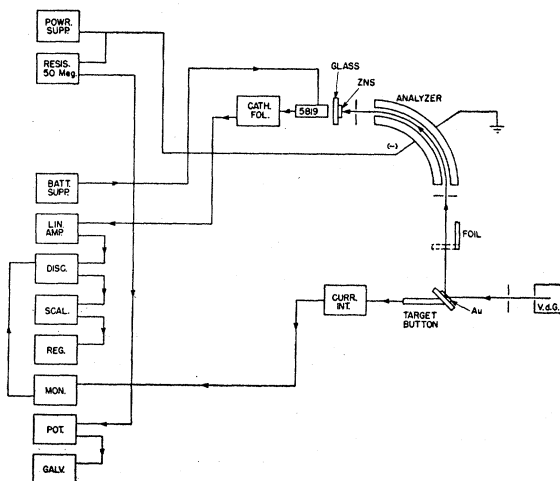


Fig. 1. Schematic diagram of the apparatus and a plan of the electrical circuits. The arrows indicate the flow of information.

¹² I wish to thank Dr. Raymond Zirkle and Dr. William Bloom for their kind permission to use the accelerator, and Mrs. L. Wang for preparing the mica foils.

¹³ S. D. Warshaw, *Rev. Sci. Instr.* **20**, 623 (1949).

¹⁴ The accelerator, model AH, was built by the High Voltage Engineering Company, Cambridge, Massachusetts.

¹⁵ Buechner, Van de Graaff, Spurduto, McIntosh, and Burrill *et al.*, *Rev. Sci. Instr.* **18**, 754 (1947).

rated layer of gold approximately 0.0076 mg/cm² thick. The thickness of the gold layer was determined from the evaporation geometry. The beryllium button was insulated from the rest of the apparatus, and the current from the button to ground was used to monitor the proton current. Those protons scattered through a ninety degree angle either (a) passed through an electrostatic analyzer, giving the initial energy E_0 ,¹⁶ or (b) first passed through the metal foil between the button and the entrance slit of the analyzer, giving the final energy E_f . The scattered protons, rather than the direct beam from the Van de Graaff, were used for the dE/dx measurements, since there was sufficient intensity and thin foils were more liable to rupture in the direct beam.

The analyzer thus measured either the energy distribution with its peak at E_0 of the protons scattered from the gold layer of the target button, or if the metal foil was placed in the beam, the energy spread with its peak at E_f of the proton beam after penetrating the foil. The most probable energy loss is then $E_0 - E_f$.¹⁷

The apparatus for supporting the foils was arranged so that two foils could be placed in the vacuum system at once. A stop was provided so that positioning the foils perpendicular to the proton beam could be easily done once the equipment was aligned with respect to the proton beam.

A schematic diagram of the apparatus and electronic control circuits is shown in Fig. 1.

For proton energies between 400 and 800 kev, a 90° electrostatic analyzer with a mean radius of 25.400 cm was used. With the slit positions and charging arrangement used in the present work the value of the constant k ¹⁸ of the analyzer was determined to be 19.77 with a standard deviation of 0.11.¹⁹ For energies between 800 and 1400 kev, a 90° cylindrical electrostatic analyzer with a mean radius of 15.167 cm and a constant of 45.23 was used.¹⁸

The exit port of the analyzer housing was closed by a glass disk, covered on the inside by an evaporated layer of aluminum. The metal was removed from an area one-quarter inch wide and one inch high directly in back of the exit slit. This transparent area was first covered with a very thin layer of silicone grease and then covered with a single layer of phosphorescent ZnS crystals (RCA type 33-Z-20A) having an average diameter of 15μ .

The aluminum coating was used to diminish the pulses produced in the phototube by discharges in the analyzer at high voltages.

¹⁶ Equal to $(196/198) E_A$, where E_A is the energy of the protons leaving the accelerator.

¹⁷ It can be shown that the average energy loss is also equal to $E_0 - E_f$.

¹⁸ The analyzer constant k is given by $k = W_0 / (V_1 - V_2)$, where W_0 is the kinetic energy, in electron kilovolts, of a proton as it leaves the foil, in a region of zero (ground) potential; and V_1 and V_2 are the potentials which, when applied to the inner and outer plates of the analyzer, will cause a proton to pass through the instrument and through the defining exit slit.

¹⁹ Allison, Skaggs, and Smith, *Phys. Rev.* **54**, 171 (1938).

A type 5819 photoelectron multiplier tube was placed directly against the outer surfaces of the glass disk to detect the scintillations caused by the protons hitting the ZnS crystals. The phototube was powered by batteries. The pulses emitted from the phototube after passing through a cathode follower stage entered a linear amplifier of conventional design,²⁰ where the average signal pulse height was increased to about twenty volts. The pulses then passed to a Schmitt discriminator circuit²¹ and thence to a scaler and register.

The current from the proton beam which struck the Be target button was measured by a current integrator and used to monitor the proton counts. To eliminate errors due to unsteadiness in the output current of the Van de Graaff accelerator, the discriminator circuit was turned off after a predetermined amount of current had struck the button. The time for each run was measured as a check on the constancy of output from the accelerator.

The deflecting voltage for the electrostatic analyzer was supplied by a 50-kv voltage doubler circuit of conventional design.²² The analyzer plate voltage was measured by determining the current drain through a 50 megohm ± 0.1 percent resistor stack in parallel with the analyzer plate by means of a 50 ohm ± 0.01 percent shunt and potentiometer (Rubicon type 2703). The potentiometer was also used to measure the analyzing magnet current by means of a shunt in the circuit as a check on the constancy of the energy of the beam from the Van de Graaff accelerator.

III. PROCEDURE OF MEASUREMENT

The Van de Graaff accelerator was prepared for a run by adjusting the analyzing magnet of the accelerator to its proper value and then varying the other parameters of the machine (charging current, corona point spacing, focus, etc.) until a steady beam of the desired energy was produced. The vertical position of the emergent beam was then adjusted for the varying type of foil holder used.

When the equipment had been adjusted for maximum geometrical transmission for each run, the analyzer voltage was first roughly adjusted for maximum counting rate with the foil either in or out of the beam. This value of the voltage was determined by means of the potentiometer. The potentiometer was then set at a value approximately 0.3 kev higher than the maximum, and successive runs were made for equal current hitting the target button while lowering the analyzer voltage 0.05 kev each time (Fig. 2). The total number of protons and the duration of each run was recorded. Usually a duration of ~ 0.5 min produced enough counts so that the statistical error was sufficiently low. The error signal of

²⁰ T. E. Cranshaw and J. A. Harvey, *Can. J. Research* **26**, 243 (1948).

²¹ W. C. Elmore and M. Sands, *Electronics* (McGraw-Hill Book Company, Inc., New York, 1949), pp. 99-103.

²² The 0-50 kv DC Power Supply Model No. 2008 was manufactured by the Beta Electric Corporation, New York, New York.

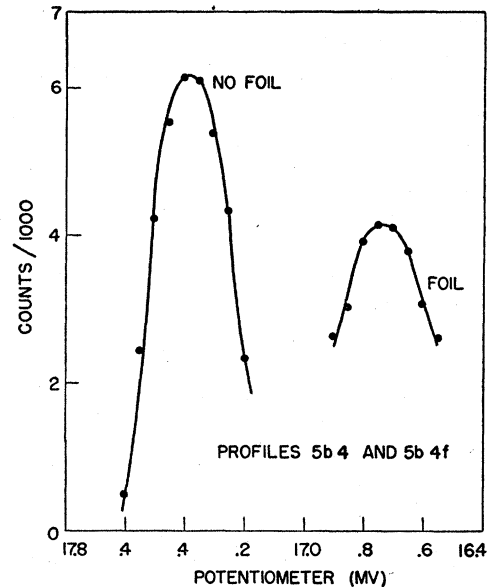


Fig. 2. Representative E_0 and E_f profiles. The abscissas can be converted into kev by multiplying by 45.23.

the potentiometer was measured on a galvanometer equipped with an optical lever. The potentiometer was compared frequently with the standard cell to prevent a significant drift. The successive counts as a function of the analyzer voltage gave an energy profile of the beam entering the analyzer. The position of the foil was then changed and the process repeated. It was found that the shape of the profile was independent of the order in which the successive points were taken.

A check was made of the duration for each run. It was considered that the machine was operating with sufficient steadiness if the time for each run was constant to within approximately 10 percent. Points for which the variation was appreciably greater were re-run.

IV. ERROR ANALYSIS

Part of the spread in the energy profile of E_0 and E_f is caused by the fact that the analyzer has a finite resolution, which is necessary in order to have a reasonable transmission through the instrument. The "25-cm" analyzer had fixed entrance and exit slits resulting in a theoretical limit of resolution of $0.008E$, where E is the energy of the ions traversing the instrument. Measurements of energy profiles with no foil in the beam showed an energy spread which agreed with that predicted.

In this experiment the "15-cm" analyzer slit width was usually 1 mm or less, corresponding to a resolution of 0.32 percent.

The resistor stack in parallel with the analyzer plate was composed of ten 5 ± 0.005 megohm resistors (Shallcross Type XR5). The shunt in series with the stack to measure the current drain through the resistance was 50 ± 0.05 ohms (Rubicon, Type B, No. 1015 resistance box).

TABLE I. The proton energy loss as a function of energy. The values for proton energies above 200 kev are taken from the solid curves of Fig. 3(a) and (b). The values below 200 kev are taken from Warsaw's curves (reference 9).

Proton energy (kev)	dE/dx (keV/(mg/cm ²))				
	Be	Mica	Al	Cu	Au
25	546.0				
50	617.0				
75	640.0				
100	615.0				
150	521.0				
200	467.5				
250	433.0				
300	405.0				
350	380.8	312.0			
400	360.0	286.5			
450	342.0	266.5			
500	325.0	250.0			
550	310.8	236.0			
600	297.5	224.0			
650	284.2	213.5			
700	272.5	204.0			
750	266.3	196.0			
800	250.9	189.0			
850	241.0	182.0			
900	231.7	176.5			
950	223.0	171.0			
1000	215.0	165.0			
1050	206.3	160.0			
1100	198.0	154.5			
1150	190.8	150.0			
1200	184.5	146.0			
1250	178.8	143.0			
1300	173.3	140.0			
1350	168.5	137.0			
			422.0	185.0	61.0
			439.0	212.0	77.0
			416.0	223.5	87.0
			366.0	228.0	90.0
			333.5	222.0	88.5
			314.5	212.0	84.5
			297.0	200.0	80.5
			283.0	189.5	77.2
			271.0	180.5	74.5
			260.0	172.5	72.0
			250.0	166.0	69.5
			241.0	160.0	67.5
			233.0	154.0	66.0
			223.5	148.5	64.5
			217.0	144.0	63.5
			210.0	140.0	62.0
			202.5	136.0	60.2
			196.0	132.0	59.0
			189.5	128.7	58.5
			183.0	126.0	58.0
			177.0	122.0	57.0
			171.0	119.5	56.5
			166.5	117.0	55.2
			162.0	114.7	54.7
			157.5	112.5	54.0
			154.0	110.0	53.5
			150.5	109.0	
			146.5	107.0	

Since it was necessary to measure only the ratio and not the magnitude of each of these resistors to check the accuracy of the analyzer voltage measurements, the two resistors were placed in a bridge circuit with other known resistors. The ratio of the two was found to be $1.0019 \pm 0.0012 \times 10^6$.

The individual settings of the potentiometer could be easily estimated to within 0.01 percent of the range. The guaranteed accuracy of the potentiometer is 0.1 percent, but the actual adjustment is much closer. As a further check, the potentiometer was compared with two other similar instruments using a current meter with an accuracy of 0.5 percent (Weston Model 622 milliammeter). All potentiometers were in agreement within the accuracy of the meter.

The error signal of the potentiometer was registered continually on a "spot light" galvanometer (Rubicon No. 3404HB). The 0.05-kev analyzer voltage steps used in obtaining a profile corresponded to a galvanometer displacement of one cm. By means of a fine control on the autotransformer in the primary circuit of the high voltage power supply, the galvanometer movement during a run was kept constant to within one mm. This corresponded to a fluctuation at the analyzer plate of ± 0.114 kev.

The ac ripple of the analyzer voltage supply was measured and found to have a peak to peak value of 0.08 percent.

The energy spread of the accelerator beam was kept as small as possible to reduce the width of the energy profiles. This spread was largely determined by the width of the accelerator exit slit. The effect of this slit can be easily estimated by considering the change in the radius dr of a beam of protons with slightly different energies traversing the analyzing magnet through which it moves in an arc with a 17-inch radius. If we consider dr as the displacement at the slit due to an energy difference dE , we shall have a good first-order approximation. From elementary principles we have

$$dr = d[(2mE)^{1/2}C/eH] = (r/2E)dE.$$

The Van de Graaff exit slit width was approximately 0.5 mm. For such a slit opening, the energy spread will be a constant percentage of the energy equal to

$$dE/E = 2dr/r = 2.05/17(2.54) = 0.23 \text{ percent.}$$

The effect of this energy spread will be merely to increase the width of the energy profile, unless, during the course of a run, the energy distribution of the protons within this spread should change. To estimate the magnitude of this effect a series of measurements of $\Delta E/\Delta x$ at the same proton energy was made with resulting values unchanged to within the limit of error arising from other causes.

It would seem reasonable to assign an uncertainty in the determination of E_0 due to this cause of 10 percent of the energy spread, or

$$\Delta E_0/E_0 = 0.1(dE_0/E_0) = 0.00023E_0.$$

The proton pulse-height distribution was of such a shape that it was possible to set the discriminator to such a value that the noise pulses could be eliminated and yet have no effect on the proton pulses.

The position of the maximum was determined by finding the axis of symmetry of the energy profile. The agreement between several such estimations and the calculated value of the mean agreed to within 0.113 kev. Accordingly the value for the uncertainty in determining the maximum of the profile is taken to be ± 0.113 kev. The error in determining the maximum of the profile due to the finite number of counts for each point can be shown to be negligible providing each point of the curve includes at least a thousand counts. This is almost always the case in this experiment.

Since the solid angle at the foils defined by the analyzer exit slit was approximately 0.00077 steradian, any errors due to multiple scattering in the foil should be negligible.

There seemed to be no regular folding of the foils, which would reveal itself in the form of a double peak in the E_f energy profile. Although some wrinkles were observed, the foils appear as though the greater portion of the foil is flat and deviates, perhaps, within $\pm 3^\circ$ from the plane of the foil holder, which is perpendicular to the proton beam. This wrinkling would result in a slight shift of the final energy profile toward the low energy

end, giving a too high value for $E_0 - E_f$. This effect results in a measured value for E_f which is too low by 0.068 ± 0.068 percent. Since the foils are kept in a vacuum and moved in and out of the beam slowly, the effect due to wrinkling should be the same for all measurements with the same foil.

The energy loss in the foils varied from 13 kev to 78 kev for the metallic foils and up to 138 kev in the case of the mica foil. By the use of two foils of differing thicknesses (except for Be and mica) the energy loss was kept to a value of approximately 10 percent of the initial proton energy. Although, for very thin foils and low specific ionization, the energy loss profile is asymmetric, the proton energies and foil thicknesses used in this experiment result in profiles which from theoretical considerations should be symmetrical.²³ That this is so may be seen from the profile shown in Fig. 2. An analytical analysis of the experimentally obtained E_0 and E_f profiles shows them to be very nearly symmetrical.

The foils were placed in the vacuum system immediately before use so as to avoid the formation of oil layers on the foils. The arrangement used for evaporation²⁴ produced a very nearly uniform deposition of metal both on the Parlodion coated slides and the platinum foils. The variation of the foil thickness at various points on the glass slide and the variation of thickness between the platinum foil and the glass slides was calculated to be less than ± 0.25 percent.

The area of deposition on the platinum foil was measured to within an error of ± 0.21 percent.

The platinum foils were weighed before and after the evaporation with a microbalance.²⁵ The sensitivity of the balance was measured periodically and found to be approximately $2 \mu\text{g}/\text{division}$. By averaging the results of several weighings, it was possible to make a weight determination to within $\pm 1 \mu\text{g}$.

Several "control" evaporations (no metal evaporated) were made and the difference in foil weights before and after the operation differed within a few micrograms. The error in determining the amount of evaporated metal ranged from ± 1 percent for $200 \mu\text{g}$ of deposited metal to ± 0.5 percent for amounts up to 1 milligram.

The surface density of the beryllium and mica foils was determined by weighing a portion of the foil and then determining its area. A photographic enlargement was made of the foil specimen resulting in an area of approximately 100 cm^2 which was determined accurately by means of a planimeter. The magnification of the image was found by enlarging known lengths at various positions covered by the foil to minimize local distortion of the photograph. The planimeter readings were found to be reproducible to within one percent. The weight

determination in the case of the beryllium foil was subject to a considerable error (~ 15 percent) since the piece of foil weighed was, unfortunately, quite small. The absolute value of dE/dx for the beryllium curve may contain this much error, and the good agreement between the results of this experiment and other determinations may be fortuitous. The relative values between points at different proton energies are not, of course, affected by the weight determination. The error in weighing the mica foils because of its large weight (1.715 mg) is less than one percent. The mica used is muscovite.

The metal foils used in this experiment seem to be fairly smooth. An estimate of the roughness can be made from the shape of the energy loss profile. Assuming that all of the spread in traversing the foil is due to roughness, the relative variation in thickness appears to be 7 percent. It is unlikely that the variation is other than random—as can be seen from shadow photographs of similarly prepared foils²⁶—and a symmetrical broadening of the E_f profile is to be expected.

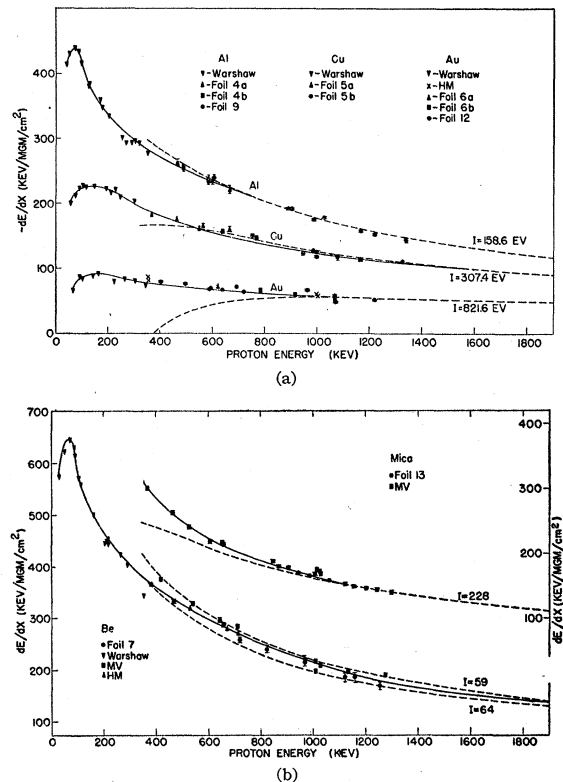


FIG. 3(a) and (b). Proton energy loss as a function of energy. The experimental curves are solid. The dashed lines are calculated from Eq. (1). The references to other determinations of dE/dx are S. D. Warsaw, Phys. Rev. **76**, 1759 (1949); HM, T. Huus and C. P. Madsen, Phys. Rev. **76**, 323 (1949); MV, C. P. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 1782 (1948). The error in the Be measurements due to the uncertainty in weighing is not shown. Numerical values of dE/dx can conveniently be read from Table I.

²³ L. Landau, J. Phys. U.S.S.R. **8**, 201 (1944).

²⁴ J. Strong, *Procedures in Experimental Physics* (Prentice-Hall, Inc., New York, 1945), p. 198.

²⁵ Type FDJ, manufactured by Wm. Ainsworth and Sons, Inc., Denver, Colorado.

²⁶ R. C. Williams and P. C. Backus, J. Appl. Phys. **20**, 98 (1949).

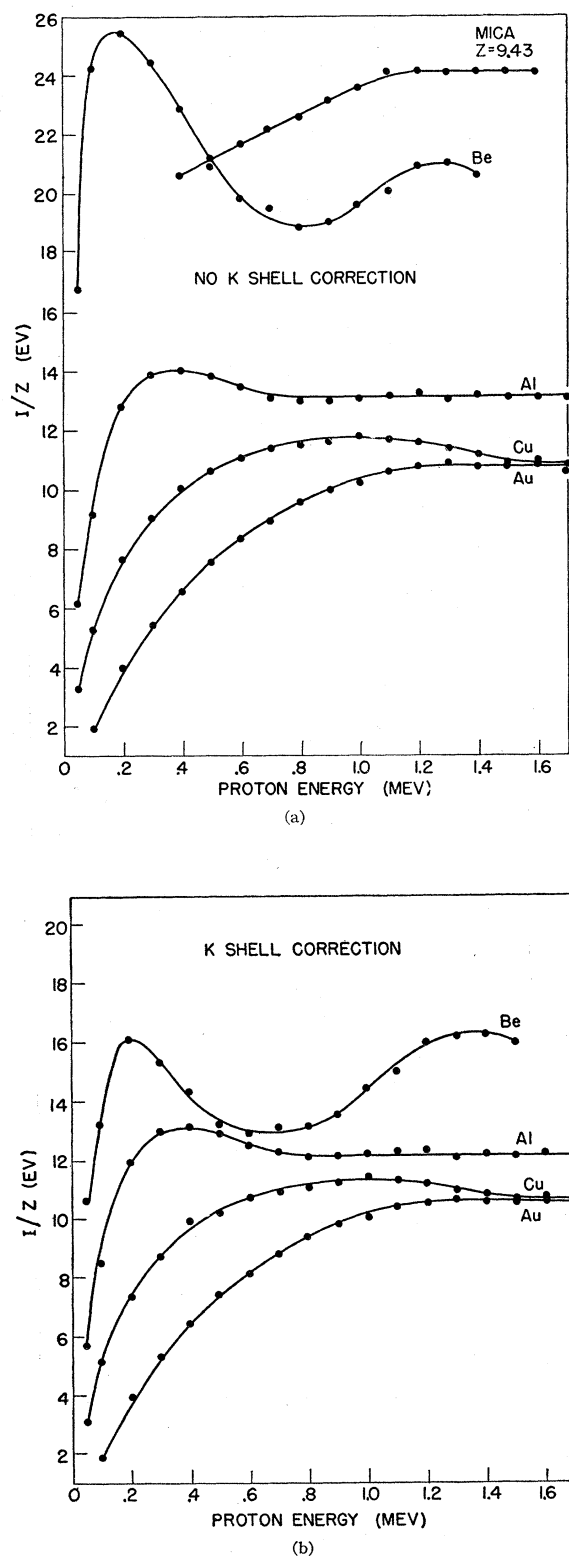


FIG. 4. The average excitation potential. (a) The values of I were obtained from the data of Table I using Eq. (1) of the text with C set equal to zero; (b) The values of I were obtained from the data of Table I using Eq. (1) of the text.

V. RESULTS

The results of the experimental determinations of $\Delta E/\Delta x$ are given in Table I and Fig. 3(a) and (b). The curves are seen to join smoothly at the lower energy end with the results of Warsaw. At the high energy end the curves fit smoothly with the formulas of Bethe⁴ and Bloch.⁵ The experimental curves are shown solid. The dashed curves were calculated using the formula

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2 N}{mv^2} \left[Z \ln \frac{2mv^2}{I} - C_k(\eta) \right]; \quad \eta = \frac{mv^2}{2(Z-0.3)^2 Ry} \quad (1)$$

Although this formula is valid only for large values of η ,²⁷ nevertheless this formula was used as a means of smoothly extrapolating from the energies used here to much higher energies where C_k approaches zero. The value of I for aluminum was obtained by Bloembergen and van Heerden using 70-Mev protons.²⁸ The dashed curve then represents a very reasonable extrapolation to higher energies. The value of I for copper was determined by Mather and Segrè using 340-Mev protons.²⁹ The value of I for gold was chosen to fit the experimental curve. Reference 1 contains a collation of the determinations of I for various elements.

The value of the average excitation potential I calculated from the smoothed curves as a function of the proton energy is given in Figs. 4(a) and (b). The calculated value of I with and without the K shell corrections is given. The rise in the value of I for Be and Al at the lower proton energies has no physical significance. It arises from the fact that the logarithmic term in the theoretical formula causes it to approach $-\infty$ and hence to cross the experimental curve. At this point the value of I calculated from the experimental curve using the theoretical formula must again equal the value of I for high proton energies.

The expression for $\Delta E/\Delta x$ may be written

$$\Delta E/\Delta x = \alpha(V_f - V_0)/\Delta x,$$

where α is a constant, Δx is the thickness and V_0 and V_f are the potentiometer readings corresponding to the maxima of the E_0 and E_f profiles, respectively.

The presence of a subtraction in the formula for the stopping power tends to divide the errors in measurement into two groups. The errors in the quantities being subtracted are emphasized if the difference between the quantities is small compared to the quantities themselves; as is the case in this experiment. The other errors do not exert as much influence on the final error and need not be so diligently reduced.

The largest errors in α are the result of the uncertainty in the values of the analyzer constants and

²⁷ L. M. Brown, Phys. Rev. **79**, 297 (1950).

²⁸ N. Bloembergen and P. J. van Heerden, Phys. Rev. **83**, 561 (1951).

²⁹ R. Mather and E. Segrè, Phys. Rev. **84**, 191 (1951).

resolutions; the uncertainties in the parameters of the 25-cm instrument being about two times those for the 15-cm analyzer. The largest error in Δx arises from the error in foil weighing and the uncertainty in the backing removal. The only estimation of the backing error that can be made is an upper bound obtained from considering the errors involved in weighing. There is reason to believe that the actual error is quite small, but the upper bound will be used in computing the over-all error. The uncertainty in α and Δx , however, affects the experimental results only by changing the scale of the curve and alters the shape only as a second-order effect.

The most important errors in determining V_0 and V_f are the analyzer voltage ripple and the uncertainty in determining the mode. The voltage ripple results in the observed curve being a "moving average" of the actual curve.³⁰ Since the actual curves should be nearly symmetrical, the averaging effect of the voltage ripple should cause a small variation in the position of the maximum.

Since the accuracy of $\Delta E/\Delta x$ depends so much on the difference between V_0 and V_f , it is not possible to give one value for the accuracy which is true for each point

on a curve. The accuracy of the individual points in Fig. 3 is given by the vertical lines at each point.

The author wishes to thank Professor Samuel K. Allison for suggesting this problem, for extending the facilities of his laboratory, and for his constant advice and encouragement without which this work would not have been completed. Grateful acknowledgment is also made of the assistance of Gordon DuFloth, Peter Weyl, and others of the Kevatron Group; and to George Moulton, Charles E. Soderquist, and other members of the Institute of Radiobiology and Biophysics.

Note added in proof:—Since this paper was submitted for publication, private communications have been received from C. B. Madsen, reporting further dE/dx measurements at Copenhagen, and from A. B. Chilton and J. N. Cooper at Ohio State University giving stopping powers of copper and nickel for protons. Madsen's data include stopping powers for 2000-kev protons, and using them (directly, for beryllium, mica, and gold, and with some adjustment to our curve for copper and aluminum) it is concluded that the following are the best values at present for 2000 kev. Be, 140; mica, 102; Al, 116; Cu, 75; Au, 43. These should be more reliable than the theoretically extrapolated points above 1350 kev of Fig. 3. The data of Chilton and Cooper on copper lie in the proton energy range 406–1050 kev; their values of dE/dx are about 4 percent lower, on the average, than those reported here.

³⁰ B. Van der Pol and H. Bremmer, *Operational Calculus* (Cambridge University Press, Cambridge, 1950), pp. 293–300.