Assuming this agreement to signify that the reaction proceeds indeed, for $h\nu > \sim 25$ Mev, via C^{12} and Be^{8} levels near the above energies, we are forced to conclude that it suddenly ceases to go through the levels involved at lower energies which should energetically be favored.

This situation is easily understood[4] if one assumes (a) $E1$ absorption, in O^{16}, in this energy range, (b) charge-independence of nuclear forces, and (c) one or more levels of $T=1$ in C^{12} near 16 Mev. Assumption (c) is plausible, inasmuch as the carbon levels[5] at 15.09 Mev and 16.07 Mev correspond probably to the ground and first excited states of B^{12}, respectively. $E1$ absorption in O^{16}, a self-mirrored nucleus, should lead only to $T=1$ states, which cannot rapidly decay by α's except to $T=1$ levels in C^{12}. The latter can decay by α-emission through small $T=0$ "admixtures," if no other particle emission competes.

Six events appear to go by a cascade mechanism as predicted in reference 4 ($E_C^* \cong 26.5$ Mev, $E_{Be}^* \cong 17$ Mev); 3 of these are, however, also compatible with $E_C^* = 16$ Mev.

The present data do not enable one to resolve the peak near 16 Mev into two peaks separated by only 1 Mev, nor to determine the anomalously small *true* width which these ought to have. This would be required to prove fully the correctness of our explanation.

A further problem is raised if more detailed experiments confirm that for $8 < E_T < 10$ Mev the level at 13 Mev is strongly preferred to lower-lying C^{12} levels. Since for $E < 15$ Mev all levels have $T=0$, this preference cannot be attributed to charge independence. It is as mysterious as the preferential α-decay of the 16.07-Mev state to the 3-Mev level of Be^{8}, assumed here and observed in $C^{12}(\gamma, \alpha)Be^{8}$,[6] and other reactions.[5]

An interesting consequence of our hypotheses is that the O^{16} levels above 25 Mev suggested by $\sigma(h\nu)$ of reference 1 ought to have $J=1^-$, $T=1$.

We hope to present a more detailed report, based on improved statistics, in the near future.

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Movable Critical Points of a Nonlinear Wave Equation

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A MOVABLE critical point of nonlinear differential equation is a branch point or an essential singularity, the position of which depends on the constants of integration and so is not fixed. Since the presence of movable critical points greatly complicates the solution of an equation, it is of some interest to see if known necessary conditions for their absence[1] are violated in a particular case. We consider here the nonlinear wave equation discussed by Schiff[2] and Thirring:[3]

$$\Box\phi - \kappa^2\phi - \eta\phi^3 = 0. \tag{1}$$

Both Schiff and Thirring are satisfied with approximate solutions, because of the impossibility of separating variables. Heisenberg[4] considers a simplified form of Eq. (1), in which a single Cartesian coordinate x and the time t are the independent variables. He finds Lorentz-invariant solutions by introducing a single new variable $s = t^2 - x^2$. With $\eta = 0$, he shows that Eq. (1) reduces to Bessel's equation of order zero in the variable $\kappa s^{\frac{1}{2}}$.

With $\eta \neq 0$, we now show that Eq. (1) is not free of movable critical points. The substitution $\phi = (8s/|\eta|)^{\frac{1}{2}} f(s)$ puts Eq. (1) in one of Ince's standard forms:[5]

$$f'' = -2f'/s \pm 2f^3 - (1+\kappa^2 s)f/4s^2, \tag{2}$$

where the upper sign is for $\eta < 0$ and the lower sign for $\eta > 0$. It is apparent that the necessary condition for the absence of movable critical points is not satisfied if $\eta > 0$. With $\eta < 0$, the condition is satisfied only if there exists a function $q(s)$ such that:[5]

$$f'' = -3qf' + 2f^3 - (q' + 2q^2)f. \tag{3}$$

Comparison of Eqs. (2) and (3) shows that no such function $q(s)$ exists. There is then no existence proof for the solution of Heisenberg's one-dimensional Lorentz-invariant simplification of Eq. (1).

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Synchrocyclotron Production and Properties of Magnesium 28†

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SHELINE and Johnson[1] have recently reported producing Mg^{28} with a half-life of 21.3±0.2 hours by the reactions $Mg^{26}(\alpha, 2p)Mg^{28}$ and $Si^{30}(\gamma, 2p)Mg^{28}$. Prior to learning of these results, we had bombarded a target of silicic acid with 350-Mev protons in the Carnegie Institute of Technology synchrocyclotron in a search for the product of the reaction $Si^{30}(p, 3p)Mg^{28}$. We confirm the identification of this nuclide.

The target consisted of 2 grams of silicic acid wrapped in aluminum foil. Ten milligrams of magnesium carrier were added, and the silica was leached out four times with 6*N* NaOH. The residue was dissolved in HCl, buffered to *p*H∼5 with acetic acid and sodium acetate, and repeatedly scavenged by addition of aluminum carrier and precipitation with 8-hydroxyquinoline. Magnesium was precipitated by the addition of ammonia and 8-hydroxyquinoline, dissolved with HNO_3, and precipitated twice as $MgNH_4PO_4 \cdot 6H_2O$.

We derive a half-life for Mg^{28} of 22.1±0.3 hours from a decay curve covering over seven half-lives. Since this differs significantly from the value of Sheline and Johnson, we mention that the nonlinearity corrections for the Geiger counter used were determined carefully by the method of multiple paired sources.[2] Aluminum fractions, separated from the magnesium as the 8-hydroxyquinolate, exhibited a 2.4-minute half-life characteristic of Al^{28}.

Aluminum absorption curves of the Mg^{28}–Al^{28} equilibrium mixture could be resolved into only two electronic components of ranges 65 and 1450 mg/cm². The latter corresponds well with the 2.86-Mev[3] beta of Al^{28}; and the former, which represents a maximum beta-energy of 0.3 Mev, we assign to Mg^{28}.

Both Mg^{28} and Al^{28} have a comparative lifetime (ft) of $\sim 10^5$ seconds, corresponding to allowed transitions. The unobserved ground-state transition between Al^{28} and Si^{28} is presumably second forbidden, and the ground-state transition between Mg^{28} and Al^{28} would be expected to be similarly forbidden. Hence the decay of Mg^{28} must be to an excited state or states of Al^{28}, with consequent gamma-emission. Accordingly, we examined the gamma-radiation from the equilibrium mixture with a NaI(Tl) scintillation spectrometer. In addition to the known 1.78-Mev[3] gamma of Al^{28}, several other gammas were found to be present, with energies extending to at least 2.6 Mev. The total disintegration energy of Mg^{28} thus appears to be at least 2.9 Mev. Al^{28} is known[4] to have about a dozen excited states within ∼3 Mev of its ground state. Further work is needed to establish the complex disintegration scheme; we have been forced to discontinue our experiments temporarily by a shutdown of the cyclotron.

In each of two one-hour bombardments the yield of Mg^{28} was about 8×10^5 disintegrations per minute. The circulating current

was believed to be ~0.2 microampere, but the effective bombarding current may have been severalfold greater because of multiple traversals of the target. On the basis of comparative yields, the cross section appears to be $\sim 10^{-4}$ barn.

Sheline and Johnson have mentioned the potential significance of Mg^{28} as a tracer, particularly in photosynthesis and general biochemistry. It should also be of value in the investigation of metallurgical systems, of geochemical problems, and of organomagnesium compounds such as Grignard reagents. While the above yield is not great, it is ample for producing a tracer for experiments of several days duration. We plan to seek target materials and conditions that will give higher yields and to develop chemical procedures for isolating Mg^{28} in carrier-free condition.

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The Density Distribution within Two Meters of Large Air Shower Axes*

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Details of the lateral structure of individual air showers have been observed[1] by using a 20-channel electron-pulse ionization chamber which took 20 density samples from each shower. The geometry is shown in plan in Fig. 1, where each area represents the region from which an individual collecting wire gathered electrons. The pulses were separately amplified, displayed on 20 cathode-ray tubes, and photographed. The last stage of amplification was made nonlinear so that a large range of densities could be accurately measured. Amplifier calibrations were made by pulsing the shell of the chamber and stability of gain proved satisfactory. The absolute value of ionization was determined in the customary manner from Po α-particle ionization. Precautions such as purification of the argon, experimental and theoretical evaluation of the pick-up between detection areas, etc., were taken. The recording system was actuated by a fourfold coincidence in a simple Geiger-counter array.

During 109 hours of operation at an altitude of 280 meters, events were recorded in which the density distributions over the array of wires in the chamber were of the following types: (1) flat with only small fluctuations from the average (consistent with those expected from the statistics of independent events) (events 3 and 4); (2) increasing or decreasing functions (event 9); (3) markedly peaked (events 5 and 8); and (4) flat with rather marked and systematic fluctuations from the average (event 13) (see Figs. 2 and 3). These events have been interpreted, respectively, to be the result of showers whose axes hit (1) relatively far away from the chamber; (2) near the end of the chamber with small

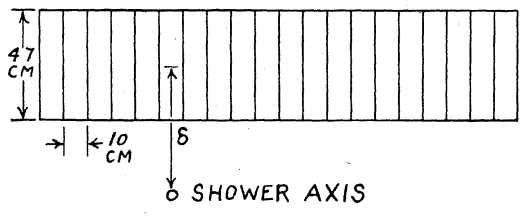

Fig. 1. Plan of the detection areas.

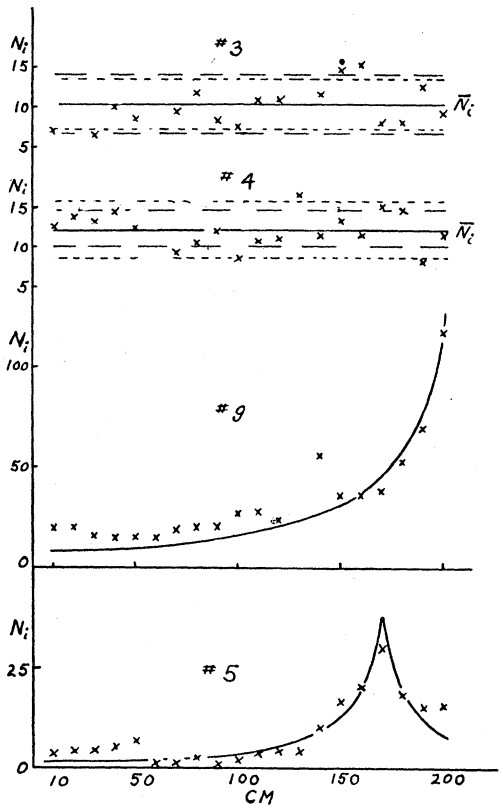

Fig. 2. Individual showers; number of electrons N_i within the detection areas *versus* position of the areas. In 3 and 4 the dotted lines represent $(\bar{N}_i)^{\frac{1}{2}}$ and the dashed lines σ, the rms deviation from the average. The curve in 9 represents a shower of local size $\Pi = 3.5 \times 10^4$ electrons with $\delta = 28.5$ cm. The curve in 5 represents $\Pi = 5.6 \times 10^3$ with $\delta = 23.5$ cm.

values of δ; (3) inside the chamber collecting area; and (4) inside the chamber with multiple shower cores due to more than one initiating π^0 meson. These four classifications are not sharply defined and showers of intermediate types were observed.

The first interest in these events is that of viewing for the first time a detailed "display" of an air shower. The scientific interest is twofold: (a) what is the shape of the lateral distribution function for a single cascade shower, and (b) what is the evidence for a multiplicity of cores, especially a multiplicity attributable to a multiplicity of π^0 mesons? In answering (a) we are forced to assume a distribution and look for consistency because of the finite detector areas, the transition effect in the top of the chamber, and the variation in δ (see Fig. 1) from event to event. As a first approximation, the chamber response curves fitted to each event were constructed (including transition effect) from lateral distribution functions that were obtained from existing shower theory results. The distributions were obtained as functions of electron and photon energies for air and for distances within 10 meters of the shower axis.[1] The electron distributions are perhaps dependent upon better approximations than those used by Molière, (e.g., the number of low energy electrons is certainly treated better) but the resultant total distribution function was the same as that found by Molière.[2] The experimental results appear to require no drastic revision of our total distribution function.

In answering (b) it is necessary to consider the expected non-Poissonian fluctuations in the density distribution near the axis of a single-cored shower and also the effect of π^0 decay. It has been demonstrated[1] that neither of the two considerations provides a likely explanation of the auxiliary peaks in events 5 and 9. Events