superimposed upon the much larger scale dark-field crystal pattern of the third crystal. Figure 2 shows a microdensitometer tracing of some of the fringes. To convince ourselves that these fringes were interference fringes due to the geometry of the instrument, this geometry has been slightly changed and the fringe behavior as a function of the changes investigated. It was found that within certain limitations a rotation of the third crystal by $2\frac{1}{2}°$ produced an 8° relative rotation of the fringes. The rotation of the fringes has been measured from a fixed direction on the crystal itself so that no ambiguity exists between fringe rotation and crystal rotation. We carried out some experiments changing the intercrystal distance between the first and second crystal and found that the fringe spacing varied as a function of that distance, as was to be expected.

The fringes to date are rather fugitive and hard to find, but it is believed that improved crystals will eliminate some of this difficulty.

A complete record of these experiments will be published at a later date.

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Cyclotron Resonances, Magnetoresistance, and Brillouin Zones in Semiconductors

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ONE of the basic problems in the band theory of solids is to determine the shapes of the energy surfaces in the Brillouin zone. A possible solution is furnished by cyclotron resonances at low temperatures (say 10°K) in weakly doped germanium: For this situation, (1) Maxwell statistics can be used, (2) the interactions of the carriers are unimportant so that a description in terms of single carrier momenta in the Brillouin zone is good, and (3) the collision frequency ν is so much less than $\omega=2\pi f$ for 1.25-cm waves[1] that inertial effects dominate, and the dependence of ν upon position in the Brillouin zone is unimportant.

Some typical situations in which resonance might be observed are illustrated in Fig. 1. The E's and v's vary as $\exp(i\omega t)$ so that the conductivity $nq\mu$ is complex. The standard transverse and longitudinal magnetoresistance configurations are represented in (t) and (l), a combination of (t) and the Hall effect in (tH), and circular polarization in (c).

$$\mu_t(\omega, H)=v_x/E_x, \tag{1}$$

$$\mu_H(\omega, H)=cE_y/HE_x. \tag{2}$$

$$\mu_{tH}(\omega, H)=v_y/E_y=\mu_t(\omega, H)\{1+[H\mu_H(\omega, H)/c]^2\}^{-1}. \tag{3}$$

Some of the resonances may be illustrated by spherical energy surfaces with a single (relaxation) frequency ν and $\omega_H=qH/m^*c$. We find that μ_t, μ_H, and μ_l are independent of H:

$$\mu_t=\mu_H=\mu_l=q/\{m^*(\nu+i\omega)\}, \tag{4}$$

$$\mu_{tH}=q(\nu+i\omega)/\{m^*[(\nu+i\omega)^2+\omega_H^2]\}, \tag{5}$$

$$\mu_c=q[\nu+i(\pm\omega+\omega_H)]/\{m^*[(\nu+i\omega)^2+\omega_H^2]\}. \tag{6}$$

The plus sign in μ_c holds when the field rotation and cyclotron orbit have the same direction. If $\omega>10\nu$, as may be achieved in Ge at low temperatures, the resonance peaks observed at $\omega_H^2=\omega^2-\nu^2$ will be within 1 percent of that proper for m^* alone. If ν is a function of $\mathcal{E}$, then the mobility expressions should be based on finding v_x, v_y as functions of E_x and E_y and averaging over the

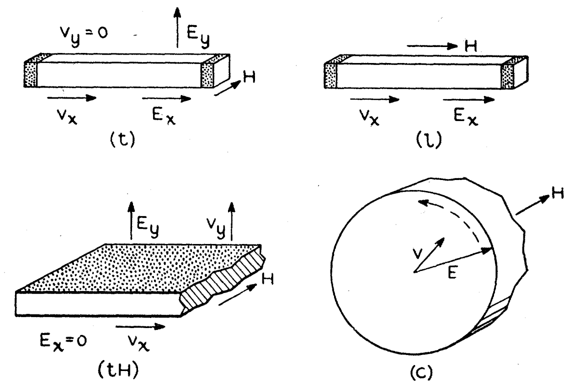

FIG. 1. Some typical situations for observing cyclotron resonances: (t), conventional transverse magnetoresistance configuration; (l), conventional longitudinal magnetoresistance configuration; (tH), a transverse situation in which Hall effect contributes to resistance; (c), circularly polarized electric field.

carriers with a weighting factor of $\mathcal{E}$; for a constant mean free path, this will lead to the customary factors of $8/3\pi$, etc. and to a small magnetoresistance in μ_t. This averaging does not affect the conclusion that if $\omega=\omega_H$, μ_c reduces to $\mu_c(0, 0)$ for the wave rotating in the cyclotron direction.

Herman's calculations[2] suggest that the energy surface for the conduction band in Ge consists of six ellipsoids of revolution lying on [100] directions with a longitudinal mass m_1 and transverse mass m_2. The valence band is probably triply degenerate with surfaces of three sheets of nonellipsoidal shapes.[3] For these sheets we can define "tubes," each having its characteristic mass m_α and corresponding cyclotron frequency.[4]

For an ellipsoidal energy surface given by

$$\mathcal{E}=(P_x^2/2m_x)+(P_y^2/2m_y)+(P_z^2/2m_z) \tag{7}$$

and a magnetic field with direction cosines α, β, λ, the effective cyclotron mass is

$$m^*=[m_xm_ym_z/(m_x\alpha^2+m_y\beta^2+m_z\gamma^2)]^{\frac{1}{2}}. \tag{8}$$

Separate resonances should be observed for each different orientation of ellipsoid to H. Thus, for H parallel to [100] for the six ellipsoids, m^* will equal $(m_1m_2)^{\frac{1}{2}}$ four times and m_2 twice; there will be no longitudinal resonance. For H parallel to [111], $m^*=m_2[3m_1/(m_1+2m_2)]^{\frac{1}{2}}$ six times, and for (l) we find

$$\mu_l=(2m_1+m_2)[(\nu+i\omega)^2+\omega_H^2 9m_1m_2/(2m_1+m_2)(m_1+2m_2)] \div 3m_1m_2(\nu+i\omega)[(\nu+i\omega)^2+\omega_H^2]. \tag{9}$$

Similar, but generally more complex, expressions apply to other cases.

Evidently, if the surfaces are ellipsoids, the determination of the resonance field for several conditions will give a unique determination of the mass parameters and hence of the energy surface shapes.

For the triply degenerate surface, a distribution of masses from zero (at the conical contact of the outer surfaces) to infinity will be present. If the inner surface is nearly spherical, a strong isolated resonance will occur. For this and the doubly degenerate case, it appears likely that the predicted resonance behavior will require difficult numerical calculations. However, it also appears probable that a numerical fit based on the three parameters[3] will be unique.

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