Higher Spin Polarizations in Nuclear Reactions

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In a previous letter[1] a general theorem was reported for the polarization resulting from nuclear reactions. In essence, the polarization is just the expectation value in the scattered wave of the tensor of rank unity formed by the spin operator $\mathbf{i}'$ of the outgoing particle. In general, however, an outgoing particle will have nonzero irreducible tensor moments up to a maximum tensor rank given by $2i'$. For example, a deuteron produced in a reaction will possess a Legendre second moment $[3(i_z')^2-i'(i'+1)]$ which will usually differ from zero. Lakin and Wolfenstein[2] have recently pointed out the possible importance of the deuteron second-rank tensor in analyzing reactions.

We wish to point out that a completely general result can be written down for the expectation value of any irreducible tensor $T_\kappa^{(q)}$ of rank q and component κ formed from the spin operator $\mathbf{i}'$. All sums over magnetic quantum numbers can be performed, since these are essentially geometrical in nature; and the final result is expressible completely in terms of the nuclear scattering matrix, the Racah coefficients, and the X coefficient introduced by Fano and Racah.[3,4] The previously communicated results on the polarization appear as a special case of this general result corresponding to $q=1$. The expression of Blatt and Biedenharn[5] for the angular distribution of scattering and reaction cross sections is also obtained immediately by setting $q=0$.

The expectation value of the tensor $T_\kappa^{(q)}$ can be written

$$\langle T_\kappa^{(q)}\rangle=\frac{\pi^{\frac{1}{2}}\lambda\!\!\!^{-}{}_\alpha^2[(2i-q)!(2i+q+1)!]^{\frac{1}{2}}}{2(2I+1)(2i+1)(2i')!}\times P_8\left(\frac{i'}{[i'(i'+1)]^{\frac{1}{2}}}\right)\sum_{L=0}^{\infty}A_L Y_{L\kappa}(\theta,\phi),\quad(1)$$

where i, i', I, and I' are the spins of the bombarding particle, outgoing particle, target nucleus, and residual nucleus, respectively, and P_q is the usual Legendre polynomial.

The expression for A_L is

$$\begin{aligned}A_L=\sum &\, i^{l_2-l_1-l_2'+l_1'}(-1)^{I'-i'-s+L+J_1-s_1'-l_2'}(2L+1)^{-\frac{1}{2}}\\ &\times[(2f+1)(2l_1+1)(2l_2+1)(2l_1'+1)(2l_2'+1)]^{\frac{1}{2}}\\ &\times[(2s_1'+1)(2s_2'+1)]^{\frac{1}{2}}(2J_1+1)(2J_2+1)\\ &\times[\delta(\alpha,\alpha')\delta(s_1',s)\delta(l_1',l_1)-S(\alpha's_1'l_1';\alpha sl_1;J_1\pi_1)]^*\\ &\times[\delta(\alpha,\alpha')\delta(s_2',s)\delta(l_2',l_2)-S(\alpha's_2'l_2';\alpha sl_2;J_2\pi_2)]\\ &\times(l_1'l_2'00/l_1'l_2'L0)(l_1l_200/l_1l_2f0)(fq0\kappa/fqL\kappa)\\ &\times W(l_1J_1l_2J_2;sf)W(i's_1'i's_2';I'q)\\ &\times X(J_1l_1's_1';J_2l_2's_2';fLq),\quad(2)\end{aligned}$$

where S is the nuclear scattering matrix, (l_1l_200/l_1l_2L0) the usual Clebsch-Gordon coefficient, J the total angular momentum, π the parity, s the initial channel spin, l the incident orbital angular momentum, and all final quantities are primed. The sum is over $J_1J_2l_1l_2l_1'l_2'\pi_1\pi_2s_1's_2's$ and f.

From conservation of parity and the properties of the Clebsch-Gordon coefficient it is easily seen that the sum over f reduces to a single term, $f=L$, for the angular distribution ($q=0$) or polarization ($q=1$) problem.

A generalization of the rules for the complexity of angular distributions results from the properties of the Racah and X coefficients. If there is a largest effective initial orbital angular momentum l, a largest final momentum l', or a largest total angular momentum J, there will be a largest value of L in Eq. (1) given by the simultaneous conditions

$$\begin{aligned}L&\leq 2l';\,2l+q;\,2J+q &&(q\text{ even}),\\ &\leq 2l';\,2l+q-1;\,2J+q-1 &&(q\text{ odd}).\end{aligned}\quad(3)$$

Details of the preceding calculations will appear in a paper to be submitted to this journal shortly. The writer wishes to thank Dr. Ugo Fano for permission to see the manuscript by Fano and Racah in advance of publication and for the information that the X coefficient was also introduced by Wigner in an earlier unpublished manuscript.

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[3] U. Fano, National Bureau of Standards Report 1214, p. 48 (unpublished).
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Double Meson Photoproduction*

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In this note we present preliminary results of a study of the pair production of mesons by gamma-rays incident on protons. The three processes considered are

$$\begin{aligned}&(a)\quad \gamma+p\rightarrow p'+\pi^++\pi^-,\\ &(b)\quad \gamma+p\rightarrow p'+\pi^0+\pi^{0\prime},\\ &(c)\quad \gamma+p\rightarrow n+\pi^++\pi^0.\end{aligned}$$

The desirability of investigating the double meson photoproduction process in the pseudoscalar theory stems from the results of Watson and Lepore[1] and Lévy,[2] wherein it is seen that the exchange of a meson pair gives rise to a major contribution to the nucleon-nucleon interaction. The term in the meson-nucleon interaction Hamiltonian that is important for the pair processes may be separated by the Dyson[3] or Foldy[4] transformations. The interaction in the pseudoscalar theory is written as

$$if\int\bar{\psi}\gamma_5\tau_\alpha\phi_\alpha\psi d\mathbf{r},\quad(1)$$

where the notation of reference 3 is used. The meson-nucleon interaction in the transformed Hamiltonian can be separated into one large term that is bilinear in the meson field,

$$M(f^2/2M^2)\int\bar{\psi}\psi\phi^2 d\mathbf{r},\quad(2)$$

plus other terms, one of which is the usual derivative coupling form. Lévy[2] has indicated the important role of the pair term, Eq. (2), in harmonizing the predictions of the pseudoscalar meson theory with the observed properties of the low energy neutron-proton interaction. On the other hand, the sharp increase in the meson-nucleon scattering cross sections as a function of meson energy in the range 60–150 Mev, as observed at Chicago, would seem to indicate that the pair term predicts too much energy-independent S wave scattering.[3]

The process of double meson photoproduction provides a direct check on the role of the pair term. For incident gamma-rays of energy <1 Bev the nucleon motion may be treated nonrelativistically, and it is easy to see that process (a) will dominate in magnitude over (b) and (c) in this energy region. This is because the gamma-ray can be absorbed by the charged π-mesons in process (a), whereas it can only be absorbed by the proton in process (b). In process (c) the pair term, Eq. (2), does not operate since it contains no mechanism for changing the nucleon charge. All other terms in the transformed interaction Hamiltonian which can contribute to these processes are small of order μ/M relative to the pair term. It is readily calculated that

$$\sigma(\pi^0,\pi^0)\sim\sigma(\pi^+,\pi^0)\sim(\mu/M)^2\sigma(\pi^+,\pi^-),$$

where $\mu/M=0.15$ is the meson-nucleon mass ratio.

The process (a) will be accessible for experimental study shortly when accelerators producing gamma-ray beams are operating in the 400-Mev region. The threshold for this process is 322 Mev. It can be observed by detecting π^- mesons emerging from a hydrogen target under gamma-bombardment.[5]