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Interactions of μ-Mesons*

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The energy spectrum of electrons knocked on by μ-mesons has been measured over a wide range of energies. It is found that the theoretical cross section as derived by Bhabha fits the data for knock-on energies of a few Mev up to energies of the order of a Bev. From these data it is concluded that any particle structure that might exist does not appreciably affect collisions with impact parameters of 6×10^{-13} cm or more.

It is found that the theoretical cross section for direct pair production by μ-mesons is of the right order of magnitude. From the experiment an upper limit of 2×10^{-29} cm^2 nucleon is set for the cross section for the production by μ-mesons of a penetrating secondary of the type originally reported by Braddick and Hensby.

I. INTRODUCTION

SINCE the discovery of the μ-meson many experiments have been done to determine the μ-electron collision cross section.[1–7] With a few exceptions,[1,2,7] the purpose of most of these experiments was to determine the average number of collision and shower electrons in equilibrium with the μ-mesons in dense materials. Most of these "equilibrium" electrons have energies of only a few Mev. It has been the purpose of the present experiment to determine the μ-electron collision cross section up to as high momentum transfers as possible. The results of the experiment have been compared with the formula derived by Bhabha[8] on the assumption that both the μ-meson and μ-electron are point Dirac particles. It was thought that some evidence of particle are point Dirac particles. It was thought that some evidence of particle structure might appear in deviations from the formula for high momentum transfers.

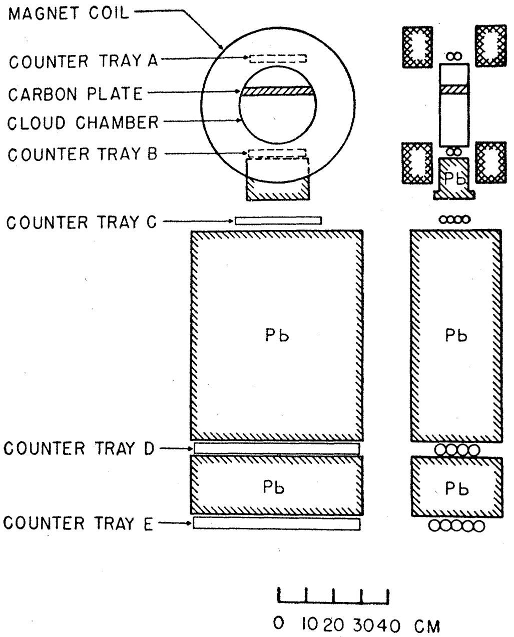

FIG. 1. Experimental arrangement of the cloud chamber and its associated triggering counters.

The experiments of Braddick[9,10] and collaborators on cosmic rays underground indicate that μ-mesons produce penetrating secondary particles with a rather large cross section. Recently other workers[11,12] have done experiments which seem to show that the cross section derived from at least some of Braddick *et al.*'s experiments is too large.

II. EXPERIMENTAL METHOD

The experimental apparatus consisted of a cylindrical cloud chamber in a magnetic field of about 1400 gauss. The chamber contained a one-inch carbon plate in which the measured secondaries were produced. The chamber was operated with the usual alcohol-water mixture in about 1.15 atmospheres of argon. It was illuminated from the side with a single xenon filled flash lamp obtained from the Amglo Corporation. Stereoscopic photographs were taken and used in the momentum measurement.

The position of the chamber and magnet relative to the triggering system of Geiger-Mueller counters is shown in Fig. 1. In order to trigger the chamber a single counter had to be discharged in trays *A*, *B*, *C*, *D*, and *E*. This triggering arrangement required that at least one particle traverse about 1000 g/cm^2 of lead. For μ-mesons this condition required a minimum energy of about 1.5 Bev. A few extensive showers did trigger the apparatus; however, the vast majority of the expansions were

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[9] H. J. J. Braddick and G. S. Hensby, Nature **14**, 1012 (1939).

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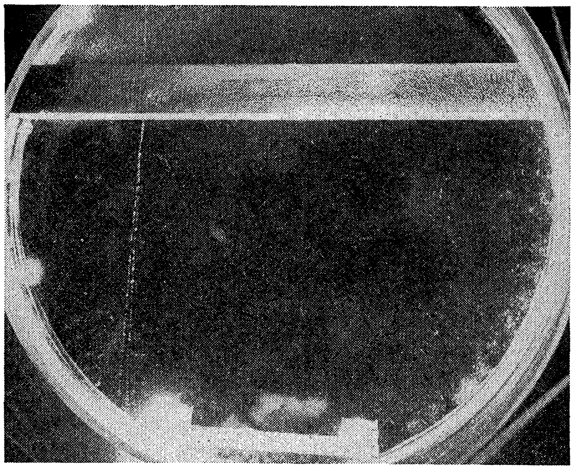

FIG. 2. A knock-on electron of about 230-Mev energy is produced by a single particle traversing the cloud chamber.

triggered by single particles. Almost all of these particles must have been μ-mesons. In the observation of the traversal of about 200 000 g/cm^2 of material inside the chamber not a single penetrating shower was found originating in the chamber. No heavily ionizing particles were ever observed to originate near the path of the triggering particle.

The magnetic field was measured by means of a flip coil. The field varied between 1300 to 1600 gauss from the middle to the side of the chamber and between 1300 to 1200 from the central portion to the front or back of the illuminated region. The radial variation was corrected for in each individual measurement by using the value of the magnetic field strength at the midpoint of the measured track.

The temperature of the room in which the chamber was operated was maintained to within about ±1° Fahrenheit. Some of the time the fast tracks in the chamber appeared to have distortions caused by thermal gradients in the chamber. The average energy of the mesons triggering the apparatus was about 7 Bev and these particles were not appreciably bent by the magnetic field. Consequently the deflection of the secondary particles was measured relative to the primary particle. This method of measurement eliminated to a large extent the effects of turbulence. Distortion was always worse very close to the plate in the chamber. In this region the primary and secondary particles are very close together and consequently their tracks undergo almost the same distortions. In the case of the higher energy secondaries where the inaccuracy produced by distortion would be great, the angle of emission of the secondary electrons is very small, and the secondary track remains close to the primary track. An example of a high energy knock-on is shown in Fig. 2.

III. EXPERIMENTAL RESULTS

The first experiment done will be called experiment *A*. The procedure was to trigger on single particles passing through the chamber and then measure the energy of each knock-on secondary which appeared to emerge from the carbon plate.

The results of experiment *A* are given in Table I. The theoretical results with which the experiment is compared is the result of averaging the Bhabha formula over the portion of the meson spectrum accepted by the apparatus.[13] The μ-meson spectrum used is that one obtained by Glaser *et al.*[14] The comparison of experiment and theory appears to be rather good up to energies of about 100 Mev. However, for knock-ons with energies greater than 100 Mev there appears to be a possible discrepancy. For secondary electrons with energies greater than 100 Mev about 25 particles were expected and 39 were found. This deviation could be a statistical one. However, it was considered necessary to make further checks and to improve the experiment if possible.

The experimental arrangement was altered in the following ways. The carbon plate was raised by one inch in order to give more track-length on which to measure curvature. Tray *B* was removed from the coincidence system. It was thought that mesons producing high energy knock-ons in the chamber might record more efficiently because the secondary might strike a counter in tray *B*, even though the meson missed tray *B* but did hit the remaining trays.

The results of experiment *B* are given in Table II. It may be seen that the agreement between experiment and theory is quite good up to the highest knock-on energies measured. An integral spectrum of the results obtained from experiment *B* is plotted in Fig. 3 and compared with the theoretical results. In Fig. 4 the results of experiments *A* and *B* are combined and compared with theory. A normalization to the theoretical curve was made at about 50 Mev. This energy was chosen because possible experimental errors at this energy would be common to the higher energies. The

TABLE I. Results of experiment *A*.

Energy range in Mev	Number of knock-ons Theoretical	Experimental
2–4	228	185
4–8	181	173
8–16	128	163
16–32	80	94
32–64	45	63
64–100	17	21
100–200	14.7	22
200–300	4.4	9
300–600	3.6	5
600–1000	1	2
>1000	1	1

Total number of traversals = 11 300.
Total number of knock-ons = 1152.

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[13] For high energy knock-ons, only that portion of the spectrum for which the collision was kinematically possible was used in the averaging.

[14] Glaser, Hamermesh, and Safonov, Phys. Rev. **80**, 625 (1950).

Table II. Results of experiment B.

Energy range in Mev	Number of knock-ons Theoretical	Experimental
2–4	207	
4–8	164	136
8–16	115	101
16–32	72	65
32–64	40	55
64–100	16	23
100–200	13	13
200–300	4	5
300–600	3.3	5
600–1000	1	1
>1000	1	3

Total number of traversals = 10 248.
Total number of knock-ons = 1 095.

Table III. Results of experiment C.

No. traversals	No. knock-ons	No. knock-ons traversing $\frac{1}{2}$-in. plate	Electronic shower	+Knock-on secondaries
3519	321	13	33	9

agreement between theory and experiment is good up to knock-on energies of the order of 1 Bev.

IV. OTHER EXPERIMENTAL RESULTS

In this experiment no cases of scatterings of greater than 5° of the primary particle were observed. Two cases of smaller scatterings were observed but these were rather doubtful because of turbulence in the chamber. From the number of grams of carbon traversed, the cross section for scattering of high energy μ-mesons through more than 5° must be less than 2×10^{-29} cm^2/nucleon.

In the course of experiment A, a block of lead one inch thick was placed above the chamber. Many electronic showers appeared to originate in the lead. In eight thousand traversals with the lead above the chamber, eight electron-positron pairs were observed to originate in the carbon close to the trajectory of the μ-meson. It seems very likely that many of these pairs are the result of the conversion of γ-rays from small bremsstrahlung or knock-on showers originating above the chamber. In the course of 13 000 carbon plate traversals with no lead above the chamber only two cases of apparent direct pair production by the incident μ-meson were observed. By averaging the formula for the direct pair production cross section[15] over the incident μ-meson spectrum, it was found that in examining 13 000 traversals one would expect to find 3.5 detectable pairs. The calculation was made on the assumption that 50 Mev total pair energy would be the minimum pair energy efficiently detected.

V. ASSOCIATED PENETRATING PARTICLES

From the results of Braddick *et al.*[9,10] one would expect occasionally to find an "associated penetrating particle" formed in the carbon plate. Such an occurrence should appear very similar to a high energy knock-on electron.

For a knock-on electron the energy, E^1 is given as a function of the angle of emission with respect to the meson direction (θ), and momentum and mass of the meson (p, μ) by the following formula:

$$\frac{E^1}{2m_ec^2}=\frac{(cp)^2\cos^2\theta}{[m_ec^2+((cp)^2+(\mu c^2)^2)^{\frac{1}{2}}]^2-(cp)^2\cos^2\theta}.$$

By expressing energies in units of $2m_ec^2$ and neglecting small terms, it is found that

$$(E^1+1)\sin^2\theta+E^1\left[\left(\frac{\mu c}{p}\right)^2+\frac{1}{cp}\right]=1,$$

or

$$E^1\theta^2\cong1.$$

Braddick and collaborators have found that the penetrating secondary particles have an energy of the order of 1 Bev and a mean square angle of emission of about 1.6×10^{-2} radians^2. Such a particle would have a value of $E^1\theta_{\text{proj}}^2$ of about 8 on the average, which is many times larger than the theoretical limit for a knock-on electron.

All secondary particles with energy greater than 50 Mev have been collected and their values of $E^2\theta_{\text{proj}}^2$ calculated. The results are given in a histogram in Fig. 5. Some of the particles appear to have $E^1\theta^2_{\text{proj}}$ which are greater than the theoretical limit. All except one of these discrepancies is readily explainable by multiple scattering in the carbon plate. One particle of measured momentum of 950^{+500}_{-200} Mev/c was emitted at an angle of 4°. Such an angle of emission would require a multiple scattering of 7 or 8 times the rms scattering angle. The probability of a single scattering of an electron by this amount is about 5×10^{-3}. From this one event in the course of the traversal of 94 000 g/cm^2 of carbon

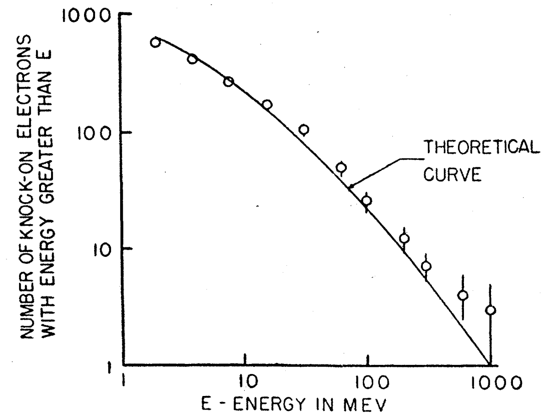

Fig. 3. The integral spectrum of knock-on electrons as derived from experiment B is compared with the theoretical curve.

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[15] H. J. Bhabha, Proc. Roy. Soc. (London) **A152**, 559 (1935).

an upper limit for the associated penetrating particle cross section of about 2×10^{-29} cm^2/nucleon can be set. Another experiment was done to check on the existence of associated penetrating particles. The one-inch carbon plate was replaced by a one-inch lead plate and a one-half-inch lead plate was placed below it in the chamber. Thirty-five hundred traversals of the one-inch plate were observed with this arrangement. The results of the experiment are given in Table III. The results are quite different from the results with the carbon plate. Many small cascade showers were observed which were presumably the result of a collision or radiation processes in the lead. About 3 percent of the single secondaries emerging from the lead were positively charged. These particles are presumably positron remnants of small showers in the lead. Thirteen of the knock-on secondaries of the μ-mesons traversed the one-half-inch lead plate without shower production. Most of these particles scattered considerably in traversing the lead. However, one or two showed scatterings of only six or seven degrees. These particles appeared to be moderate energy (50–150 Mev) knock-on electrons. There was no evidence for other processes occurring. Thus this experiment is also consistent with an upper limit of 2×10^{-29} cm^2/nucleon for the production of associated penetrating particles by μ-mesons.

VI. DISCUSSION OF RESULTS

The μ-meson-electron scattering experiment has verified the knock-on formula over a wide range of energy of knock-on electrons. The spin orbit term in the formula is certainly verified but the spin-spin interaction term is too small to be detectable.

A limit can be set on the magnetic moment of the μ-meson. From the lack of deviation from the theoretically calculated knock-on formula, it can be said that the magnetic moment of the μ-meson cannot be more than nine times the normal Dirac moment. The bremsstrahlung cross section is quite sensitive to the magnetic moment of the particle. From the recent burst experiment of Driggers,[16] it can be shown that the magnetic moment of the μ-meson cannot differ from the normal moment by more than about 40 percent.

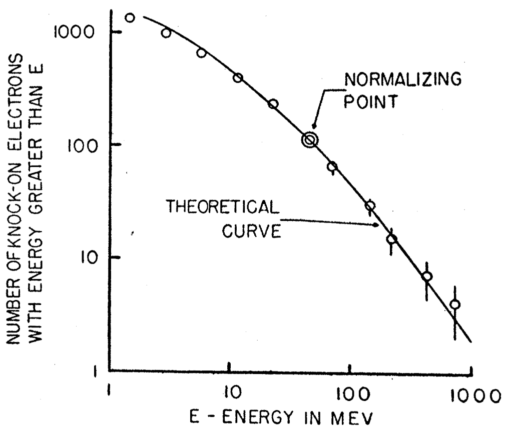

FIG. 4. The experimental integral spectrum obtained from experiments A and B is compared with the theoretical spectrum.

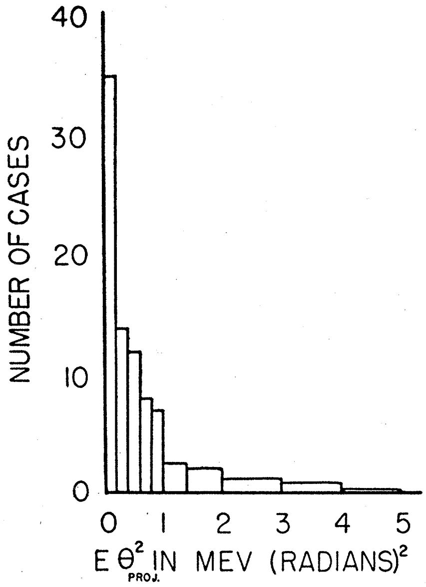

FIG. 5. A histogram showing the number of secondaries with a given value of $E\theta^2_{\text{proj}}$.

No evidence of particle structure was found in the experiment. From the results, however, limits can be set as to the dimensions of any particle structure. It can be shown that collisions in which momentum, $p_{\text{c.m.}}$, is transferred in the center-of-mass system have an average impact parameter of $\hbar/p_{\text{c.m.}}$. The momentum change in the center-of-mass system can be calculated from the energy change in the laboratory system as follows. Consider the difference of the momentum-energy four-vectors of the electron before and after the collision. Using the fact that the energy of the electron in the center-of-mass system is not changed in the collision and that the length of the four-vector is invariant to Lorentz transformation, it is found that

$$(cp_{\text{c.m.}}-cp^1{}_{\text{c.m.}})^2=(cp_{\text{lab}})^2-(E_{\text{lab}}-m_ec^2)^2$$

or

$$c\Delta p_{\text{c.m.}}=[2m_ec^2(E_{\text{lab}}-m_ec^2)]^{\frac{1}{2}}.$$

No large deviations from the theory were found for momentum transfers of the order of 1 Bev/c, and consequently it can be concluded that no structure effects are felt at impact parameters of greater than 6×10^{-13} cm.

The search for secondary particles other than electrons yielded only one case of a secondary which apparently could not be explained as an elastically scattered electron. From the experiments using carbon and lead as the producing material, it was concluded that the cross section for the production of a penetrating

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[16] F. E. Driggers, Phys. Rev. 87, 1080 (1952).

secondary could not be more than about 2×10^{-29} cm^2/nucleon. If the cross section were as high as 5×10^{-29} cm^2/nucleon, the chances of obtaining the present results would be only about three percent.

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Dipolar Broadening of Magnetic Resonance Lines in Magnetically Diluted Crystals

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Calculations are given for the frequency moments associated with the dipolar broadening of magnetic resonance lines in crystals having lattice points populated at random by identical paramagnetic ions or nuclei. It is found that for fractional magnetic population $f>0.1$ the line shape is approximately Gaussian with a width proportional to $f^{\frac{1}{2}}$; for $f<0.01$ the line shape is approximately Lorentzian with a width proportional to f.

We have solved rigorously by the method of moments the problem of magnetic dipolar broadening of magnetic resonance lines in crystals having lattice points populated at random by identical paramagnetic ions or nuclei, the remaining equivalent lattice sites being populated by diamagnetic ions or nuclei with spin zero. Such a situation often occurs in paramagnetic resonance research by design in order to improve resolution, or it may arise naturally as in crystals with color centers or as in nuclear resonance experiments when only one of the isotopes has a magnetic moment. The dipolar broadening problem when all lattice sites are populated identically has been solved by Van Vleck.[1] For random population there have been published two apparently conflicting arguments, one[2] suggesting that the line width is proportional to the square root of the concentration of magnetic systems, the other[3] suggesting that at low concentrations the width is directly proportional to the concentration and that the line has a Lorentz shape. The result of our exact treatment is that in the absence of exchange and hyperfine interaction the line width for random occupancy is proportional to the square root of the concentration, if more than about 0.1 of the lattice sites are filled magnetically, while for appreciably lower concentrations the width is proportional to the concentration.

The result follows easily from the equations of Van Vleck in the paper cited. We consider first the second moment. From his Eqs. (9) and (10) the mean square deviation of the frequency from the Larmor value may be written

$$\langle\Delta\nu^2\rangle_{\mathrm{Av}}=[S(S+1)/3N\hbar^2]{\textstyle\sum_{j,k}}'\,B_{jk}^2, \tag{1}$$

where N is the actual number of magnetic ions and

$$B_{jk}=-3g^2\mu_B{}^2r_{jk}{}^{-3}[\tfrac{3}{2}\gamma_{jk}{}^2-\tfrac{1}{2}], \tag{2}$$

γ_{jk} being the direction cosine of $\mathbf{r}_{jk}$ with the static field. If f is the probability that a lattice site is occupied by a magnetic system, the number of k sites occupied at a displacement $\mathbf{r}_{jk}$ from the occupied j sites is, after summing over j, simply Nf, as N sites j are occupied and f is the probability that any other site is occupied. Hence the sum $\sum_{jk}'$ over all occupied sites ($j\neq k$) is just $Nf\sum_k'$, where we now sum over *all* k sites, whether or not occupied. Thus Eq. (1) becomes

$$\langle\Delta\nu^2\rangle_{\mathrm{Av}}=[S(S+1)/3\hbar^2]f{\textstyle\sum_{k}}'\,B_{jk}^2, \tag{3}$$

where the sum is over all lattice sites, so that the second moment is always proportional to the concentration.[4] It is necessary, however, to examine the fourth moment before any conclusions may be drawn regarding the apparent width of the line. Van Vleck's Eq. (24) for the fourth moment involves sums of the form

$$\text{(I)}\ {\textstyle\sum_{jk}}'\,B_{jk}^4;\quad \text{(II)}\ {\textstyle\sum_{jkl}}'\,B_{jk}^2B_{jl}^2;$$

and

$$\text{(III)}\ {\textstyle\sum_{jkl}}'\,B_{jk}^2B_{jl}B_{kl}.$$

It is readily seen that these sums over randomly filled sites are equal to the following sums over all sites (with

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[1] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).

[2] Whitmer, Weidner, Hsiang, and Weiss, Phys. Rev. **74**, 1478 (1948).

[3] P. W. Anderson, Phys. Rev. **82**, 342 (1951).

[4] It may be noted that this differs from the result one would obtain if the effect of dilution were to expand the magnetic lattice uniformly.

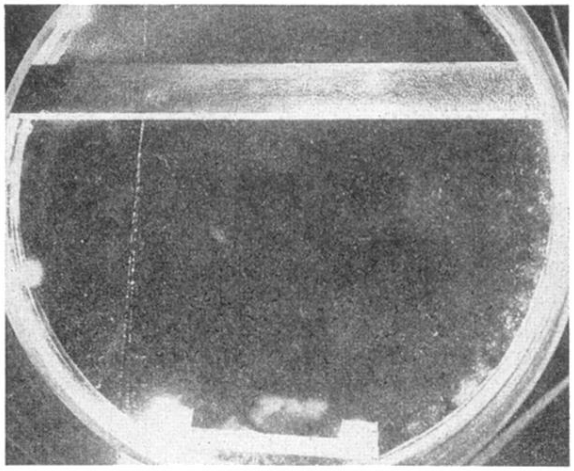

FIG. 2. A knock-on electron of about 230-Mev energy is produced by a single particle traversing the cloud chamber.