Range Distribution of μ-Mesons from π-Meson Decay in Photographic Emulsion*

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The lengths of μ-meson tracks from 1000 π-meson decays have been measured in two Ilford C-2 200-micron thick emulsions, which had been exposed by R. Sagane in a spiral orbit spectrometer to the Berkeley cyclotron. Shrinkage factors obtained from this analysis were used to calculate the true ranges of the μ-meson tracks. The distribution of true ranges meets statistical tests for a Gaussian distribution; it has a mean of 597 ± 1 microns and a standard deviation of 29.1 ± 0.7 microns. Since decay in flight and soft photon emission accompanying decay have negligible effects in this study, the straggling parameter for 4.1 Mev μ-mesons in photographic emulsion can be obtained; it is 4.86 ± 0.12 percent. The effect of experimental uncertainties on these values can be about 0.1 percent, which is slightly less than the statistical limits imposed by the finite number of events.

I. INTRODUCTION

THE present investigation was undertaken to study the range straggling in photographic emulsion of μ-mesons from π-meson decay. Emission of gamma-rays accompanying π-μ decay has been proposed[1-3] as a mechanism to explain the occurrence of anomalously short μ-meson tracks.[4-8] Since experiment and theory indicate that the magnitude of this effect is small, the μ-mesons ejected from π-mesons at rest[9] should be essentially monoenergetic. It is of interest to determine if an asymmetry can be detected in the range distribution. Thus, if experiment indicates a symmetrical distribution, it is reasonable to assume that the μ-mesons are essentially monoenergetic; one can then obtain the straggling parameter for 4.1-Mev μ-mesons in photographic emulsion. A good value for this parameter can be found since the π-meson flux from the Berkeley cyclotron is large enough to permit study of statistically significant numbers of events in single plates.

II. EXPERIMENTAL METHOD

Ilford C-2 200-micron thick emulsions, which had been exposed in a spiral orbit spectrometer to the Berkeley cyclotron, were made available by R. Sagane. About 800 positive π-μ decays with the μ stopping in the emulsion were obtained in each of two plates. The ranges of 750 μ-mesons were measured in the first plate; 250 were measured in the second. All measurements were made with a Cooke, Troughton, and Simms M4005 microscope, using the two stage micrometer screws as standards and using the fine focus control to measure depths. An eyepiece reticle, of 62.1 microns projected length, was calibrated against these micrometer screws. The reticle was used to measure straight segments of the tracks; the micrometers were used to step off successive segments of highly curved portions, to measure sections including large-angle scatterings, and to obtain the residual length after the last full reticle. In all cases the x-y plane projection of a segment was corrected to give the length by using $S^2(\Delta z)^2$, with the shrinkage factor S taken to be 2.7 from the literature.[10] It was impossible to determine this shrinkage factor by direct measurement as the plates had been processed before the program was initiated.

The ranges thus obtained were analyzed statistically to determine whether correlation existed between the z projection of a track and the range ascribed to it. Such a correlation would occur if an incorrect value had been used for the shrinkage factor. This correlation was found to exist; a regression equation was constructed from geometrical considerations and the coefficient was calculated for each plate. From these coefficients it was possible to obtain corrected values for the shrinkage factors. These corrected shrinkage factors and the microscope readings for the segments of each track were used to calculate the true range of the track. These true ranges and their z projections were statistically analyzed by the previous method to show that a significant correlation between them did not exist.

III. ANALYSIS

Geometrical Considerations

The true range r_t of a track may be represented by

$$r_t=\sum_i[\Delta x_i^2+\Delta y_i^2+(S+\delta)^2\Delta z_i^2]^{\frac{1}{2}}, \tag{1}$$

where i denotes the segments of the track, $S=2.7$ (the

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[8] W. Riezler and A. Rudloff, Z. Naturforsch. **7a**, 570 (1952).
[9] The probability of decay in flight of a π-meson of a few hundred kev is negligibly small, as comparison of mean lifetime with stopping time indicates. This is substantiated by a study of 3600 π^- mesons which stopped in photographic emulsions. No π-μ decays were observed among the 3600 π^- endings, which experimentally confirms the small probability of π-meson decay in flight in photographic emulsion.
[10] J. Rotblat and C. T. Tai, Nature **164**, 835 (1949).

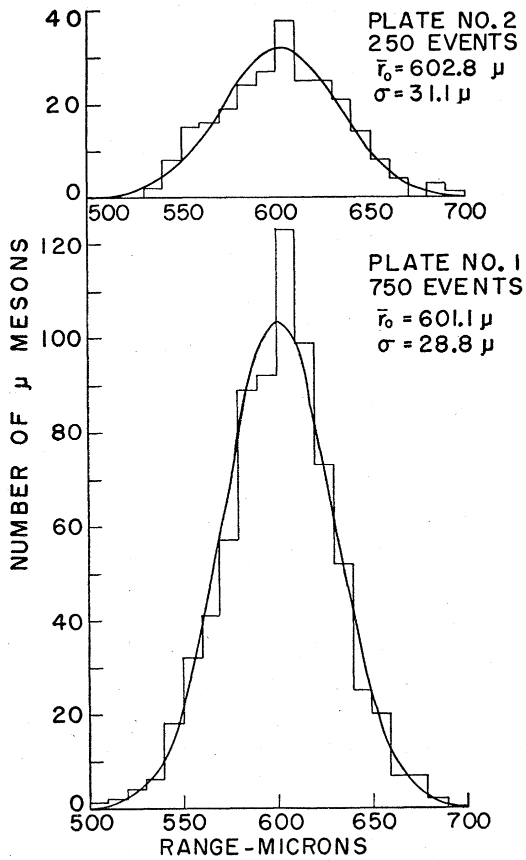

FIG. 1. Histograms of original ranges in each plate. In each case the experimental distribution and the normal curve have the same mean and standard deviation.

shrinkage factor originally assumed), and $S+\delta$ is the true shrinkage factor. The original range r_0 of the track is given by the expression

$$r_0=\sum_i[\Delta x_i^2+\Delta y_i^2+S^2\Delta z_i^2]^{\frac{1}{2}}. \tag{2}$$

Expanding Eq. (1) and neglecting higher order terms, one obtains

$$r_t=r_0+\sum_i\frac{(2S\delta+\delta^2)\Delta z_i^2}{2[\Delta x_i^2+\Delta y_i^2+S^2\Delta z_i^2]^{\frac{1}{2}}}. \tag{3}$$

Since most of the segments are measured with the 62.1-micron reticle, and the contribution of $S^2(\Delta z)^2$ to the radical is small, the approximation is made that

$$[\Delta x_i^2+\Delta y_i^2+S^2\Delta z_i^2]^{\frac{1}{2}}=[(62.1)^2+(2.7)^2\langle\Delta z^2\rangle_{\mathrm{Av}}]^{\frac{1}{2}}. \tag{4}$$

Thus, one obtains

$$r_0=r_t+\beta\psi, \tag{5}$$

where

$$\beta=\frac{-(2S\delta+\delta^2)}{2[(62.1)^2+(2.7)^2\langle\Delta z^2\rangle_{\mathrm{Av}}]^{\frac{1}{2}}},\quad \psi=\sum_i\Delta z_i^2. \tag{6}$$

Statistical Considerations

Owing to the geometrical considerations above, the regression equation is assumed to be of the form

$$r_0=r_t+\beta\psi. \tag{7}$$

The regression coefficients β are estimated by making the least squares fit of the analytic function $y=a+\beta x$ to the 750 experimental points ($x=\psi$, $y=r_0$) from the first plate, and again to the 250 experimental points from the second plate.[11] The correction δ to the original shrinkage factor S is obtained for each plate by equating the geometrically derived expression for β (6) to the statistically estimated value.

Experimental Considerations

The measurement of the vertical component of a track segment introduces the largest uncertainty in the range determination. This uncertainty is reduced if the emulsion thickness is small with respect to the μ-meson ranges; hence, 200-micron thick emulsions were advantageous. Any systematic error in the fine focus calibration is eliminated by using the shrinkage factor obtained from the regression calculations. Because of the large variation in grain density at the π-μ junction in Ilford C-2 emulsions, this type of emulsion was desirable for this study.

The reproducibility of measurement on a single track was checked by repeated measurement of randomly selected tracks. From these data, the standard deviation of the error was estimated to be 3 microns. Other errors in the range of a track occur due to shrinkage fluctuations throughout the emulsion, the estimation of arc lengths by chords, and the variation of lateral magnification with depth in the emulsion (caused by the different indices of refraction for gelatin and immersion oil). The net effect of these errors can be reasonably estimated to be less than 1 percent on a single track, hence less than 0.1 percent on the distribution parameters, which is slightly less than the statistical limits imposed by the finite number of events.

IV. EXPERIMENTAL RESULTS

Histograms of the original ranges are shown in Fig. 1. The hypothesis that the experimental distribution of ranges in each plate can be represented by a normal curve was tested by comparing the third and fourth moments of this distribution with the moments of a normal frequency curve having the same mean and standard deviation.[12] For the first plate, the deviations of the distribution moments from the normal curve moments are less than one would expect in approximately 30 percent of samples drawn from a known normal distribution; for the second plate, the deviations

[11] A. M. Mood, *Introduction to the Theory of Statistics* (McGraw-Hill Book Company, Inc., New York, 1950).
[12] R. A. Fisher, *Statistical Methods for Research Workers* (Hafner Publishing Company, New York, 1950), eleventh edition.

are less than one would expect in approximately 40 percent of such samples. These results are thus consistent with the hypothesis that the distributions are normal. There are two reasons for making these tests, even though the original ranges are known to be incorrect. First, the application of the statistical analysis used for the determination of the corrected shrinkage factors is not valid unless the original ranges are normally distributed. Second, it should be shown that the normal distribution of true ranges finally obtained is not merely due to the fortuitous choice of a shrinkage factor.

The corrected shrinkage factors obtained by the method outlined are

$$S+\delta=2.46\pm0.14 \quad \text{(first plate)},$$
$$S+\delta=1.89\pm0.24 \quad \text{(second plate)}.$$

The difference in these shrinkage factors is attributed to the difference in the processing and treatment of the plates. The second plate was continually under immersion oil during the long scanning period, and stored at a high humidity. An independent measurement showed the thickness of a dry emulsion changed by about 20 percent after storage at a high humidity and then absorption of immersion oil.

Figure 2 is a histogram of the 1000 true ranges, with the corresponding normal frequency curve superimposed. The same analytic test for goodness-of-fit between the distribution of true ranges and the normal curve was made by comparing their moments. The emission of gamma-rays accompanying π-meson decay would cause shorter μ-meson ranges and lead to a negative third moment for the distribution. The experimentally determined third moment of the distribution is negative, but it is only 1.26 standard deviations removed from zero, i.e., of samples drawn from a known normal distribution, approximately 20 percent would have larger third moment values. Thus, this experimentally determined asymmetry cannot be called statistically significant. The deviation of the fourth moment from the normal curve value is less than would be expected from approximately 80 percent of samples from a known normal distribution. Thus, there is no statistically valid evidence for rejecting the hypothesis that the distribution of μ-meson ranges from π-meson decay in photographic emulsion is a normal distribution.

Statistical analysis of the true ranges and their z projections shows that there is no significant correlation between them.

The mean of the distribution of true ranges is 597 ± 1 microns. The standard deviation of this distribution is 29.1 ± 0.7 microns.

V. CONCLUSIONS

1. The range distribution of 1000 μ-mesons from π-meson decay in photographic emulsions can be considered normal, within the statistical limits. In particular, the third moment of this distribution indicates no statistically significant asymmetry.[13]

2. In this work the experimental errors approach the statistical uncertainties in magnitude. These statistical uncertainties are inversely proportional to the square root of the number of events. Therefore, this study appears to give the distribution parameters with as high precision as can be reasonably obtained using present emulsion techniques. The values for these parameters are[14]

$$\bar{r}=597\pm1 \text{ microns}, \quad \sigma=29.1\pm0.7 \text{ microns}.$$

3. Since there is no statistically significant departure from normality in the range distribution, the μ-mesons were essentially monoenergetic. Thus the straggling parameter can be obtained for 4.1 Mev μ-mesons in

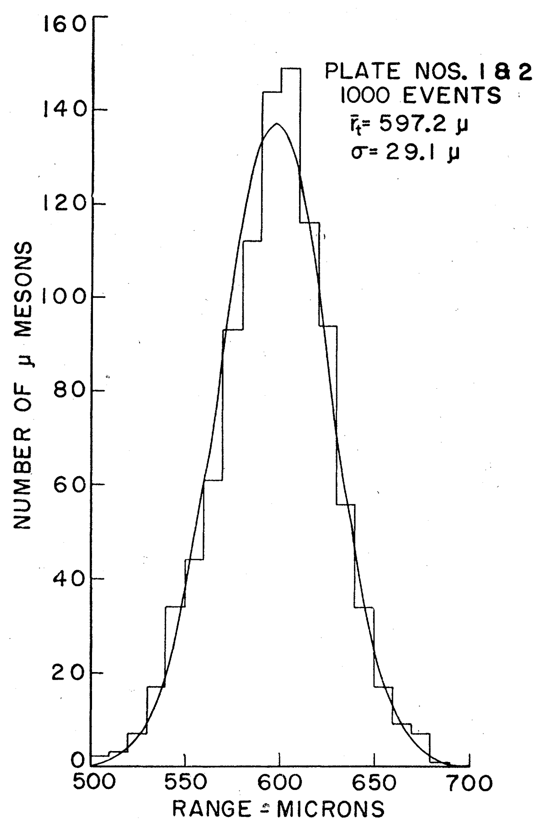

FIG. 2. Histogram of true ranges. The distributions of true ranges in each plate were combined by direct addition. The experimental distribution and the normal curve have the same mean and standard deviation.

[13] See reference 7. These workers report an extended non-Gaussian "tail" on the short end of the range distribution, from a study of 300 μ-meson ranges. No evidence of such a tail has been found in this investigation of 1000 μ-meson ranges. The inherent disadvantages in their use of thick electron-sensitive emulsions may account for this discrepancy.

[14] See reference 5. Barkas gives the values $\sigma=26.5$ microns and $P=4.4$ percent. He normalized range distributions of 163 μ-mesons in three plates to a mean of 600 microns and combined them to obtain his distribution. The mean ranges in the three plates were 591.1 ± 2.3 microns, 609.8 ± 2.5 microns, and 600.0 ± 2.3 microns.

photographic emulsion. This parameter is[15]

$$P=(29.1\pm0.7)/(598.5\pm1)=4.86\pm0.12 \text{ percent.}$$

We are indebted to Dr. R. Sagane for making his photographic plates available to us. Professor J. K. Knipp has provided invaluable stimulation, advice, and discussion in the process of this work. The suggestions and criticism of Professor Oscar Kempthorne of the Iowa State College Department of Statistics have been extremely welcome and useful.

[15] If the emulsion thickness t is less than the range r of a μ-meson, the longer tracks have a higher probability of escaping from the emulsion. The probability of a μ-meson which originates in the emulsion also stopping in it is $t/2r$. Thus, if the μ-meson ranges would be normally distributed in an infinite emulsion, their distribution in a thin emulsion would be $(K/r)\exp[-(r-\bar{r})^2/2\sigma^2]$. It can be shown that this distribution can be approximated by a normal distribution with the same standard deviation and a displaced mean, $K'\exp[-(r-\bar{r}-\Delta)^2/2\sigma^2]$, with more accuracy than could be detected in this study, by expanding K/r in a Taylor's series about $\bar{r}$ and showing that the series expansion of $K'\exp[\{2(r-\bar{r})\Delta-\Delta^2\}/2\sigma^2]$ sufficiently approximates this Taylor's series when $\Delta=-\sigma^2/\bar{r}$. Thus, an experimentally obtained range distribution has a mean about 1.5 microns less than the value to be expected in infinite emulsion.

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Total Compton and Pair Production Cross Sections at 19.5 Mev*

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Total cross sections of 19 elements were measured at 19.5 Mev by the photonuclear detector method. The penetration of betatron bremsstrahlung (maximum energy, 20.4 Mev) through the absorbers was determined by the $C^{12}(\gamma,n)C^{11}$ reaction (threshold, 18.7 Mev) in polyethylene. Simultaneously operated monitor-detector Geiger units, which canceled timing and intensity fluctuation errors, recorded the carbon-11 activity. Percent standard deviations averaged 0.6 for most elements; the estimated systematic error was 0.4 percent. The narrow energy band width permitted an accurate calculation of the correction for the proportion of counts due to secondary radiation.

From the absorption data a value of the Compton cross section was determined. The integrated Klein-Nishina result fell within the 2 percent experimental error calculated by adding linearly to the statistical errors, an estimated 50 percent uncertainty in the published photonuclear cross sections employed in the determination. The cross sections for pair production found from the experiment σ_e, are related to the theoretical values σ_t, as derived by the Born approximation (Bethe-Heitler formula including screening), by $(\sigma_t-\sigma_e)/\sigma_t=(1.55\pm0.1)\times10^{-6}Z^2$, to a first approximation.

I. SUMMARY OF THE THEORY OF ABSORPTION OF HIGH ENERGY QUANTA

Of the major processes by which high energy photons interact with matter[1]—Compton effect, pair production, triplet production, atomic photoelectric effect, and photonuclear reactions—the first two account for nearly all of the total absorption cross section in all elements at high relativistic energies.

The differential cross section for the Compton effect was calculated by Klein and Nishina on the assumption that the interacting electrons have zero binding energy.[2] The integration is elementary,[3] and the result has been verified at many low energies.[4]

The cross section for the production of pairs in the nuclear field has been derived, with the aid of the Born approximation, by Bethe and Heitler.[5] The treatment included the effect of screening of the nuclear potential by the atomic electrons based on the assumption of a Thomas-Fermi atomic model. The pair cross section was recalculated by Maximon, Davies, and Bethe, using Coulomb wave functions.[6]

Triplet production, or the creation of pairs in the field of an electron, resulting in the ejection of the three particles has been studied by Wheeler and Lamb[7] in a manner similar to the analysis of nuclear pair production by Bethe and Heitler. The cross section per atom is approximately $1/Z$ that of pair production; this ratio varies little with energy or Z.

The relativistic theory of high energy photoelectric absorption in the K shell was developed rigorously by

* Work performed under the auspices of the U. S. Atomic Energy Commission. A preliminary report was given to the American Physical Society, Berkeley meeting (December 27–29, 1951).

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[1] A more complete discussion of these absorption processes may be found in A. I. Berman, Los Alamos Document, LAMD 1088 (unpublished).

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