

$\mathcal{H}_{SL}$  were obtained by transcribing the nonrelativistic form of the Breit fine structure Hamiltonian<sup>2</sup> for the electromagnetic interaction between two electrons to apply to the interaction between a neutron and an electron, neglecting a term in the neutron momentum  $\mathbf{p}_n$ , smaller by a factor  $\sim m/M_p$  than the electron momentum term in Eq. (3). Equation (2), when expanded, includes besides the usual dipolar energy a  $\delta$ -function term necessary to obtain agreement with experiment.<sup>3</sup> Both Eqs. (2) and (3) may be derived in an elementary way from classical electromagnetic theory.

Consider the differential scattering cross section in center-of-mass coordinates in Born approximation for the scattering of a monochromatic beam of neutrons by a molecule or small crystal. The Born approximation is expected to be valid here for scatterers of linear dimensions less than about 20A independent of neutron wavelength but may well be valid for much larger scatterers. The matrix elements of Eq. (1) involved are between wave functions of the system neutron plus scatterer with no interaction. When integration is carried out over the neutron space and spin coordinates, and an average over initial neutron spin states is performed, the differential cross section may be written

$$\sigma(\omega) = \left( \frac{R g_n e^2}{2mc^2} \right)^2 \sum_b \frac{k_b}{k_a} \{ [I - \mathbf{e}_{ba} \mathbf{e}_{ba}] : [\mathbf{P}_{ab}^S \mathbf{P}_{ba}^S - i(\mathbf{P}_{ab}^S \mathbf{P}_{ba}^L + \mathbf{P}_{ab}^L \mathbf{P}_{ba}^S)] - \mathbf{P}_{ab}^L \cdot \mathbf{P}_{ba}^L \}, \quad (4)$$

corresponding to an unpolarized incident beam. In Eq. (4) we have used the abbreviation

$$\mathbf{P}_{ba} = \int \psi_b^* \mathbf{P}(\mathbf{K}_{ba}) \psi_a d\tau \quad (5)$$

for matrix elements of the operators

$$\mathbf{P}^{(S)}(\mathbf{K}) = \sum_j e^{i\mathbf{K} \cdot \mathbf{r}_j} \mathbf{s}_j, \quad (6)$$

$$\mathbf{P}^{(L)}(\mathbf{K}) = \sum_j e^{i\mathbf{K} \cdot \mathbf{r}_j} \mathbf{K} \times (\mathbf{p}_j / \hbar), \quad (7)$$

with respect to wave functions  $\psi$  of the scatterer. The subscripts  $a$  and  $b$  refer, respectively, to initial and final states;  $R$  is the ratio of the reduced mass to the proton mass;  $\mathbf{k}$  is the neutron wave vector,  $\mathbf{K}_{ba} = \mathbf{k}_a - \mathbf{k}_b = K_{ba} \mathbf{e}_{ba}$ ;  $\mathbf{I}$  is the unit dyadic;  $k_a$  and  $k_b$  are related by conservation of energy.

Equation (4), or analogous expressions for cases in which the incident neutron beam is polarized and/or the scattered beam is analyzed, should be useful for testing electronic wave functions of a magnetic scatterer, especially in regions frequented by valence electrons. Assuming that any bound-state electronic eigenfunction of the scatterer can be expressed as a linear combination of anti-symmetrized products of orthonormal one-electron orbitals  $\phi$ , e.g., molecular or crystal orbitals, the electronic part of the matrix elements (5) is expressible as a linear combination of one-electron matrix elements of the types

$$(\phi' | e^{i\mathbf{K} \cdot \mathbf{r}} | \phi) \quad \text{and} \quad (\phi' | e^{i\mathbf{K} \cdot \mathbf{r}} \nabla | \phi). \quad (8, 9)$$

The  $\phi$ 's are commonly approximated as a linear combination of orthonormal atomic orbitals<sup>4</sup>  $u_{nl}(r) Y_l^m(\theta, \varphi) \chi_{m_s}(\sigma)$ , in which case (5) is ultimately expressible as a linear combination of matrix elements (8, 9) in which the  $\phi$ 's are atomic orbitals about the same center. These can be evaluated quite generally except for radial integrals:

$$\begin{aligned} (n'l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} | nlm_s) &= (m_s' | s | m_s) \\ &\quad \times \int_0^\infty u_{n'l'}(l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} | lm) u_{nr} r^2 dr \\ (n'l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} \nabla | nlm_s) &= \delta(m_s' m_s) \\ &\quad \times \int_0^\infty u_{n'l'}(l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} \nabla | lm) u_{nr} r^2 dr, \end{aligned}$$

in which the angular matrix elements  $(l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} | lm)$  and  $(l'm_s' | e^{i\mathbf{K} \cdot \mathbf{r}} \nabla | lm)$  involve linear combinations of spherical Bessel functions  $j_p(Kr)$  with coefficients depending on the direction of  $\mathbf{K}$  times 1,  $1/r$ , or  $\partial/\partial r$ ; tables of these matrix elements for  $l', l \leq 3$  will be published elsewhere.

For free atomic scatters closed shells yield no contribution to Eq. (4) and, if spin interactions are neglected in the atomic Hamiltonian, neither do the cross terms in Eq. (4).

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<sup>1</sup> For example, O. Halpern and M. H. Johnson, Phys. Rev. 55, 898 (1939); G. T. Trammel, Bull. Am. Phys. Soc. 28, No. 1, 26 (1953).

<sup>2</sup> G. Breit, Phys. Rev. 34, 553 (1929).

<sup>3</sup> Shull, Wollan, and Strauser, Phys. Rev. 81, 483 (1951).

<sup>4</sup> See, for example, E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951).

## Total Cross Sections for 400-Mev Neutrons\*

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THOUGH the present program of total neutron cross-section measurements at this laboratory has not yet been completed, it was thought of interest to report the preliminary results already available.

The experiment consists of good geometry transmission measurements in the well-collimated neutron beam of the University of Chicago synchrocyclotron. A novel threshold detector is used to define a narrower energy spread of the effective neutron beam than is available directly from the cyclotron. This detector is a counter telescope, consisting of two scintillation counters and a Čerenkov counter,<sup>1</sup> set to observe recoil protons scattered from polyethylene in the neutron beam. The average effective energy of the neutron beam is estimated to be about 400 Mev.

The hydrogen cross section was determined by measuring the attenuations of the neutron beam by samples of graphite, benzene, and cyclohexane containing the same thicknesses of carbon. This technique conveniently permits measuring the effect of different amounts of hydrogen in approximately the same geometry.

The cross sections that have been determined are as follows, in barns:

Hydrogen	0.0336 ± 0.001
Carbon	0.298 ± 0.003
Copper	1.190 ± 0.015
Cadmium	1.84 ± 0.03
Lead	2.88 ± 0.04
Thorium	3.23 ± 0.05
Uranium	3.26 ± 0.05

The errors shown are statistical errors of counting and the values given are to be regarded as tentative pending a more complete study of possible systematic errors in the measurement, and a more accurate determination of the neutron energy spectrum.

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<sup>1</sup> John Marshall, Phys. Rev. 86, 685 (1952).

## Comparison of Gamma-Ray Reflections from the (550) and (310) Planes of Quartz

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MEASUREMENTS with the curved crystal focusing gamma-ray spectrometer<sup>1-3</sup> to date have been made using a thin lamina of quartz cut so that the (310) planes are perpendicular to the faces of the lamina. Although the (310) planes are the most convenient for establishing a link between the region of x-radiation and the region of gamma-radiation, the use of these reflecting planes places rather severe limitations on the precision of the instrument in the energy region above 500 kev. These limitations are as follows: (1) The Bragg angle for selective reflection varies as  $E^{-1}$ , and for first-order reflections is less than 20 minutes of arc for 1-Mev gamma-radiation. An accuracy of better than 1 or