Current Increase at Constant Amplification Factor in Steady Corona with Coaxial Cylindrical Geometry*

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(Received December 5, 1952)

It has long been known that above onset of steady positive wire corona with coaxial cylindrical geometry, Townsend's relation of 1914 relating current i and over-voltage, $V-Vg$, of the form $V-Vg=Bi/V$ applies over a considerable range. This law is deduced assuming a constant potential across a zone of limited constant ionizing radius. Such assumption implies a constant Townsend amplification factor for ionization by collision, a fact just recently experimentally established by Colli, Facchini, and Gatti. The corona current i on this theory, however, increases many times despite constant amplification factor. It is shown that this occurs by slight temporary raising of the amplification factor producing a rapid proliferation of ionizing sequences over the anode surface, increasing current and space charge density, but maintaining essentially constant amplification factor. This action has a bearing on the spread of the normal glow discharge over the cathode.

SINCE 1914 the equation of Townsend[1] for corona currents with anode in coaxial cylindrical geometry above onset of steady corona has been justified experimentally within fairly wide limits of potential variation irrespective of the gas filling or corona mechanism.[2] In rough form this reads,

$$V-V_g=\frac{piA^2}{2KV}\log(A/a), \qquad (1)$$

where V_g is the observed corona threshold, V the applied potential, K the reduced mobility, p the gas pressure, A and a the radii of cathode and anode cylinders, respectively, and i the current. Since space charge density n of carriers beyond the ionizing zone from anode surface at $r=a$ to $r=r_{0g}$ where ionization ceases is given by

$$n=\frac{ip\log(A/a)}{2KV}=\frac{p\log(A/a)}{2KV}\frac{dq}{dt}, \qquad (2)$$

with n constant across the gap, it is clear that the overvoltage $V-V_g$ in Eq. (1) serves only to overcome and move the positive ion space charge and that the corona operates at V_g. If this is correct then the ion amplification factor

$$\exp\int_{r_{0g}}^{a}\alpha dr$$

must remain sensibly constant over the region where this law applies, for at V_g the threshold condition is

$$\gamma\exp\int_{r_{0g}}^{a}\alpha dr=1, \qquad (3)$$

whether γ stands for a cathode mechanism or its equivalent in the photoelectric gas process characteristic of fast Geiger counters. While this constancy of the amplification factor has been tacitly assumed, it has not been strikingly brought home until the recent work of Colli, Facchini, and Gatti[3] who actually observed this align-constancy in A and $A+10^{-3}$ CO_2 mixture coaxial counters above corona threshold at various pressures above 200 mm. This constancy permits the use of such tubes as proportional α- and β-ray counters above background signal since the size of the short pulse produced $<10\mu$sec is clearly discernable against the slower background fluctuations.

This observation strikingly brings out an apparent paradox. For according to Eq. (1) the current i, or rate of charge production in the active zone, increases rapidly [proportional to $V(V-V_g)$], as V increases while the amplification factor is constant. Careful study shows, that the contradiction arises from assuming that the only way in which rate of ion production can increase is through change in the amplification factor. In reality ion production can be augmented, (a) by increasing the amplification factor and/or γ as well as the time rate of successive events by increasing V_g, *or*, (b) by keeping the amplification factor constant and merely increasing the number of multiplying sequences corresponding to the condition

$$\gamma\exp\int_{r_{0g}}^{a}\alpha dr=1$$

per unit area of electrode surface. Changes by mechanism (a) would involve increasing the potential for active ionization above V_g by ΔV_g and increasing r_{0g}. If the increase were of any magnitude over an extended voltage regime this would at once be detected through failure of the Townsend law and of measurements such as those of Colli, Facchini, and Gatti. It is also energetically an uneconomical mechanism for it involves increase in potential energy, and thus more degradation loss. To

* Studies underlying his have been supported by the U. S. Office of Naval Research.
[1] J. S. Townsend and P. J. Edmunds, Phil. Mag. **28**, 789 (1914); H. S. Townsend, Phil. Mag. **28**, 83 (1914); *Electricity in Gases* (Oxford University Press, London, 1915), p. 376 ff. H. F. Boulind, Phil. Mag. **18**, 909 (1934).
[2] Sven Werner, Z. Physik **90**, 356 (1934); **92**, 705 (1934); C. G. Miller and L. B. Loeb, J. Appl. Phys. **22**, 499 (1951).
[3] Colli and Facchini, and Gatti, Rev. Sci. Instr. **23**, 621 (1952).

understand the process (b), it is essential to state that the amplification factor and the quantities actually determining V_g are not the product of the ideal average experimental Townsend functions γ and

$$\exp\int_{r_{0g}}^{a}\alpha dr$$

in Eq. (3). Instead they are values somewhat larger, for at the ideal limit,[4] as Wijsman has shown, the discharge could hardly be self-sustaining, because of statistical fluctuations and here the space charges also augment interruption. Furtheremore it takes very small, (practically imperceptible on an observational scale), changes in V_g to give a quantity

$$\gamma\exp\int_{r_{0g}}^{a}\alpha dr>1.$$

Assume, for example, that such a change is produced when V is suddenly increased by a small amount ΔV above its previous value. At such an increase the first effect with a previous constant ion production dq/dt, will be to clear away space charge near the wire more rapidly than before. The active voltage in ion production, V_g, will thus instantaneously be increased by ΔV_g which may be quite small. At once

$$\gamma\exp\int_{r_{0g}}^{a}\alpha dr$$

will become greater than unity and carrier production as well as carrier densities will rise through multiplication of the number of avalanche sequences per unit surface area of anode. This will in time augment the space charge density near the temporarily extended r_{0g} and wipe out the increase ΔV_g, thus restoring the operating potential to V_g but having augmented the number of uninterrupted sequences

$$\exp\int_{r_{0g}}^{a}\alpha dr$$

per unit area of anode.

[4] R. A. Wijsman, Phys. Rev. **75**, 833 (1949).

Thus space charge will be maintained despite decreased time of traverse and dq/dt, or i, will be increased. V_g and

$$\exp\int_{r_{0g}}^{a}\alpha dr$$

will however in large measure be unchanged except for a small instantaneous fluctuation, lasting perhaps some 10μsec.

On such a picture the introduction of 10^4 new electrons by an α particle track in 10^{-8} sec will cause an instantaneous increase in current for some μsec after which the excess space charge must reduce the current until it dissipates from the neighborhood of the anode. The observed current should rise to a sharp peak in a μsec and then sharply fall below its steady value eventually recovering its steady value after some 10μsec. This is just what Colli, Facchini, and Gatti[3] observed. Returning to the steady corona at V_g it is clear that the increase of current produced by proliferation of avalanche sequences per unit area of anode will continue with increasing V until the space charge density with increasing potential is reduced so that the width of the ionizing zone from a to r_{0g} irreversibly increases, or until secondary processes involving crossing times or other effects become inadequate to increase dq/dt as fast as needed. At such values of V the Townsend law begins to fail and amplification factors increase. The region over which amplification factors are constant can thus be expected to be quite extensive in potential range, as observed. There is in this explanation a vague analog to the action of the normal cathode fall in glow discharges. There however the space charge density increasing over the cathode causes a spreading of the glow over the still uncovered cathode surface by virtue of the repulsion between the various more intensive γ-conditioned unit avalanche sequences.[5] With the coaxial counter the anode wire is covered with glow over its whole surface because of the diffusive nature of the photo-ionization in the gas. Avalanches are shorter and less intense so that the space charge density is less especially with small A and rapid clearing. Increase of current can then readily occur by increase in avalanche processes per unit area of anode surface.

[5] L. B. Loeb, J. Appl. Phys. **19**, 882 (1948); W. Finkelnburg and S. M. Segal, Phys. Rev. **83**, 582 (1951).