Relative Energy Levels of Some P Shell Nuclei in Intermediate Coupling*†

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The relative energy levels of the low states for configurations consisting of 2, 3, 4, 8, 9, and 10 nucleons in the P shell are shown for all values of the spin orbit to interaction energy ratio by means of second-order perturbation calculations in both extreme coupling schemes. The interaction potential is assumed to be a linear combination of ordinary, and space, spin, and charge exchange terms. Coulomb forces are ignored so that the charge quantum number T is a constant of the motion. Comparison is made with the first few excited states of some light nuclei. There is some evidence that the spin orbit energy is relatively more important for N^{14} than for Li^6.

I. INTRODUCTION

THE individual particle model with strong spin orbit coupling[1] has been successful in explaining the "magic numbers" of atomic nuclei and in explaining the ground state angular momenta. The success of this theory is dependent on an increase in the spin orbit parameter with atomic number. For very heavy nuclei the energy levels are described by the jj coupling scheme. For very light nuclei the observed splittings of what appear to be multiplet states indicates that the energy levels may be better explained by Russell-Saunders coupling.

The purpose of the present paper is to show the relative energy levels of light, P-shell nuclei assuming phenomenological central and spin-orbit interactions for all ratios of the spin-orbit to central interaction values. This is done by performing second-order perturbation calculations at the two coupling extremes. Hummel and Inglis[2] have studied the intermediate coupling states of Li^7 using both Wigner and Majorana interactions. The calculations given here differ essentially from those of Hummel and Inglis for this nucleus in the form assumed for the interaction.

TABLE I. Diagonal elements of the interaction energy for the jj states $(3/2)^u(1/2)^v$ for $n=u+v=2, 3, 4, 8, 9, 10$.

u	v	T	J	Energy	u	v	T	J	Energy
2	0	0	3	$5.00K$	8	2	0	1	$79.22K$
		1	0	2.87			1	0	77.35
		0	1	0.33	7	3	0	2	81.42
		1	2	0.33			0	1	80.89
1	1	0	2	5.00			1	2	78.09
		0	1	4.47			1	1	75.42
		1	2	1.67	6	4	0	3	81.42
		1	1	−1.00			1	0	79.29
0	2	0	1	2.80			0	1	76.75
		1	0	0.93			1	2	76.75
3	0	1/2	3/2	$9.51K$	8	1	1/2	1/2	$63.32K$
		1/2	7/2	7.98	7	2	1/2	3/2	66.16
		1/2	5/2	4.74			1/2	5/2	65.22
		3/2	3/2	2.23			1/2	3/2	62.26
		1/2	1/2	0.73			1/2	1/2	60.94
							3/2	3/2	59.65
4	0	0	0	$18.73K$					
		0	2	15.83	8	0	0	0	$49.90K$
		0	4	14.11	7	1	0	2	53.51
		1	3	11.34			0	1	49.89
		1	2	11.20			1	2	48.68
		0	2	7.42			1	1	47.03
		1	1	6.48					
		3	0	4.66					

II. THEORY

A. General

When a number of particles possessing spin and orbital angular momenta are allowed to interact they can in general couple their spins and orbital angular momenta in many ways to form different states. For atomic nuclei the energy differences between states arising from the same configuration can be as large as or larger than the energy differences between states arising from different configurations. Hence, a study of the relative energies of the states belonging to the lowest configuration is not expected to explain all of the low-lying levels for a given nucleus. However, for the nuclei from $A=6$ through $A=14$ one might expect the first few excited states to belong to configurations in which the P shell is partially filled.

The problem is to calculate the average value of the total energy

$$E=\sum_{ik}\tfrac{1}{2}V_{ik}+\sum_i a\mathbf{l}_i\cdot\mathbf{s}_i+\text{constant},$$

for the possible states of the P-shell configurations. In this formula V_{ik} represents the interaction between the ith and kth nucleons, $a\mathbf{l}_i\cdot\mathbf{s}_i$ represents the spin orbit energy of the ith nucleon and the summations are taken over all nucleons in the P shell. In what follows the spin orbit parameter a is treated as a constant for any given nucleus.

V_{ik} is, of course, one of the unsolved problems of nuclear physics. We shall assume

$$V_{ik}=(W+MP_x{}^{ik}+BP_\sigma{}^{ik}-HP_\tau{}^{ik})J(r_{ik}),$$

where $P_x{}^{ik}$, $P_\sigma{}^{ik}$, and $P_\tau{}^{ik}$ are the well-known space, spin, and charge exchange operators, respectively. Then

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[1] M. Mayer, Phys. Rev. 78, 16 (1950).

[2] H. Hummel and D. Inglis, Phys. Rev. 81, 910 (1951).

if $J(r_{ik})$ represents the correct triplet nucleon-nucleon force, the low energy scattering data supplies the conditions:

$$W+M+B+H=1, \quad W+M-B-H=0.6.$$

In the interest of eliminating from the final results all arbitrary constants except the spin orbit to interaction parameter ratio the assumption is made that this form of the interaction should also satisfy the saturation requirements of the nuclear force.[3] With this added condition one obtains the rather uncertain values:

$$W=-0.13, \quad M=0.93, \quad B=0.46, \quad H=-0.26.$$

B. Russell-Saunders Coupling

Assuming the spin orbit parameter to be zero the various states can be labeled by means of the total spin and orbital angular momentum quantum numbers of the protons and neutrons separately. Or, with charge independent forces, the states may be labeled by the total isotopic quantum number T, as well as the total spin S and the total angular momentum L. These constants of the motion are still not sufficient to completely specify the wave functions for all possible states. Wigner[4] has pointed out that, for the ordinary and space exchange terms of the interaction, the space part of the wave function should belong to a definite representation of the permutation group. This representation is described by means of the partition numbers $[\alpha] = [\alpha_1, \alpha_2, \cdots]$. In the presence of the spin and charge exchange terms these wave functions are very good approximations to the true wave functions.[5] (The off-diagonal elements of the interaction energy using these wave functions are multiples of K only—see below.) The labels T, S, L, and $[\alpha]$ are sufficient to specify completely the wave functions for all P-shell states except for the case of six nucleons. We shall use the notation ${}^{ab}A(\alpha)$, where $a=2T+1$, $b=2S+1$, and $A=S, P, D, \cdots$ specifies the total orbital angular momentum.

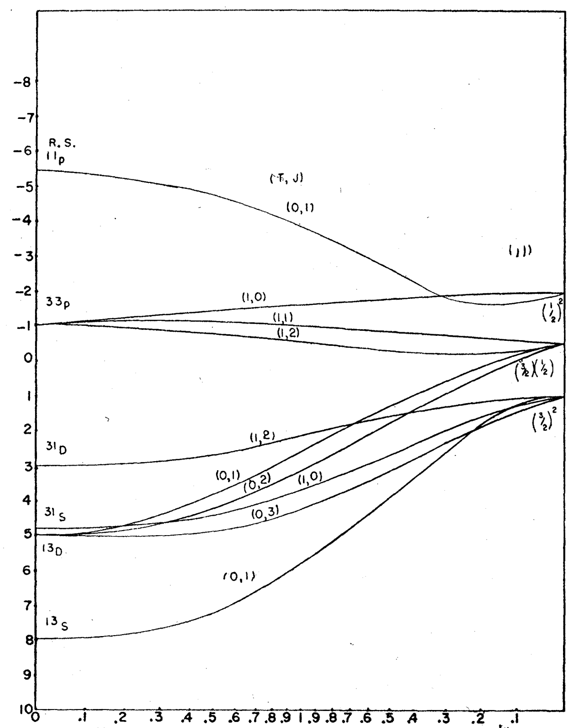

FIG. 1. $\bar{V}/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for $2P$-shell nucleons. $\chi=a/K$.

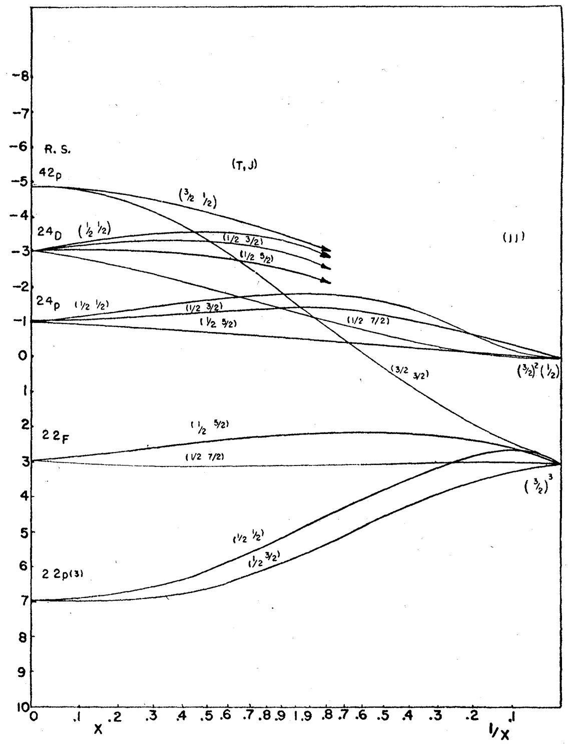

FIG. 2. $(\bar{V}-9K)/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for $3P$-shell nucleons. $\chi=a/2K$.

Feenberg and Phillips[5] have calculated the interaction energies for the "low" partitions for the P-shell nuclei. Writing the three components of the single particle wave functions for P-shell nucleons in the form

$$P_{\pm 1}=(i/\sqrt{2})R(r)(x\pm iy), \quad P_0=R(r)z,$$

the average interaction energy appears as a linear combination,

$$\bar{V}=\omega L+\gamma K,$$

[3] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, New York, 1948), p. 217.
[4] E. Wigner, Phys. Rev. **51**, 106 (1937).
[5] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

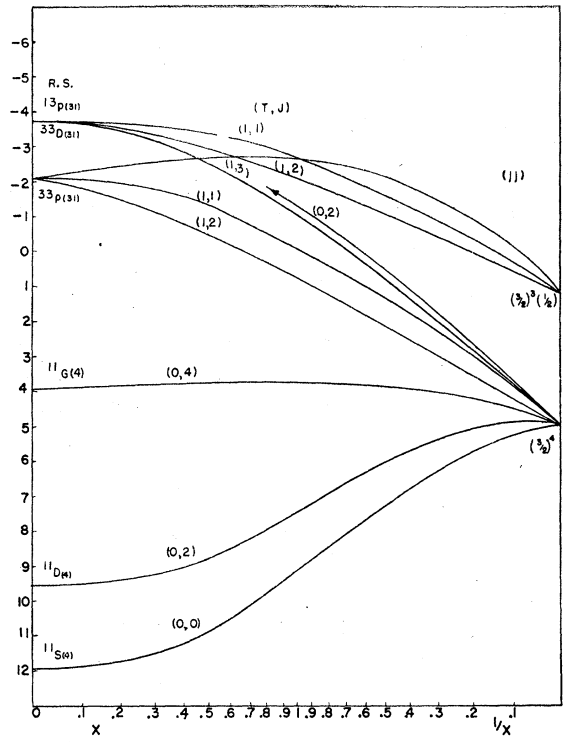

FIG. 3. $(\bar{V}-20K)/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for $4P$-shell nucleons. $\chi=2a/5K$.

of the two integrals:

$$L=\int\cdots\int x_1^2x_2^2R^2(r_1)R^2(r_2)J(r_{12})dv_1dv_2,$$

$$K=\int\cdots\int x_1x_2y_1y_2R^2(r_1)R^2(r_2)J(r_{12})dv_1dv_2.$$

L is approximately six times K, so that the relative energy values of the Russell-Saunders multiplets can be obtained in units of K. These multiplets are split by the spin orbit energy into different states of total angular momentum $\mathbf{J}=\mathbf{L}+\mathbf{S}$.

C. *jj* Coupling

In order to take full advantage of the calculations of Feenberg and Phillips, the matrix elements of the interaction energy in the jj coupling extreme can be calculated by expressing the jj wave functions in terms of the Russell-Saunders wave functions. This transformation is readily obtained since the eigenvalues of the spin orbit matrices calculated in the Russell-Saunders limit are known. Hence, it was here necessary only to extend the calculations of Feenberg and Phillips to include all Russell-Saunders states. Table I shows the diagonal elements of the interaction energy for $L=6K$ for all jj states for 2 and 8, and for the low configuration jj states for 3, 4, 6, and 7 nucleons in the P shell.

III. INTERMEDIATE COUPLING AND DISCUSSION OF RESULTS

In order to show the transition from one coupling scheme to the other the usual practice is to plot

$$(\bar{V}-\Delta)/K(1+\chi^2)^{\frac{1}{2}}=(\bar{V}-\Delta)/ca(1+\chi^{-2})^{\frac{1}{2}}\ vs\ \chi/(1+\chi),$$

where $\chi=ca/K$. c and Δ are arbitrary constants which are chosen so that the total range and relative separation of the states are the same order of magnitude at the two extremes of the plot. For low values of a/K, one effectively plots $(\bar{V}-\Delta)/K$ *vs* χ and for low values of K/a one effectively plots $(\bar{V}-\Delta)/ca$ *vs* $1-\chi^{-1}$.

Figures 1 through 6 show the transition from Russell-Saunders to jj coupling for the low-lying levels for 2, 3, 4, 8, 9, and 10 nucleons in the P shell, respectively. The curves have been interpolated for those regions not obtainable by either second-order perturbation calculation. The Russell-Saunders perturbation calculations break down for an a/K of the order of unity. The total interaction energy for $12-n$ nucleons is the same as that for n nucleons with the addition of a constant term in K and a reversal of the sign of a. One can include the constant term in Δ so that the curves are similar for these two cases in the Russell-Saunders limit.

The first three energy levels indicated in Fig. 1 for $K/a=0.7$ show agreement with the known levels of Li^6 if $K=-1.9$ Mev. This value of K is about twice that obtained by Feenberg and Phillips by actual evaluation

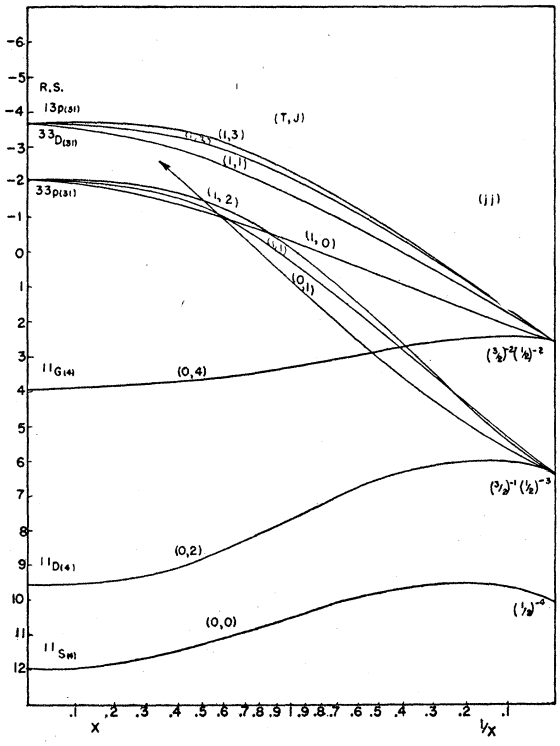

FIG. 4. $(\bar{V}-58.28K)/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for $8P$-shell nucleons. $\chi=2a/5K$.

of the integral. The second excited state of Li^6 has $T=1$ and should correspond, in the approximation for which T is a good quantum number to the ground states of He^6 and Be^6.

The lowest states for 10 nucleons in the P shell should be those of N^{14}. The energy of the first excited (2.3-Mev) state of N^{14} is approximately the same as that of the ground states of C^{14} and O^{14} when the Coulomb energy differences are taken into account. Figure 6 shows that a value of a/K at least as large as 3 is needed to explain the first excited state of N as a charge triplet. Approximately correct relative values for the first five energy levels of N^{14} can be obtained for $a/K=5$ and $K=-0.6$ Mev. The curves are not very reliable in this region, however.

It should be noted that the relative spacings of the ^{13}D, ^{13}S, ^{31}D and ^{31}S levels depend only on the conditions,

$$W+M+B+H=1, \quad W+M-B-H=0.6.$$

Any additional conditions on these constants can only change the slopes of the curves in the jj limit. For a different reason the relative positions of the lower two multiplets in Figs. 2 and 5 and the lower three multiplets in Figs. 3 and 4 are independent of the constants W, M, B, and H. This is because the ordinary and space exchange energies are the same, and the spin and charge exchange energies are zero for these multiplets.

The lower levels shown in Fig. 2 should correspond to the energy levels of Li^7 and Be^7. Actually, the total angular momenta of the ground and first excited states of Li^7 are in agreement with the splitting of a ^{22}P level. However, the relatively large separation of the second and third excited states is not explainable by the curves in Fig. 2. In addition to this difficulty, the third excited state appears to have $J=\frac{3}{2}$.[6] It is very probable that this nucleus is better described by the Inglis model.[7] The low-lying levels of Be^8 which should correspond to the curves in Fig. 3 are also probably better described by this model.

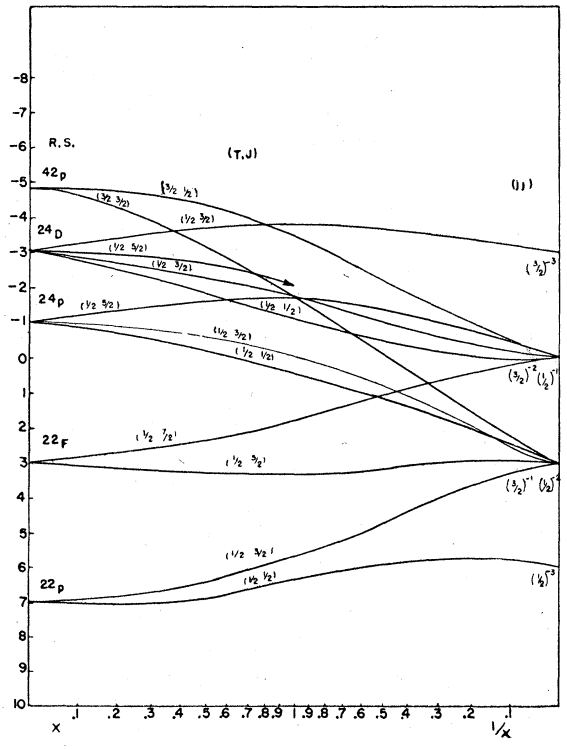

FIG. 5. $(\bar{V}-66.42K)/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for 9P-shell nucleons. $\chi=a/2K$.

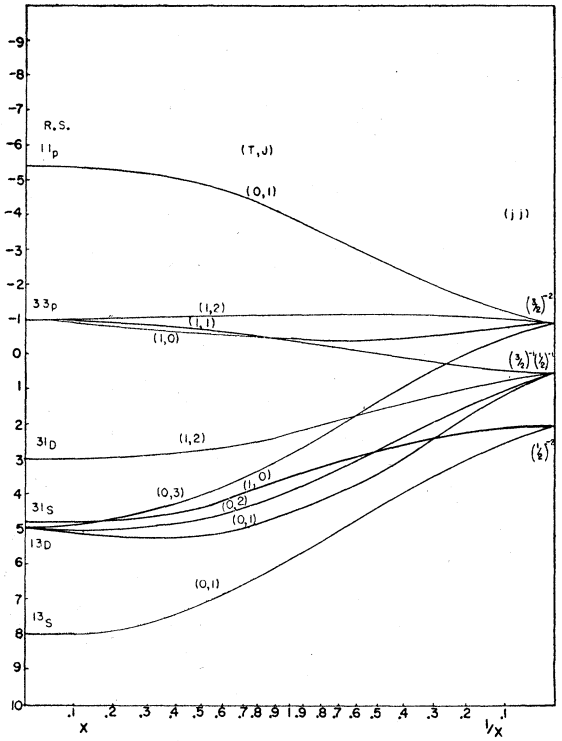

FIG. 6. $(\bar{V}-76.42K)/K(1+\chi^2)^{\frac{1}{2}}$ *vs* $\chi/(1+\chi)$ for 10P-shell nucleons. $\chi=a/K$.

No very definite conclusions can be obtained from Fig. 4. If one assumes K to be of the order of -1 Mev, then an a/K of the order of 3 gives the correct energy (4.5 Mev) for the first excited state of C^{12}. This places the ^{11}G level at approximately 11 Mev and causes all levels between 4.5 Mev and this value to have odd parity.

The two lower levels of C^{13} can be explained by Fig. 5 for $a/K=2.2$, $K=-1$ Mev. The splitting of the ^{22}F level would then be approximately 2 Mev, corresponding to the energy difference between the 5.4- and 3.9-Mev levels. The 3.1-Mev level of C^{13} has even parity and hence is not given by our calculations.

The author wishes to thank Dr. Philip Stehle for helpful discussions and criticism of this work. The kind interest shown by Dr. A. J. Allen is also much appreciated.

[6] M. Peshkin and A. J. F. Siegert, Phys. Rev. **87**, 735 (1952).
[7] D. Inglis, Phys. Rev. **85**, 492 (1952).