The function F is given by

$$F(s,q)=1-\beta_0{}^2E_e/E_m+\tfrac{1}{2}[(U_0+U_F)/2U_0]^2 [(q^2+s^2-p_e{}^2)/q^2](E_e/m_e)+\delta,$$

$$\delta=(4m_e{}^2+p_e{}^2-q^2)E_e/8m_eU_0{}^2 \text{ for spin } 0, \quad (3)$$

$$\delta=\{(4m_eE_e-s^2)E_e+(q^2-p_e{}^2)(E_e-2m_e)\}/8m_eU^2 \text{ for spin } \tfrac{1}{2}.$$

The term δ, which depends on the spin of the incident particle, is negligible unless the energy of the electron is comparable to the energy of the incident particle. The integration of Eq. (2) leads to very unwiedly expressions and is best performed numerically. As a check one can easily verify that in the limit $B=0$, f goes over into the well-known expressions of Bhabha.[15]

[15] H. J. Bhabha, Proc. Roy. Soc. (London) A164, 247 (1938).

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The Production of Heavy Mesons

L. S. KOTHARI*
Cavendish Laboratory, Cambridge, England
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Using Fermi's theory of multiple production, the production of heavy mesons in a nucleon-nucleon collision (with primary energy $<20Mc^2$) is studied. One obtains the result that heavy mesons should be produced singly about 10 times more frequently than in pairs. Creation of mesons in a meson-nucleon collision is also considered, and it is found that mesons of energy $E<Mc^2$ would, most likely, pass through nuclear matter without further creation of mesons, while for $E>2Mc^2$ the probability of multiplication is large. In a qualitative way, the results deduced seem to agree rather well with experiments.

I. INTRODUCTION

RECENTLY a number of different theories have been developed to explain the multiple production of mesons in nucleon-nucleon collisions,[1] for which quite convincing evidence now exists. Fermi[2] has proposed a theory based upon statistical considerations which seems to be in fair agreement with the experimental data so far obtained. We propose to consider here the production of heavy mesons of the basis of his theory.[3] Application of Fermi's theory to this problem implies the assumption that the heavy mesons—like the π-mesons—are "strongly" coupled to the nucleons. This assumption is fairly well born out by experiment.[4]

In the next section, the production of heavy mesons in a nucleon-nucleon collision is considered. Meson-nucleon collisions are studied in a later section. Though we have confined ourselves mainly to primary energies $E<20Mc^2$, in the last section extremely high energy collisions have also been briefly discussed.

II. NUCLEON-NUCLEON COLLISION

In high energy collisions between nucleons the production of mesons heavier than π-mesons is well established. Using Fermi's theory, we calculate the probability for the creation of a heavy meson in a collision process. The collision is studied in the center-of-mass system of the colliding particles, and it is assumed that W, the total energy in the center-of-mass system, is suddenly released in a small volume Ω,

$$\Omega=4\pi/3(\hbar/\mu c)^3(2Mc^2/W), \quad (1)$$

surrounding the nucleons. (M is the mass of the nucleon and μ is the mass of the π-meson.) One then calculates statistically the probability that a certain number of mesons are present in the final state.

With a suitable modification of Fermi's result [Eqs. (20) and (21) of reference 2], we find that the probability for r nucleons, s heavy mesons (of mass κ), and n π-mesons being present in the final state of the collision is given by

$$P(r,s,n)=\frac{2r^{\frac{3}{2}}}{(r+s)^{\frac{3}{2}}}\frac{(29.3/\omega)^{s/2}}{\alpha!}\left\{\frac{6.31}{\omega^{\frac{1}{3}}}(\omega-r-\tfrac{2}{3}s)\right\}^{\alpha} \times\exp\left\{-\frac{6.31}{\omega^{\frac{1}{3}}}(\omega-r)\right\}, \quad (2)$$

where

$$\alpha=\{3n+\tfrac{3}{2}(r+s)-\tfrac{5}{2}\},$$

$\omega=W/Mc^2$, and we have taken $\kappa/M=\frac{2}{3}$, $\mu/M=0.15$. In deriving the above result the nucleons and the heavy mesons have been treated in the nonrelativistic approximation, whereas the π-mesons have been treated in the extreme relativistic approximation. Further, the conservation of angular momentum has not been con-

* On Government of India scholarship.
[1] See Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. 73, 127 (1948); W. Heisenberg, Z. Physik 126, 509 (1949); K. A. Brueckner, Phys. Rev. 82, 590 (1951); R. Glauber, Phys. Rev. 84, 395 (1951); see also H. W. Lewis, Revs. Modern Phys. 24, 241 (1953).
[2] E. Fermi, Progr. Theoret. Phys. (Japan) 5, 570 (1950).
[3] L. S. Kothari, Jr., Nature 171, 309 (1953).
[4] A. Pais, Phys. Rev. 86, 663 (1952).

sidered, and the spins of the particles have been neglected.

In a nucleon-nucleon collision $r=2$ (neglecting nucleon pair production). Taking this value for r, we have from Eq. (2)

$$P(2, s, n)=\frac{8.5}{(2+s)^{\frac{3}{2}}}\frac{(29.3/\omega)^{s/2}}{[(6n+3s+1)/2]!}\times\left\{\frac{6.31}{\omega^{\frac{1}{3}}}(\omega-2-\tfrac{2}{3}s)\right\}^{(6n+3s+1)/2}\times\exp\left\{-\frac{6.31}{\omega^{\frac{1}{3}}}(\omega-2)\right\}. \quad (3)$$

The result calculated from this formula is given in Table I.

It will be noticed that in collisions involving primary energies up to about $20Mc^2$, the possibility of a heavy meson being observed is fairly small, though very much within the experimental reach. In nucleon-nucleus collisions the probability of observing a heavy meson will be still less, because of the fact that the heavy meson in colliding with other nucleons inside the nucleus will, most likely, change over into a π-meson. This point is discussed further in Sec. IV.

The slightly higher ratio of heavy mesons to π-mesons near the threshold energy for the creation of a heavy meson will also be observed. This arises because, according to this theory, the probability of creation of a heavy meson varies more slowly with increasing energy than does the average number of π-mesons created.

The heavy mesons will mostly be created singly, though in some collisions one may expect to get two or more of them. For primary energies in the neighborhood of $15Mc^2$, the ratio of the number of collisions in which a single heavy meson is produced to those in which a pair of heavy mesons appear would be nearly 10.

III. MESON-NUCLEON COLLISIONS

Another related problem, which is of importance, is the production of mesons in a meson-nucleon collision.

TABLE I. The relative probability for the production of heavy mesons in a nucleon-nucleon collision. ω' is the energy of the primary particle in the laboratory system divided by Mc^2. $\bar{n}_i$ ($i=0, 1, 2\cdots$) is the average number of π mesons produced when i heavy mesons are also created in the collision. $\Sigma_n P(r, n)$, ($r=0, 1, 2$) is the probability for the production of r heavy mesons. $\Sigma_n P(0, n)$ has been taken to be 100. σ is the ratio of heavy mesons to π-mesons produced in the collision. The other symbols are defined in the text.

ω	ω'	$\bar{n}_0$	$\Sigma_n P(1, n)$	$\bar{n}_1$	$\Sigma_n P(2, n)$	$\bar{n}_2$	σ
3	3.5	1.2	7				~1/17
4	7.0	2.5	9	1.0	0.5	0	1/26
5	11.5	3.5	11	2.2	1.2	0.9	1/28
6	16.9	4.4	12	2.3	1.4	1.8	1/31

Apart from the fact that this type of process has been observed in photographic emulsions,[5] it is of importance in considering nucleon-nucleus collisions. If the nucleus considered is heavy, the π-mesons produced inside the nucleus in a single event would multiply still further when they interact with other nucleons present.

The initial meson responsible for the collision may be either a π-meson or a heavy meson. We assume, however, that in the final state no heavy meson is emitted. (If in the final state a heavy meson is also present then, since we are neglecting the spins of the particles, the result for a given ω would not be much different from that of a nucleon-nucleon encounter. Some difference would arise, however, when we transform ω back into the energy of the primary particle in the laboratory frame.) This assumption implies that, for the same total energy in the center-of-mass system, the collision process is independent of the primary meson. However, when we calculate the energy of the incoming meson in the laboratory frame from a given total energy in the center-of-mass system, the two types of mesons would give different results.

The probability, that in the final state of collision, we have 1 nucleon and $(1+n)$ π-mesons is given by

$$P(1, 1+n)=\frac{3}{[(6n+7)/2]!}\left\{6.31\frac{\omega-1}{\omega^{\frac{1}{3}}}\right\}^{(6n+7)/2}\times\exp\left(-6.31\frac{\omega-1}{\omega^{\frac{1}{3}}}\right), \quad (4)$$

where n represents the number of π-mesons created in the collision. The nucleon has again been considered in the nonrelativistic approximation and the π-mesons in the extreme relativistic approximation. Table II gives, for various total energies ω, the relative probabilities for different number of mesons being produced, as calculated from Eq. (4).

On comparing these results with those of Fermi, one finds that for a given ω, the average number of mesons produced is more in the meson-nucleon collision than in the nucleon-nucleon collision. Thus, at these comparatively low energies, the multiplicity of mesons produced in a collision depends upon the nature of the primary particle. One also observes that incident mesons with energy $E<Mc^2$ will, on the whole, not produce any further mesons in a collision with a nucleon. However, the probability for mesons of energy $E>2Mc^2$ to lead to the creation of further mesons is fairly high. This seems to agree rather well with experiment.[5]

IV. DISCUSSION

So far, we have considered mainly nucleon-nucleon or meson-nucleon collisions. If an incident nucleon collides with a nucleus, then besides multiple production, plural

[5] J. G. Wilson, *Progress in Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1952), Chap. I.

production of mesons would also play a great role. In a "light" nucleus the probability of a daughter meson colliding with another nucleon inside the nucleus is comparatively small, and one would not expect much deviation from the results predicted by the multiple theory. In "heavy" nuclei, the case is different. If one assumes that in a single multiple collision the total energy is equally shared between the various particles in the center-of-mass system, then the heavy mesons have a rather small chance of coming out of the nucleus. In a subsequent collision, the probability of their transforming themselves into π-mesons is fairly large. Thus, for primary energies in the neighborhood of $15Mc^2$ it seems that one should expect a larger number of heavy mesons to come from collisions of a nucleon with the lighter nuclei.

The recoil nucleons may also produce further mesons in subsequent collisions, but for the energy range that we have considered, these will not be very significant.

Though we have so far confined ourselves to the energy region in which nucleons can be treated in the nonrelativistic approximation, it is of interest to note a few points about the high energy region, where the heavy mesons and the nucleons also have to be treated in the extreme relativistic approximation. It follows from thermodynamic considerations[2] that the ratio of the number of heavy mesons to the number of π-mesons produced in any collision is governed only by the ratio of their respective statistical weight factors. If the heavy meson has spin 1, then this ratio is $9/3=3$. For a scalar heavy meson this ratio would be $3/3=1$. The possibility of determining the spin of the heavy particle by studying the collisions appears, however, to be very small. The collisions usually observed are of the nucleon-nucleus type and are far more complicated than the simple nucleon-nucleon collisions which we have considered.

A further point to observe is that in these energy regions the final result of the collision does not depend upon the nature of the primary particle. Thus the heavy mesons could very well be produced in a collision between a high energy π-meson and a nucleon. There is some experimental evidence to show that such processes actually do occur.[6]

TABLE II. The relative probabilities for different number of mesons being produced in a meson-nucleon collision. According as the incident particle is a π-meson or a heavy meson, ω' or ω'' give the energy of the primary particle in the laboratory frame of reference, divided by Mc^2. $\bar{n}$ represents the average number of mesons created.

ω	ω'	ω''	$n=0$	1	2	3	4	5	6	$\bar{n}$
1.25	0.3		98	2						0
1.5	0.6		89	11	0					0.1
1.75	1.0	0.8	70	27	3					0.3
2.0	1.5	1.3	50	40	9	1				0.6
2.5	2.6	2.4	20	48	26	6	0			1.1
3.0	4.0	3.8	8	34	38	16	4	0		1.7
4.0	7.5	7.3	1	10	30	33	19	6	1	2.8

[6] Leighton, Wanlass, and Anderson, Phys. Rev. **89**, 148 (1953).