where $\sigma$ is the photoneutron cross section in barns; $\rho$ is the density of beryllium; $q$ is the strength of the 2.185-Mev line; $R$ is the outer diameter of the beryllium sphere; $R'$ is the diameter of the equivalent spherical cavity in the beryllium; and M.W. is the molecular weight of beryllium.

Substituting the experimental data into the above formula, we found $\sigma=4.1\times10^{-28}$ cm$^2$.

This result requires one important correction, since some photoneutrons arise from the bremsstrahlung from the 2.96-Mev $\beta$-rays. A calculation was made to determine the relative importance of this yield of neutrons compared to the yield of neutrons from the 2.185-Mev $\gamma$-ray. This depends on the relative intensities of the 0.86-Mev $\beta$-rays to the 2.96-Mev $\beta$-rays. Assuming that the branching ratios of the 0.86 and the 2.3-Mev $\beta$-branches are equal and neglecting the bremsstrahlung photoneutron yield from the 2.3-Mev $\beta$-rays compared to the 2.96-Mev $\beta$-rays, Eq. (1) is corrected to

$$\frac{\text{neutrons}}{\text{second}}=\rho q\frac{0.6}{\text{M.W.}}(R-R')\times\left[\sigma+22\times10^{-8}\times\frac{1.5}{1.1}\times\frac{1-2B}{B}\right], \quad (2)$$

TABLE I. Change in $\sigma$ caused by correction for bremsstrahlung yield of photoneutrons.

| $B$ (percent) | 0.6 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $\Delta\sigma(10^{-28}$ cm$^2)$ | −0.5 | −0.29 | −0.14 | −0.09 | −0.07 | −0.05 |

where $\sigma$ is in barns and $B$ is the above-mentioned branching ratio. The correction decreases the value of $\sigma$. Table I gives the calculated change in cross section as a function of $B$.

Table I shows that if we use Alburger's decay scheme, there is a negligible correction to the cross section. On the other hand, if we use Porter and Cook's value for $B$ of about one percent, then the cross section would be $3.8\times10^{-28}$ cm$^2$.

Comparing these values with Fig. 1, it can be seen that there is a minimum value of the cross section in the region of 2.185 Mev. Furthermore, the experimental value of about $3.9\times10^{-28}$ cm$^2\pm15$ percent is in reasonably good agreement with the predictions of the valence neutron model.

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# Fourth-Order Corrections to the Scattering of Pions by Nonrelativistic Nucleons*

J. S. BLAIR† AND G. F. CHEW
*Department of Physics, University of Illinois, Urbana, Illinois*

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Fourth-order corrections to the field theoretical phase shifts for pion nucleon scattering have been calculated for a linear and extended coupling and no nucleon recoil. The results support the use of the "Dancoff approximation," since those processes passing through intermediate states which contain three pions turn out to be much less important than processes involving only one and two pions.

## I. INTRODUCTION

A BROAD program of theoretical calculations is being carried out at the University of Illinois to see to what extent the experimental information on the pion-nucleon interaction at energies less than 1 Bev can be correlated on the basis of a Yukawa theory in which nucleon recoil is relatively unimportant. The qualitative arguments for the supposition that at these low energies the nucleon may be approximately regarded as a fixed "source" have been summarized by Blair and Chew.[1] These same arguments also lead to the conclusion that the pion-nucleon interaction is linear in the pion field and not very strong and that, therefore, an essentially weak-coupling calculational technique is in order. The purpose of this paper then is to present the results of a calculation, up to the fourth order in the coupling constant, of the pion-nucleon scattering phase shifts, when the nucleon is regarded as infinitely heavy. Our conclusions here form the basis for the more satisfactory method of calculation reported by Chew.[2] The notation of reference 2 will be used throughout.

## II. PROCEDURE

With the neglect of nucleon recoil, pions interact with nucleons only in $P$ states.[1] With charge inde-

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† Now at the Department of Physics, University of Washington, Seattle, Washington.

[1] J. S. Blair and G. F. Chew, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1952), Vol. II, p. 163 (1952).

[2] G. F. Chew, Phys. Rev. **89**, 591 (1953).

pendence, the cross sections for scattering can then be expressed in terms of four phase shifts, denoted $\alpha_{33}$, $\alpha_{31}$, $\alpha_{13}$, and $\alpha_{11}$, where the first index corresponds to isotopic spin $\frac{3}{2}$ or $\frac{1}{2}$, and the second index refers to $P_{\frac{3}{2}}$ or $P_{\frac{1}{2}}$ states, respectively.

It is convenient to evaluate the reaction (or reactance) matrix element,[3] $K_{f0}$, between plane wave states, and then relate $K_{f0}$ to the phase shifts. The reaction matrix may be separated into spin-flip and nonspin-flip contributions, so that, for example, the reaction matrix for the scattering of positive pions by protons is written

$$K_{f0}(\pi^+p)=B(\pi^+p)\cos\theta+A(\pi^+p)i\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta, \quad (1)$$

where $\theta$ is the angle between $\mathbf{k}_0$ and $\mathbf{k}_f$, the incident and final pion momenta, and $\mathbf{n}$ is the unit vector in the direction $\mathbf{k}_f\times\mathbf{k}_0$. Similarly, we will write the reaction matrix for the elastic scattering of negative pions by protons as

$$K_{f0}(\pi^-p)_{el}=B(\pi^-p)_{el}\cos\theta+A(\pi^-p)_{el}i\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta.$$

It is then easily verified that

$$K_{33}=\tfrac{1}{3}[B(\pi^+p)-A(\pi^+p)], \quad (2a)$$

$$K_{31}=\tfrac{1}{3}[B(\pi^+p)+2A(\pi^+p)], \quad (2b)$$

$$K_{13}=\tfrac{1}{2}[B(\pi^-p)_{el}-A(\pi^-p)_{el}]-\tfrac{1}{2}K_{33}, \quad (2c)$$

$$K_{11}=\tfrac{1}{2}[B(\pi^-p)_{el}+2A(\pi^-p)_{el}]-\tfrac{1}{2}K_{31}. \quad (2d)$$

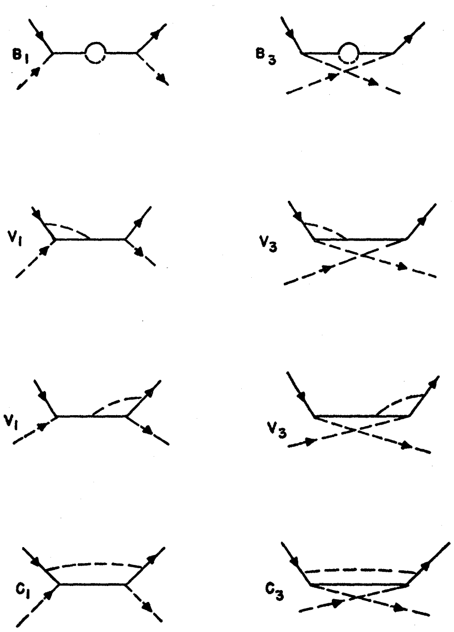


FIG. 1. Feynman diagrams for the fourth-order matrix elements in pion-nucleon scattering. The nucleon lines are solid and the pion lines dashed.

[3] Following B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950), we mean by the reaction operator $K$ a matrix which obeys equations identical in form to those obeyed by the transition matrix $T$, except that principal values are to be taken whenever poles are encountered.

The phase shifts are to be obtained from Eqs. (2) by the relation, $\tan\alpha_{ij}=-(k_0\omega_0/2\pi)K_{ij}$.

The fourth-order perturbation theory contribution to the reaction matrix is given by[4]

$$K_{f0}^{(4)}=\sum_{l,m,n\neq 0,f}\frac{H_{fn}H_{nm}H_{ml}H_{l0}}{(E_0-E_n)(E_0-E_m)(E_0-E_l)} -\tfrac{1}{2}\sum_{l,m}\frac{H_{fm}H_{m0}}{(E_0-E_m)}\left\{\frac{H_{0l}H_{l0}}{(E_0-E_l)^2}+\frac{H_{fl}H_{lf}}{(E_f-E_l)^2}\right\} -\tfrac{1}{2}\sum_{l,m}\frac{H_{fm}H_{m0}}{(E_0-E_m)^2}\left\{\frac{H_{0l}H_{l0}}{E_0-E_l}+\frac{H_{fl}H_{lf}}{E_f-E_l}\right\}, \quad (3)$$

where one is to take principal parts of any integrals over intermediate momenta. It is apparent that the second bracket is a part of the nucleon self-energy; this term compensates for a self-energy buried in the expression on the first line. The first bracket is part of the factor renormalizing $f^2$:

The interaction Hamiltonian density is taken to be[5]

$$H(\mathbf{x})=(4\pi)^{\frac{1}{2}}(f/\mu)\sum_{i=1}^{3}\rho(x)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\tau_i\varphi_i(\mathbf{x}), \quad (4)$$

where the symbols are conventional. The Fourier transform of the source density $\rho(x)$ will be denoted by $v(k)$, which is normalized to unity at $k=0$. There are eight possible diagrams describing the ordering of transitions and these are sketched in Fig. 1.[6] The evaluation of the matrix elements is straightforward.

## III. RESULTS

The resulting phase shifts through fourth order may be written as

$$\tan\alpha_{33}=2y(1-\delta+2\Delta_-+4\Delta_+), \quad (5a)$$

$$\tan\alpha_{31}=\tan\alpha_{13}=-y(1-\delta-\Delta_-+\Delta_+), \quad (5b, c)$$

$$\tan\alpha_{11}=-4y(1-\delta-4\Delta_--2\Delta_+), \quad (5d)$$

where

$$y=(2/3)(f/\mu)^2(k_0^3/\omega_0)v^2(k_0), \quad (6)$$

$$\delta=(16/3\pi)(f/\mu)^2\int^{\infty}dk(k^4/\omega^3)v^2(k), \quad (7)$$

$$\Delta_\pm=(2/3\pi)(f/\mu)^2\int^{\infty}dk(k^4/\omega^2)[\omega_0/(\omega\pm\omega_0)]v^2(k). \quad (8)$$

Even though all contributions are finite, it is desirable to separate out those terms which merely renormalize

[4] See, for example, E. Corinaldesi and G. Field, Phil. Mag. **40**, 1159 (1949).

[5] W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946), p. 12.

[6] Our notation here follows Ashkin, Simon, and Marshak, Progr. Theoret. Phys. **5**, 684 (1950), who have considered fourth-order scattering in the relativistic theory.

the second-order calculation. Thus, the recurring quantity $(1-\delta)$ should be interpreted as a factor renormalizing $f^2$. It may be noted that $(9/16)\delta$ is simply the weak coupling probability for a one pion configuration in the self-field of a single physical nucleon.

The above results can be compared with those resulting from the "Dancoff approximation:"[2]

$$\tan\alpha_{33}=2y(1-2\Delta_-)^{-1}, \tag{9a}$$

$$\tan\alpha_{31}=\tan\alpha_{13}=-y(1+\Delta_-)^{-1}, \tag{9b, c}$$

$$\tan\alpha_{11}=-4y(1+4\Delta_-)^{-1}. \tag{9d}$$

It is readily seen that the expansion of Eqs. (9) to fourth order in $f^2$ is equivalent to Eqs. (5) except for the contributions of $\Delta_+$. The terms in $\Delta_-$ arise only from diagrams in which the number of pions in the second intermediate state is one (i.e., the diagrams of Fig. 1 with subscript 1); those in $\Delta_+$ appear when this number is three (i.e., diagrams with subscript 3). A measure of the validity of the "Dancoff approximation" then is the relative magnitude of $\Delta_-$ and $\Delta_+$. It is qualitatively expected that $\Delta_-$ should outweigh $\Delta_+$ because of the possible smallness of the denominator in $\Delta_-$.

The term $\Delta_-/f^2$ is shown as the solid curve in Fig. 2, as a function of $\omega_0/\mu$ for the case of a "square" momentum distribution $v(k)$, with several choices of $\omega_{\max}$.[2] $\Delta_+/f^2$ is similarly plotted in Fig. 2 and in the range shown is seen to be substantially smaller than $\Delta_-/f^2$. On the other hand, if the cut-off energy is made very large compared to $\omega_0$ (i.e., the source radius is made very small compared to the pion wavelength) it is seen by inspection of formula (8) that $\Delta_+\rightarrow\Delta_-$ and the Dancoff approximation loses its validity. Fortunately, it appears that the experimental data, up to now at least, can be fitted with a relatively low cut-off. Using the values of $f^2=0.2$ and $\omega_{\max}=3.2\mu$, deduced in reference 2, it is seen that the $\Delta_+$ contributions to formulas (5) are generally less than 20 percent in the low energy region, while at the same time the con-

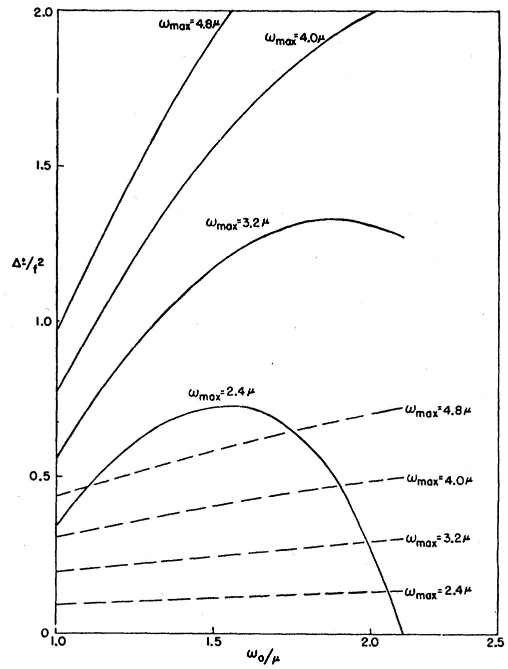


FIG. 2. The quantities, $\Delta_\pm$, given by Eq. (8), as functions of $\omega_0$ for various choices of $\omega_{\max}$. The solid curves represent $\Delta_-/f^2$ and the dashed curves $\Delta_+/f^2$.

tributions from $\Delta_-$ are large. (It is perhaps worth noting that in the important phase shifts, $\alpha_{33}$ and $\alpha_{11}$, the effect of $\Delta_+$ reinforces that of $\Delta_-$; so even if the cutoff were too large to allow the neglect of $\Delta_+$, the fourth-order contributions would still be in the right direction to fit experiment.)

In conclusion, we believe that our results here support the validity of the Dancoff approximation, *provided that intermediate virtual pions at very high energy do not play a dominant role in the scattering.* This is another way of saying that as a source of pions the nucleon must effectively be extended over a region appreciably larger than its own Compton wavelength.