Production of Polarized Particles in Nuclear Reactions*

A. SIMON AND T. A. WELTON
Oak Ridge National Laboratory, Oak Ridge, Tennessee
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A general calculation of the polarization resulting from nuclear reactions is made by means of the S matrix and Racah formalisms. All sums over magnetic quantum numbers are performed; and the resulting polarization is expressed as a series in associated Legendre polynomials, each coefficient being manifestly real. All selection rules follow immediately from the requirements for the nonvanishing of the Racah and X coefficients. The only restriction required is to a two-body break-up.

Higher spin tensor moments are required for the complete specification of the state of polarization of a beam of particles of spin greater than $\frac{1}{2}$. A general expression is given for arbitrary spin moments resulting from a nuclear reaction. The result is expressed as a series in the spherical harmonics with all coefficients being manifestly real. The previous results of Blatt and Biedenharn for the angular distribution follow as a special case of the result. A generalization of the Eisner-Sachs rules for the complexity of angular distributions is also given.

I. INTRODUCTION

SCHWINGER[1] has shown that polarized neutrons may be obtained by the elastic scattering of neutrons from He^4. This is a special case of a more general theorem, first stated by Blin-Stoyle,[2] to the effect that, under suitable conditions, the products of any nuclear reaction will be polarized. As a result it should be possible to use resonant charged particle reactions to obtain directly (rather than by an intervening elastic scattering) high intensity beams of polarized neutrons, with energies variable over a considerable range.

In Sec. II of this paper, we write down a *general* expression for the polarization of particles emerging from a reaction (the term reaction includes the special case of elastic scattering). The expression includes the elements of the nuclear scattering matrix as well as a sum over magnetic quantum numbers, since the incident beam and target nucleus are taken to be unpolarized. The sums over the magnetic quantum numbers are essentially geometrical in nature, since the elements of the nuclear scattering matrix are independent of all magnetic quantum numbers. These sums are eliminated in Sec. III, and the final result is expressed completely in terms of the nuclear scattering matrix and the Racah coefficients.

From the properties of the Racah coefficients, it is possible to read off the selection rules governing the polarization. These are listed in Sec. IV.

In Sec. V two illustrative examples are given. In one case the nuclear parameters given by Peshkin and Siegert[3] for the $Li^6(n,\alpha)H^3$ reaction are used to estimate the polarization of the emitted tritons. In the second case the formula given by Lepore[4] for the polarization of neutrons resulting from elastic scattering from He^4 is rederived using the general result. A rough rule of thumb is also given in this section for the angles at which polarization is most likely to be observed.

If the particle produced in a nuclear reaction has a spin, complete information on its final state can be obtained by giving all its irreducible spin tensor moments. In essence the angular distribution is the expectation value in the scattered wave of the spin tensor of rank zero; namely, unity. The polarization, similarly, is simply the expectation value of the tensor of rank unity formed by the spin operator $\mathbf{i}'$. In general, however, an outgoing particle of spin i' will have nonzero irreducible tensor moments up to a maximum tensor rank given by $2i'$. For example, a deuteron produced in a reaction will have a Legendre second moment $[3(i_z')^2-i'(i'+1)]$ which will usually differ from zero. Lakin and Wolfenstein[5] have recently pointed out the possible importance of the deuteron second-rank tensor in analyzing reactions.

A reduction of the magnetic sums, similar to that for the polarization, should be possible for this generalized spin tensor expectation value. This reduction is performed in Sec. VI. The resulting general expression yields the previous formula for the polarization upon specializing to tensor rank unity. The expression of Blatt and Biedenharn[6] for the angular distribution of scattering and reaction cross sections is also obtained immediately by specializing to tensor rank zero.

A generalization of the rules for the complexity of angular distributions results from the properties of the Racah and X coefficients. These rules are given in Sec. VI.

It is interesting to note that even a single level of definite J ($\neq\frac{1}{2}$ or 0) and parity will produce polarization if more than one subchannel (l value or final

* This paper is based on work performed for the U. S. Atomic Energy Commission at the Oak Ridge National Laboratory.

[1] J. Schwinger, Phys. Rev. **69**, 681 (1946).

[2] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **64**, 700 (1951). An explicit formula for the polarization of spin $\frac{1}{2}$ particles is given in this paper. A previous Letter to the Editor of *The Physical Review* was submitted before the authors became aware of this earlier work.

[3] M. Peshkin and A. J. F. Siegert, Phys. Rev. **87**, 735 (1952).

[4] J. V. Lepore, Phys. Rev. **79**, 137 (1950).

[5] W. Lakin and L. Wolfenstein, Bull. Am. Phys. Soc. **28**, No. 1, 36 (1953).

[6] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. **24**, 258 (1952); references to this paper will be designated by BB.

channel spin) contribute. Several known reactions offer promise. In particular, analyses[7] of the angular distributions of (p,n) neutrons resulting from resonances in C^{13}, B^{11}, Li^7, and H^3 indicate that p waves or higher and opposite parities are interfering. It, in fact, seems likely that many neutron sources, hitherto considered unpolarized, may actually exhibit partial polarization. Such partial polarization could introduce systematic errors in the measurement of differential scattering cross sections for neutrons, if care is not taken.

The present paper is restricted to the case of an unpolarized initial beam and target. The general case of the spin tensor moments resulting from an arbitrarily polarized initial beam has been solved and will be reported on at a later time.

II. NOTATION AND GENERAL EXPRESSION FOR THE POLARIZATION

In the following sections the notation of BB will be closely adhered to. We consider the reaction

$$a+X=Y+b, \tag{2.1}$$

in which particle a collides with nucleus X. Particle b emerges at an angle θ to the direction of the incident beam, and Y is the residual nucleus. All quantities are measured in the center-of-gravity system. As in BB, *the formulas derived below are applicable to any collision process in which two particles collide and two particles emerge.*

The system before collision is described by the channel index α which defines the type of incoming particle (neutron, proton, etc.) as well as the state of the target nucleus, the channel spin s, and l the orbital angular momentum. The channel spin s is the total spin angular momentum in the entrance channel and is formed by the vector addition of the intrinsic spin i of the incoming particle and the spin I of the target nucleus. The state of the system after the reaction will be described by primed quantities.

The asymptotic form of the outgoing wave can be written [see BB (3.12)]

$$\psi(\alpha's')=i\lambda\!\!\!^{-}_{\alpha}\left(\frac{v_\alpha}{v_{\alpha'}}\right)^{\frac{1}{2}}\frac{\exp(ik_{\alpha'}r_{\alpha'})}{r_{\alpha'}}\phi_{\alpha'} \times\sum_{m_s'=-s'}^{s'} q(\alpha's'm_s';\alpha s m_s;\theta,\varphi)\chi(s'm_s'), \tag{2.2}$$

where v_α and k_α are the relative velocity and wave number, respectively, in entrance channel α; $\phi_{\alpha'}$ is the product of internal wave functions of nucleus Y and particle b corresponding to the specification α'; $\chi_{s'm_s'}$ is the final channel spin wave function; and[8]

$$\begin{aligned}q(\alpha's'm_s';\alpha s m_s;\theta,\varphi) &=(\pi)^{\frac{1}{2}}\sum_{J=0}^{\infty}\sum_{l=|J-s|}^{J+s}{}'\sum_{l'=|J-s'|}^{J+s'}{}'\sum_{\pi=\pm1} i^{l-l'}(2l+1)^{\frac{1}{2}}\\ &\quad\times(ls0m_s|lsJm_s)(l's'\mu'm_s'|l's'Jm_s)\\ &\quad\times[\delta(\alpha',\alpha)\delta(s',s)\delta(l',l)\\ &\qquad -S(\alpha's'l';\alpha sl;J\pi)]Y_{l',\mu'}(\theta,\varphi).\end{aligned} \tag{2.3}$$

The quantity $(ls0m_s|lsJm_s)$ is the Clebsch-Gordan coefficient defined as in Condon and Shortley.[9] With this choice of phase, we have $Y_{l,-m}=(-1)^m Y_{l,m}^*$. Here $S(\alpha's'l';\alpha sl;J\pi)$ is an element of the scattering matrix representing the probability of a colliding system having a total angular momentum J and a parity π to go from an incident state described by the quantities αsl to a final state described by $\alpha's'l'$. The good quantum numbers for this reaction are Jm_J and π. The prime on the summation symbol for the orbital angular momenta indicates that only those values of l and l' are to be chosen which will satisfy the parity condition. For pure elastic scattering S is related to the phase shift δ by the relation $S=\exp(2i\delta)$. Note that Eq. (2.2) is written for a definite incident channel spin s and channel spin direction m_s. The differential cross section for the process $\alpha\rightarrow\alpha'$ can be written

$$d\sigma_{\alpha'\alpha}=\sum_{s=|I-i|}^{I+i}\sum_{s'=|I'-i'|}^{I'+i'}\sum_{m_s=-s}^{s}\sum_{m_s'=-s'}^{s'}\frac{\lambda\!\!\!^{-}_{\alpha}{}^2}{(2I+1)(2i+1)} \times|q(\alpha's'm_s';\alpha s m_s;\theta,\varphi)|^2 d\Omega. \tag{2.4}$$

Since the scattering matrix is independent of magnetic quantum numbers, the sums over all magnetic quantum numbers are essentially geometrical in character and can be performed without any detailed knowledge of the collision process. The elegant reduction of these sums has been performed in BB. A similar reduction will be performed for the case of the polarization (as well as any higher spin moments of interest).

We define the *differential polarization* in the outgoing channel α' to be

$$d\mathbf{P}_{\alpha'\alpha}=\langle\sum(\psi(\alpha's')|\mathbf{i}'|\psi(\alpha's''))r_{\alpha'}^2 v_\alpha^{-1}v_{\alpha'}\rangle_{\mathrm{Av}}d\Omega, \tag{2.5}$$

where the sum is over all final channel spins and final channel spin directions and an average is taken over the initial states. Here $\mathbf{i}'$ is the spin operator for the outgoing particle b whose polarization is to be measured. The *polarization* will be defined to be

$$f(\theta)=(i')^{-1}d|\mathbf{P}|/d\Omega,$$

[7] H. B. Willard, private communication.

[8] Note that the separation of the final wave function into a channel spin function and an internal function is an idealization. Since, however, only angular properties of the wave functions are of interest, the final answer will be unaffected.

[9] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951).

and such that f has a maximum value of unity for a completely polarized beam.

It is more convenient in the calculations to evaluate first the expectation values of those linear combinations of the components of the spin operator $\mathbf{i}'$ which transform as spherical harmonics. In particular, for the polarization, we will evaluate the quantities $T_\kappa^{(1)}$ where

$$T_{\pm 1}^{(1)}=\mp(i_x'\pm i i_y')/[2i'(i'+1)]^{\frac{1}{2}},\qquad T_0^{(1)}=i_z'/[i'(i'+1)]^{\frac{1}{2}}. \tag{2.6}$$

These definitions are chosen so as to agree with the definition of a tensor given by Racah.[10]

Using Eqs. (2.6), (2.5), (2.3), and (2.2), we can now write the polarization intensity as

$$\begin{aligned}dT_{\kappa,\alpha'\alpha}^{(1)}=&\pi\lambda_\alpha^2(2I+1)^{-1}(2i+1)^{-1}\sum i^{l_2-l_1+l_1'-l_2'}\\&\times[(2l_1+1)(2l_2+1)]^{\frac{1}{2}}[\delta(\alpha',\alpha)\delta(s_1' s)\delta(l_1',l_1)\\&-S(\alpha's_1'l_1';\alpha sl_1;J_1\pi_1)]^*[\delta(\alpha',\alpha)\delta(s_2',s)\delta(l_2',l_2)\\&-S(\alpha's_2'l_2';\alpha sl_2;J_2\pi_2)]d\Omega\\&\times\sum_{m_s}\sum_{m_1'}\sum_{m_2'}[(l_1s0m_s|l_1sJ_1m_s)(l_2s0m_s|l_2sJ_2m_s)\\&\times(l_1's_1'\mu_1'm_1'|l_1's_1'J_1m_s)(l_2's_2'\mu_2'm_2'|l_2's_2'J_2m_s)\\&\times Y^*_{l_1'\mu_1'}Y_{l_2'\mu_2'}(\chi(s_1'm_1')|T_\kappa^{(1)}|\chi(s_2'm_2'))],\end{aligned}\tag{2.7}$$

where the first sum is over the quantities $J_1J_2\pi_1\pi_2l_1l_2l_1'l_2's_1's_2'$ and s. For simplicity we have written $m_{s1}'=m_1'$; $m_{s2}'=m_2'$.

The expression in the square brackets may be expressed entirely in terms of Clebsch-Gordan coefficients by the use of two theorems. One relates the product of two spherical harmonics to a linear superposition of spherical harmonics.

$$\begin{aligned}Y^*_{l_1\mu_1}Y_{l_2\mu_2}=&(-1)^{\mu_1}\sum_{L=|l_1-l_2|}^{l_1+l_2}\left[\frac{(2l_1+1)(2l_2+1)}{4\pi(2L+1)}\right]^{\frac{1}{2}}\\&\times(l_1l_200|l_1l_2L0)(l_1l_2-\mu_1\mu_2|l_1l_2L\mu_2-\mu_1)Y_{L,\mu_2-\mu_1}.\end{aligned}\tag{2.8}$$

The other expresses the matrix element of $T_\kappa^{(1)}$ in terms of the Clebsch-Gordan and Racah coefficients. By the use of the Eckart-Wigner theorem and the Racah formalism we obtain the following result, which is derived in Appendix A:

$$\begin{aligned}(\chi(s_1'm_1')|T_\kappa^{(1)}|\chi(s_2'm_2'))=&(-1)^{I'-i'+m_1'-\kappa}[(2i'+1)(2s_1'+1)(2s_2'+1)/(3)]^{\frac{1}{2}}\\&\times(s_1's_2'-m_1'm_2'|s_1's_2'1-\kappa)W(i's_1'i's_2';I'1),\end{aligned}\tag{2.9}$$

where $W(abcd;ef)$ is the usual Racah coefficient.

Combining Eqs. (2.7), (2.8), and (2.9) we obtain

$$\begin{aligned}dT_{\kappa,\alpha'\alpha}^{(1)}=&[\lambda_\alpha^2/(2I+1)(2i+1)][\pi(2i'+1)/12]^{\frac{1}{2}}\\&\times\sum(-1)^{I'-i'-\kappa}i^{l_2-l_1-l_2'+l_1'}[\delta(\alpha',\alpha)\delta(s_1',s)\delta(l_1',l_1)\\&-S(\alpha's_1'l_1';\alpha sl_1;J_1\pi_1)]^*[\delta(\alpha'\alpha)\delta(s_2's)\delta(l_2'l_2)\\&-S(\alpha's_2'l_2';\alpha sl_2;J_2\pi_2)][(2l_1+1)(2l_2+1)\\&\times(2l_1'+1)(2l_2'+1)(2s_1'+1)(2s_2'+1)]^{\frac{1}{2}}(2L+1)^{-\frac{1}{2}}\\&\times(l_1'l_2'00|l_1'l_2'L0)W(i's_1'i's_2';I'1)Y_{L,\kappa}(\theta,\varphi)d\Omega\\&\times\sum_{m_s}\sum_{m_1'}\sum_{m_2'}[(-1)^{m_s}(l_1s0m_s|l_1sJ_1m_s)\\&\times(l_2s0m_s|l_2sJ_2m_s)(l_1's_1'\mu_1'm_1'|l_1's_1'J_1m_s)\\&\times(l_2's_2'\mu_2'm_2'|l_2's_2'J_2m_s)(l_1'l_2'-\mu_1'\mu_2'|l_1'l_2'L\kappa)\\&\times(s_1's_2'-m_1'm_2'|s_1's_2'1-\kappa)],\end{aligned}\tag{2.10}$$

where the sum now includes the index L. Note that by the properties of the Clebsch-Gordan coefficients, $\mu_1'=m_s-m_1'$, $\mu_2'=m_s-m_2'$ and $m_1'-m_2'=\kappa$. Hence the sum m_2' is purely formal and actually reduces to only one term: $m_2'=m_1'-\kappa$.

The remaining geometrical sums over the magnetic quantum numbers can be eliminated by the use of some Racah identities which have been recently summarized in a review paper by Biedenharn *et al.*[11]

III. ELIMINATION OF THE MAGNETIC SUMS

By use of BBR (1) and (18) we find that

$$\begin{aligned}(l_1s0m_s|l_1sJ_1m_s)(l_2s0m_s|l_2sJ_2m_s)=&(-1)^{s+2J_1-m_s}[(2J_1+1)(2J_2+1)]^{\frac{1}{2}}\sum_f[(l_1l_200|l_1l_2f0)\\&\times(J_1J_2-m_sm_s|J_1J_2f0)W(l_1J_1l_2J_2;sf)].\end{aligned}$$

Using this reduction once again, there results

$$\begin{aligned}(J_1J_2m_s-m_s|J_1J_2f0)(l_1's_1'\mu_1'm_1'|l_1's_1'J_1m_s)=&(-1)^{J_1+f+s_1'-m_s+m_1'}[(2f+1)(2J_1+1)/(2l_1'+1)]^{\frac{1}{2}}\\&\times\sum_g[(2g+1)^{\frac{1}{2}}(fs_1'0m_1'|fs_1'gm_1')\\&\times(J_2g-m_sm_1'|J_2gl_1'-\mu_1')W(J_2fl_1's_1';J_1g)].\end{aligned}$$

The sum over m_s by BBR (1) can now be written as

$$\sum_{m_s}[(gJ_2-m_1'm_s|gJ_2l_1'\mu_1')(l_1'l_2'\mu_1'-\mu_2'|l_1'l_2'L-\kappa)\times(J_2l_2'm_s-\mu_2'|J_2l_2's_2'm_2')],$$

which by BBR (19) becomes

$$=[(2l_1'+1)(2s_2'+1)]^{\frac{1}{2}}(gs_2'-m_1'm_2'|gs_2'L-\kappa)\times W(gJ_2Ll_2';l_1's_2').$$

Finally, the sum on m_1' can be put in the form

$$\sum_{m_1'}[(fs_1'0m_1'|fs_1'gm_1')(gs_2'm_1'-m_2'|gs_2'L\kappa)\times(s_1's_2'm_1'-m_2'|s_1's_2'1\kappa)],$$

which again by BBR (19) becomes

$$=[3(2g+1)]^{\frac{1}{2}}(f10\kappa|f1L\kappa)W(fs_1'Ls_2';g1).$$

[10] G. Racah, Phys. Rev. **62**, 442 (1942); references to this paper will be denoted by R.

[11] Biedenharn, Blatt, and Rose, Revs. Modern Phys. **24**, 249 (1952); references to this paper will be designated by BBR.

Hence, the entire magnetic sum can be written as

$$=(-1)^{-s-l_2'-J_2+\kappa-1}\sqrt{3}(2J_1+1)(2J_2+1)\cdot\times\sum_f(-1)^f(2f+1)^{\frac{1}{2}}(l_1l_200|l_1l_2f0)(f10\kappa|f1L\kappa)\times W(l_1J_1l_2J_2;sf)\sum_g[(-1)^g(2g+1)W(J_2fl_1's_1';J_1g)\times W(gJ_2Ll_2';l_1's_2')W(fs_1'Ls_2';g1)].$$

Finally, by BBR (14), the sum over g can be written in terms of the X function defined in Appendix B and originally introduced by Fano and Racah.[12] As a result the magnetic sum becomes

$$=(-1)^{-s-s_1'+J_1-l_2'+L+\kappa}\sqrt{3}(2J_1+1)(2J_2+1)\times\sum_f[(2f+1)^{\frac{1}{2}}(l_1l_200|l_1l_2f0)(f10\kappa|f1L\kappa)\times W(l_1J_1l_2J_2;sf)X(J_1l_1's_1';J_2l_2's_2';fL1). \quad (3.1)$$

In the particular case of the polarization tensor, which is of rank one, the sum over f reduces to a single term which is $f=L$. To see this, recall that by conservation of parity $l_1+l_2+l_1'+l_2'=$ even integer. In addition, the Clebsch-Gordan coefficient $(ab00|abc0)$ vanishes unless $a+b+c=$ even integer [see BBR (5)]. The corresponding coefficients in Eqs. (3.1) and (2.10) now show that $L+f=$ even integer. However, by the properties of $(f10\kappa|f1L\kappa)$ we must have $f=L$ or $f=L\pm1$. Hence there follows the result $f=L$.

The final result for the polarization intensity may now be written in a very simple form by noting that the entire dependence on κ of dP_κ is contained in the terms $(L10\kappa|L1L\kappa)Y_{L,\kappa}$ where we have set f equal to L. Hence

$$dP_z\equiv dP_0=0,$$

$$dP_x=(dP_{-1}-dP_1)/\sqrt{2}\sim\tfrac{1}{2}(Y_{L,-1}+Y_{L,1})\sim -i\sin\varphi\bar{P}_L{}^1(\theta),$$

$$dP_y=i(dP_1+dP_{-1})/\sqrt{2}\sim-\tfrac{1}{2}i(Y_{L,1}-Y_{L,-1})\sim i\cos\varphi\bar{P}_L{}^1(\theta),$$

where $\bar{P}_L{}^1$ is the normalized associated Legendre polynomial. This result clearly shows that the polarization is always normal to the plane formed by the directions of the incoming and outgoing particles. In terms of the unit vector $\mathbf{n}=(\mathbf{k}_\alpha\times\mathbf{k}_{\alpha'})/|\mathbf{k}_\alpha\times\mathbf{k}_{\alpha'}|$ we have the final result:

$$d\mathbf{P}_{\alpha'\alpha}=\mathbf{n}(\lambda_\alpha{}^2/4)(2I+1)^{-1}(2i+1)^{-1}[2(2i'+1)]^{\frac{1}{2}}\times\sum i^{l_2-l_1+l_1'-l_2'}\text{R.P.}[i\{\delta(\alpha',\alpha)\delta(l_1',l_1)\delta(s_1',s)-S(\alpha'l_1's_1';\alpha l_1s;J_1\pi_1)\}^*\{\delta(\alpha',\alpha)\delta(l_2',l_2)\delta(s_2',s)-S(\alpha'l_2's_2';\alpha l_2s;J_2\pi_2)\}](-1)^{I'-i'-s+l_1'+J_1-s_1'}\times[(2l_1+1)(2l_2+1)(2l_1'+1)(2l_2'+1)(2s_1'+1)\times(2s_2'+1)]^{\frac{1}{2}}(2J_1+1)(2J_2+1)(l_1l_200|l_1l_2L0)\times(l_1'l_2'00|l_1'l_2'L0)W(i's_1'i's_2';I'1)W(l_1J_1l_2J_2;sL)\times X(J_1l_1's_1';J_2l_2's_2';LL1)\bar{P}_L{}^1(\theta)d\Omega. \quad (3.2)$$

where the sum is over $J_1J_2\pi_1\pi_2l_1l_2l_1'l_2's_1's_2's$ and L. The final expression for $d\mathbf{P}$ has been written entirely in terms of real quantities by use of the symmetry property of the X coefficient for interchange of two rows (see Appendix B). This form of the expression clearly shows that the polarization is an interference phenomenon.[13]

IV. SELECTION RULES

The fact that the polarization is always normal to the scattering plane has been demonstrated in the previous paragraph. All other selection rules follow from the requirements for the nonvanishing of the Racah and Clebsch-Gordan coefficients [see BBR (5) and (13)] and are listed below:

(a) If only S waves are effective in the reaction, for either the incident or the final states, there can be no polarization.

(b) If only levels of the compound nucleus having $J=\frac{1}{2}$ and a single parity (or $J=0$ with any parity) are effective, there will be no polarization.

(c) If only channel spin 0 is effective for the final channel spin, the polarization vanishes.

(d) Polarization results from the interference of different subchannels (i.e., partial waves or final channel spins) contributing to the reaction. (The state of the residual nucleus must always be the same, of course.) Hence, if there is only a single nonzero element of the scattering matrix, the polarization will vanish.

(e) If there is no spin-orbit coupling, the polarization is zero.

(f) If there is a largest effective incident orbital wave l, final orbital wave l' or total angular momentum J, there will be a largest value of L in Eq. (3.2) given by the simultaneous conditions

$$L\leq 2l;\,2l';\,2J. \quad (4.1)$$

In addition, one must remember that L must be even if the interfering states have the same parity. These rules are identical to the rules for the limitation of the complexity of angular distributions.[14] Selection rules (a) and (b) are actually special cases of (f). Some of these rules have been given before by Wolfenstein[15] for the special case of reactions involving polarized particles of spin $\frac{1}{2}$.

[12] U. Fano and G. Racah, unpublished. See also U. Fano, National Bureau of Standards Report 1214, p. 48.

[13] F. Coester [Phys. Rev. **89**, 619 (1953)] has recently conjectured that the elements of the scattering matrix $S(s_1l;s'l';J\pi)$ having the same J and π have the same phase. If this were so, polarization could only result from the interference of levels of different J or parity and not from the possible interference of different subchannels of a single level, as stated in Sec. I. The existence or nonexistence of this type of polarization would then constitute an experimental check of Coester's hypothesis. In this connection it should also be noted that the matrix elements resulting from the use of perturbation theory or the Born approximation are all real. Hence in these approximations there can be no polarization (or higher odd tensor moments). This implies the absence of circular polarization in γ-rays resulting from nuclear reactions.

[14] C. N. Yang, Phys. Rev. **74**, 764 (1948).

[15] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

It is important to note that, if the observed angular distribution in a nuclear reaction is not isotropic, we are assured that selection rules (a) and (b) are not operating. Furthermore, if the distribution is not symmetric about 90° in the center-of-mass system, then selection rule (d) is also not operating.

In many nuclear reactions the largest incident or final orbital angular momentum is often either a p wave or a d wave. In such cases the polarization will vary with angle as $P_1^1(\theta)$ or $P_2^2(\theta)$ or a combination of these. In the absence of more detailed information, it would then seem most profitable to search for polarization in the vicinity of 45° or 135° in the center-of-gravity system.

V. ILLUSTRATIVE EXAMPLES

Any calculation of the polarization, as well as any other spin moments, requires a knowledge of the scattering matrix. In practice only a few levels contribute to a reaction at a given energy and hence the formidable sum of Eq. (3.2) reduces to only a few terms. However, even this limited information is practically nonexistent.

Recently, Peshkin, and Siegert[3] have analyzed the $Li^6(n,\alpha)H^3$ reaction and obtained some information on the nuclear parameters. In addition, Lepore[4] has given a formula for the neutron polarization resulting from He^4 scattering. We will make a numerical estimate of the polarization of the triton in the Li^6 reaction and also rederive Lepore's formula as a special case of our general result.

$Li^6(n,\alpha)H^3$ Reaction

The relative angular distribution of the tritons at 270 kev has been found to be[16]

$$I(\theta)=103+83\cos\theta+192\cos^2\theta.$$

Absorption cross-section measurements[17] show a resonance maximum of 3.1 barns, 2.5 barns of which is actually due to the resonance, at 250 kev with a width at half-maximum of about 100 kev. Peshkin and Siegert have shown that these measurements can be fitted by assuming that the resonance is of total angular momentum $J=\frac{3}{2}$ formed by a p wave with the entrance channel spin $s=\frac{1}{2}$. In addition there is interference with the two distant levels formed by s waves. One being of $s=J=\frac{1}{2}$ and the other of $s=J=\frac{3}{2}$. If we now define the corresponding scattering matrix elements in terms of the notation in reference 3, we have

$$S(1\tfrac{1}{2};1\tfrac{1}{2};\tfrac{3}{2}-)\equiv\tau/\sqrt{2},$$
$$S(0\tfrac{1}{2};0\tfrac{1}{2};\tfrac{1}{2}+)\equiv b,$$
$$S(2\tfrac{1}{2};0\tfrac{3}{2};\tfrac{3}{2}+)\equiv a.$$

The resultant angular distribution then agrees with that of Peshkin and Siegert and is

$$d\sigma(\theta)=\tfrac{1}{6}\lambda\!\!\!^{-2}[|a|^2+\tfrac{1}{2}|b|^2+\tfrac{1}{4}|\tau|^2+(1/\sqrt{2})(b^*\tau+b\tau^*)\times\cos\theta+\tfrac{3}{4}|\tau|^2\cos^2\theta]d\Omega.$$

Comparing with the experimental cross sections, we find the relative values

$$|\tau|^2=256,\quad |a|^2+\tfrac{1}{2}|b|^2=39,\quad (b^*\tau+b\tau^*)=83\sqrt{2}. \quad (5.1)$$

The expression for the polarization may now be written down almost immediately from Eq. (3.2). Since different entrance channel spins are incoherent, the only contributing term is that due to interference between the reactions denoted by τ and b. For this case, the sum on L reduces to a single term, $L=1$. Tables of the Racah coefficients have been published by Biedernharn.[18] Using these, we find

$$W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2},01)=\tfrac{1}{2},$$
$$W(1\tfrac{3}{2}0\tfrac{1}{2};\tfrac{1}{2}1)=6^{-\frac{1}{2}},$$
$$X(\tfrac{3}{2}1\tfrac{1}{2};\tfrac{1}{2}0\tfrac{1}{2};111)=(216)^{-\frac{1}{2}},$$

and the final result which is

$$d\mathbf{P}=\mathbf{n}\lambda\!\!\!^{-2}(864)^{-\frac{1}{2}}i(\tau b^*-\tau^*b)\bar{P}_1^1(\theta)d\Omega.$$

By Eq. (5.1) we have $|b|\leq(78)^{\frac{1}{2}}$ and $|\tau|=16$, and thus $|\tau^*b|\leq141.3$. Now $\tau b^*+\tau^*b=2|\tau^*b|\cos\phi=83\sqrt{2}$ where ϕ is the modulus. Hence

$$|i(\tau b^*-\tau^*b)|=2|\tau^*b|\sin\phi=[4|\tau^*b|^2-4|\tau^*b|^2\cos^2\phi]^{\frac{1}{2}}\leq[4(141.3)^2-2(83)^2]^{\frac{1}{2}}=257.1.$$

Depending on the unknown phase which is not determined by the angular distribution, there will be a polarization of the triton which may vary from zero to a maximum value given by

$$f=105\bar{P}_1^1(\theta)/[103+83\cos\theta+192\cos^2\theta]. \quad (5.2)$$

This expression has a maximum value of 0.95 at an angle $\theta=100°$. Conversely, a determination of the polarization would establish the value of this phase.

Scattering of Neutrons by He^4

The general formalism has been used to rederive Eq. (2.9) in reference 4. The only nonzero elements of the scattering matrix are those referring to S and P wave scattering. In terms of the notation of Lepore, these scattering elements are

$$1-S(0\tfrac{1}{2};0\tfrac{1}{2};\tfrac{1}{2}+)=-2i\exp(i\delta_0)\sin\delta_0,$$
$$1-S(1\tfrac{1}{2};1\tfrac{1}{2};\tfrac{1}{2}-)=-i\Gamma/(E_{\frac{1}{2}}-E-\tfrac{1}{2}i\Gamma),$$
$$1-S(1\tfrac{1}{2};1\tfrac{1}{2};\tfrac{3}{2}-)=-i\Gamma/(E_{\frac{3}{2}}-E-\tfrac{1}{2}i\Gamma).$$

Note that $\alpha'=\alpha$, $l'=l$ and $s'=s$ in all cases.

Equation (2.9) in reference 4 was found to have two misprints. The sign of the first term in the numerator should be negative. The last term in the denominator

[16] Roberts, Darlington, and Haugsnes, Phys. Rev. **82**, 299 (951).
[17] J. M. Blair and R. E. Holland (to be published).
[18] L. C. Biedenharn, Oak Ridge National Laboratory Report No. 1098, 1952.

should read in part $(x_{\frac{1}{2}}x_{\frac{3}{2}}+\frac{1}{4})$ rather than $(x_{\frac{1}{2}}+x_{\frac{3}{2}}+\frac{1}{4})$. When these corrections are made, complete agreement is reached.

VI. HIGHER SPIN MOMENTS

The previous sections have been specialized to the problem of the polarization of the outgoing particle i'. In essence this quantity is just the expectation value in the scattered wave of the tensor of rank unity formed out of the spin operators $\mathbf{i}'$. In general however, an outgoing particle will have nonzero irreducible tensor moments up to a maximum tensor rank given by $2i'$.

The previous results can be made completely general by a few slight changes. Instead of considering the expectation value of a tensor of rank unity $T_\kappa^{(1)}$, we ask for the expectation value of the general irreducible tensor operator of rank q, $T_\kappa^{(q)}$, which is formed out of the spin operators $\mathbf{i}'$.[19] We define the *differential tensor moment* to be

$$dT_{\kappa,\alpha'\alpha}^{(q)} = \langle \sum (\psi(\alpha's')|T_\kappa^{(q)}|\psi(\alpha's''))r_{\alpha'}^2 v_\alpha^{-1} v_{\alpha'}\rangle_{\text{Av}} d\Omega, \quad (6.1)$$

where the sum is over all final channel spins and final channel spin directions, and an average is taken over the initial states. By Eq. (A.2) we have

$$\begin{aligned}(\chi(s_1'm_1')|T_\kappa^{(q)}|\chi(s_2'm_2')) &= (-1)^{I'-i'-\kappa+m_1'}[(2s_1'+1)(2s_2'+1)]^{\frac{1}{2}} \\ &\times[(2i')!]^{-1}[(2i'-q)!(2i'+q+1)!/(2q+1)]^{\frac{1}{2}} \\ &\times(s_1's_2'-m_1'm_2'|s_1's_2'q-\kappa)W(i's_1'i's_2';I'q) \\ &\times P_q([i'/(i'+1)]^{\frac{1}{2}}). \quad (6.2)\end{aligned}$$

which replaces Eq. (2.9). Here P_q is the usual Legendre polynomial. The magnetic sums proceed as before with the only change being the replacement of 1 by q.

As a result, we obtain the following *general* expression for the differential tensor moment:

$$\begin{aligned}dT_{\kappa,\alpha'\alpha}^{(q)} &= \frac{(\pi)^{\frac{1}{2}}\lambda\!\!\!^{-}{}_\alpha^2[(2i'-q)!(2i'+q+1)!]^{\frac{1}{2}}}{2(2I+1)(2i+1)(2i')!} \\ &\times P_q\left(\left[\frac{i'}{i'+1}\right]^{\frac{1}{2}}\right)\sum(-1)^{I'-i'-s+J_1-s_1'+l_1'}\, i^{l_2-l_1+l_1'-l_2'} \\ &\times[\delta(\alpha',\alpha)\delta(s_1',s)\delta(l_1',l_1)-S(\alpha's_1'l_1;\alpha sl_1;J_1\pi_1)]^* \\ &\times[\delta(\alpha',\alpha)\delta(s_2',s)\delta(l_2',l_2)-S(\alpha's_2'l_2';\alpha sl_2;J_2\pi_2)] \\ &\times[(2f+1)(2l_1+1)(2l_2+1)(2l_1'+1)(2l_2'+1) \\ &\times(2s_1'+1)(2s_2'+1)/(2L+1)]^{\frac{1}{2}}(2J_1+1)(2J_2+1) \\ &\times(l_1'l_2'00|l_1'l_2'L0)(l_1l_200|l_1l_2f0)W(l_1J_1l_2J_2;sf) \\ &\times W(i's_1'i's_2';I'q)X(J_1l_1's_1';J_2l_2's_2';fLq) \\ &\times(fq0\kappa|fqL\kappa)Y_{L,\kappa}(\theta,\varphi)d\Omega, \quad (6.3)\end{aligned}$$

where the sum is over $J_1J_2\pi_1\pi_2l_1l_2l_1'l_2's_1's_2'sL$ and f. Note that for tensors of rank higher than unity, the sum over f does not necessarily reduce to a single term.

Selection rules (a), (b), and (d) of Sec. IV are now seen to apply to *all* odd values of q. Selection rules (c) and (e) apply to all values of q greater than zero.

A generalization of the rules[14] for the complexity of the angular distribution also follows from the properties of the Racah coefficients. If there is a maximum effective *final* orbital angular momentum l', a maximum *initial* orbital momentum l, or a maximum total angular momentum J, then a maximum value of L in Eq. (6.3) is given by the simultaneous conditions

$$\begin{aligned}L &\leq 2l';\ 2l+q;\ 2J+q \quad (q \text{ even}) \\ &\leq 2l';\ 2l+q-1;\ 2J+q-1 \quad (q \text{ odd})\end{aligned} \quad (6.4)$$

along with the condition that L must be even if the interfering levels have the same parity.

Equation (6.3) is the general expression for the components of the spin tensor with respect to the direction of the incident beam. It is also convenient to have these components expressed relative to the direction of scattering (θ, φ). If the incident beam direction is denoted by $\mathbf{k}$ and the scattered direction by $\mathbf{k}'$, let us choose a new coordinate system with z' axis along $\mathbf{k}'$, and with the y' axis along $\mathbf{k}\times\mathbf{k}'$. The Euler angles of this rotation are then $(\varphi, \theta, 0)$ relative to the original coordinate system.

The spin tensor operator $T_\mu^{(q)}$ in the new system is then related to the spin tensor operators $T_\kappa^{(q)}$ in the original coordinate system by the relation[20]

$$T_\mu^{(q)} = \sum_\kappa D_{\kappa,\mu}^{(q)}(\varphi,\theta,0)T_\kappa^{(q)}, \quad (6.5)$$

where $D_{\kappa,\mu}^{(q)}$ is an element of the three-dimensional rotation group. The complete dependence on κ of Eq. (6.3) is contained in the factors $(fq0\kappa|fqL\kappa)Y_{L,\kappa}(\theta,\varphi)$. Hence, the new tensor moment contains the transformed factors

$$\sum_\kappa D_{\kappa,\mu}^{(q)}(\varphi,\theta,0)Y_{L,\kappa}(\theta,\varphi)(fq0\kappa|fqL\kappa).$$

Using the relation

$$(-1)^\kappa Y_{L,\kappa}(\theta,\varphi) = [(2L+1)/4\pi]^{\frac{1}{2}}D_{-\kappa,0}^{(L)}(\varphi,\theta,0),$$

as well as the following expansion of the product of two rotation matrices,[21]

$$D_{\alpha\beta}^{(S)}(R)D_{\gamma\delta}^{(T)}(R) = \sum_U (ST\alpha\gamma|STU\alpha+\gamma) \times(ST\beta\delta|STU\beta+\delta)D^{(U)}{}_{\alpha+\gamma,\beta+\delta}(R),$$

and performing the sum over κ by R (20a), we obtain for the transformed factors

$$(-1)^q[(2L+1)/(2f+1)]^{\frac{1}{2}}(Lq0\mu|Lqf\mu) \times[(2L+1)/4\pi]^{\frac{1}{2}}D_{0,\mu}^{(f)}(\varphi,\theta,0).$$

---

[19] All tensor operators are to be defined so as to agree with the definition given in R. For definiteness we take

$$T_0^{(q)} = P_q(i_z'/[i'(i'+1)]^{\frac{1}{2}}).$$

[20] E. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren* (F. Vieweg, Braunschweig, 1931), p. 165.

[21] This is derived in reference 20, p. 203.

In addition,

$$D_{0,\mu}^{(f)}(\varphi,\theta,0)=D_{0,\mu}^{(f)}(0,\theta,0)=(-1)^{\mu}[4\pi/(2f+1)]^{\frac{1}{2}}Y_{f,\mu}(\theta,0).$$

This result yields the following simple recipe for converting Eq. (6.3) so as to give the tensor moments relative to the scattered axis.

(a) Replace the expressions $(fq0\kappa|fqL\kappa)Y_{L,\kappa}(\theta,\varphi)$ by $(Lq0\kappa|Lqf\kappa)Y_{f,\kappa}(\theta,0)$.

(b) Multiply by the factors

$$(-1)^{q+\kappa}(2L+1)(2f+1)^{-1}.$$

The component κ now refers to the scattered axis. Note carefully that the angle θ is still measured relative to the incident direction and that our result is now independent of φ. Note also that f and L have interchanged their roles since the complexity of the angular distribution is now determined by f. This clarifies the physical meaning of the parameter f and leads to an alternative form of the generalized rule for the complexity of the angular distribution. Instead of Eq. (6.4) we have

$$\begin{aligned} f&\leq 2l,\ 2J,\ 2l'+q &&(q\text{ even})\\ &\leq 2l,\ 2J,\ 2l'+q-1 &&(q\text{ odd})\end{aligned}\qquad(6.6)$$

along with the condition that f must be even if the interfering levels have the same parity.

Angular Distribution of Nuclear Reactions

It is interesting to note that the results given in BB for the angular distribution of nuclear reactions now appear immediately as a special case of our general result. Since the angular distribution is essentially the expectation value of the unit operator in the final state, we obtain this result by taking $q=\kappa=0$ in Eq. (6.3). An immediate consequence is that $f=L$ and $s_1'=s_2'=s'$. In addition, by Eqs. (B.1) and BBR (14) and (30),

$$X(J_1l_1's';J_2l_2's';LL0)=(-1)^{L+S'-J_1+l_2'}\times W(l_1'J_1l_2'J_2;s'L)[(2L+1)(2s'+1)]^{-\frac{1}{2}},$$

$$W(i's'i's';I'0)=(-1)^{i'+s'-I'}[(2i'+1)(2s'+1)]^{-\frac{1}{2}},$$

and $Y_{L,0}=[(2L+1)/4\pi]^{\frac{1}{2}}P_L(\theta)$. The result follows immediately.

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APPENDIX A. REDUCTION OF THE SPIN MATRIX ELEMENT

Consider the matrix element $(\chi(s_1m_1)|T_\kappa^{(q)}|\chi(s_2m_2))$, where s_1 and s_2 are channel spins resulting from the vector addition of the spins i and I. The magnetic quantum numbers of s_1 and s_2 are m_1 and m_2, respectively. The tensor operator $T_\kappa^{(q)}$ operates only on the particle with spin i and is so normalized that

$$T_0^{(q)}=P_q(i_z'/[i'(i'+1)]^{\frac{1}{2}}),$$

where P_q is the usual Legendre polynomial. By the use of the Wigner-Eckart theorem and the Racah formalism [see R (44) and (16′)] we have[22]

$$(\chi(s_1m_1)|T_\kappa^{(q)}|\chi(s_2m_2))=(-1)^{I-\kappa-i+m_1}\times[(2s_1+1)(2s_2+1)/(2q+1)]^{\frac{1}{2}}W(is_1is_2;Iq)\times(s_1s_2-m_1m_2|s_1s_2q-\kappa)(i\|T^{(q)}\|i),\quad(A.1)$$

where the reduced matrix element $(i\|T^{(q)}\|i)$ is independent of κ, m_1, m_2, I, s_1, and s_2. To evaluate this matrix element then, we can choose special values of the parameters which allow a solution of Eq. (A.1). Let us choose $\kappa=I=0$ and $m_1=m_2=i$. Then $s_1=s_2=i$ and

$$(\chi(ii)|T_0^{(q)}|\chi(ii))=P_q([i'/(i'+1)]^{\frac{1}{2}}),$$

$$W(iiii;0q)=(-1)^{2i-q}/(2i+1),$$

and, by BBR (1) and R (16),

$$(ii-ii|iiq0)=(-1)^{2i-q}(iii-i|iiq0)=(-1)^{2i-q}(2i)!(2q+1)^{\frac{1}{2}}[(2i-q)!(2i+q+1)!]^{-\frac{1}{2}}.$$

Substituting these in Eq. (A.1) we obtain

$$(i\|T^{(q)}\|i)=P_q([i/(i+1)]^{\frac{1}{2}})[(2i)!]^{-1}[(2i-q)!(2i+q+1)!]^{\frac{1}{2}}.$$

Hence the general result is

$$\begin{aligned}(\chi(s_1m_1)|T_\kappa^{(q)}|\chi(s_2m_2))&=(-1)^{I-\kappa-i+m_1}(2q+1)^{-\frac{1}{2}}[(2i)!]^{-1}\\&\quad\times[(2s_1+1)(2s_2+1)(2i-q)!(2i+q+1)!]^{\frac{1}{2}}\\&\quad\times W(is_1is_2;Iq)(s_1s_2-m_1m_2|s_1s_2q-\kappa)\\&\quad\times P_q([i/(i+1)]^{\frac{1}{2}}).\end{aligned}\quad(A.2)$$

For the particular value of $q=1$,

$$(\chi(s_1m_1)|T_\kappa^{(1)}|\chi(s_2m_2))=(-1)^{I-\kappa-i+m_1}W(is_1is_2;I1)\times[(2s_1+1)(2s_2+1)(2i+1)/3]^{\frac{1}{2}}\times(s_1s_2-m_1m_2|s_1s_21-\kappa).\quad(A.3)$$

APPENDIX B. SOME PROPERTIES OF THE X COEFFICIENT

The X coefficient is defined by Fano and Racah[12] in terms of Racah functions as

$$X\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=(-1)^S\sum_z[(2z+1)W(bdcg;za)\times W(dbfh;ze)W(gchf;zi)].$$

[22] For the spin moments of rank greater than zero it is necessary to specify the scattering matrix more completely than the definition given in BB (3.3) and (3.2). The phase of the channel spin wave function must be fixed. We take

$$\chi_{s,m}=\Sigma(iIm_im-m_i|iIsm)\chi_{i,m_i}\chi_{I,m-m_i}.$$

Where $S=a+b+\cdots+i$. Fano and Racah have shown than an interchange of two rows or columns multiplies X by $(-1)^S$ and that the interchange between rows and columns leaves X invariant. It should also be noted that the elements of each row and each column must form a possible triad.

For simplicity in printing, we have written the X function as $X(abc;def;ghi)$. The special value

$$X(abc;dec;gg0) = (-1)^{c+g-a-e}W(abde;cg)/[(2c+1)(2g+1)]^{\frac{1}{2}} \quad \text{(B.1)}$$

is often useful.

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The Azimuthal Distribution of Photoelectrons Produced by 0.5-Mev Polarized Photons*

FRANK L. HEREFORD AND JEROME P. KEUPER†
Department of Physics, University of Virginia, Charlottesville, Virginia
(Received January 22, 1953)

The azimuth angular distribution of photoelectrons ejected from lead by linearly polarized 0.51-Mev photons has been studied. The known correlation between polarization states of annihilation quanta was employed in preparation of the incident beam of polarized photons. Results indicate that for emission from the K shell, the most probable direction is in the plane containing the momentum and polarization vectors of the incident photon. The asymmetry in the distribution thereby produced is slightly greater than that expected from the relativistically computed differential cross section.

EARLY measurements of the spatial distribution of photoelectrons produced by linearly polarized photons were carried out by Wilson,[1] Bubb,[2] and Kirkpatrick.[3] These measurements were confined to photons of energy less than 40 kev because of the difficulty of preparing a polarized beam at higher energies. The results confirmed the $\cos^2\phi$ azimuthal distribution deduced from classical considerations by Auger and Perrin,[4] and from nonrelativistic quantum mechanics by Sommerfeld and Schur.[5] Here ϕ is the angle between the plane containing the momentum vectors of the photon and ejected photoelectron and that containing the polarization and momentum vectors of the incident photon. Thus the most probable electron emission is along the polarization vector.

This result is valid only for electrons ejected from the K shell, those from higher levels showing complete azimuthal symmetry. The K shell electrons normally comprise approximately 80 percent of all electrons ejected for moderately hard quanta.[6] Actually the computed distribution for L electrons contains a term yielding an azimuthal asymmetry.[7] However, this comcomponent represents a small fraction of the 20 percent of total intensity deriving from higher levels and can be neglected for our purposes.

For relativistic energies, Sauter[8] has computed the distribution for K electrons with the following factor in the differential cross section giving the angular distribution,

$$d\sigma \sim \frac{\sin^2\theta}{(1-\beta\cos\theta)^4}\{\tfrac{1}{4}(\gamma-1)^2(1-\beta\cos\theta) + [1/\gamma-\tfrac{1}{2}(\gamma-1)(1-\beta\cos\theta)]\cos^2\phi\}, \quad (1)$$

where β is the photoelectron velocity in units of c, $\gamma=(1-\beta^2)^{-\frac{1}{2}}$, and θ is the angle $(\mathbf{k}_0, \mathbf{p})$ between the incident photon and photoelectron. For the case of importance here $(\theta=\pi/2)$, this factor reduces to

$$d\sigma \sim \tfrac{1}{4}(\gamma-1)^2+[1/\gamma-\tfrac{1}{2}(\gamma-1)]\cos^2\phi. \quad (2)$$

In the nonrelativistic limit the second term predominates in agreement with the previously stated results. For $\beta=0.87$ (K.E.$=0.51$ Mev) the second term vanishes yielding azimuthal symmetry, and for higher energies this term is negative. Moreover, in this region the first term, containing the square of the energy, predominates. Thus for $\beta>0.87$ the preferred K electron emission is orthogonal to the polarization vector of the incident photon. It can be presumed that the 20 percent of the photoelectrons from higher levels show azimuthal symmetry for all energies, although no calculations on this point have been performed.

No experimental results in the region of higher energies were available until recently when the cross-polarization property[9] of electron-positron annihilation quanta was used to investigate the azimuthal distribution of photoelectrons ejected by the 0.51-Mev anni-

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1 C. T. R. Wilson, Proc. Roy. Soc. (London) A104, 11 (1923).
2 F. W. Bubb, Phys. Rev. 23, 137 (1924).
3 P. Kirkpatrick, Phys. Rev. 38, 1938 (1931).
4 P. Auger and F. Perrin, J. phys. et radium 8, 93 (1927).
5 A. Sommerfeld and G. Schur, Ann. Physik 4, 409 (1930).
6 W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), p. 127.
7 G. Schur, Ann. Physik 4, 433 (1930).
8 F. Sauter, Ann. Physik 11, 454 (1931).
9 J. A. Wheeler, Ann. N. Y. Acad. Sci. 48, 219 (1946).