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The Small Angle Scattering of X-Rays from Cold-Worked Solids

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The small angle scattering of x-rays or thermal neutrons from cold-worked crystals is calculated on the basis of two models, according to which the scattering arises predominantly from the density variation associated either with small cavities or with edge-type dislocations. The elastic distortions surrounding the cavities are unimportant, so that the scattering from cavities in a uniform medium of density n is the same, to a good approximation, as that from partices (in vacuum) of density n of the same size as the cavities. Thus the usual formulas of small angle scattering obtain, the scattered intensity has the familiar Gaussian-like dependence on scattering angle, and earlier results on multiple scattering may be applied. Around edge-type dislocations, on the other hand, the density variation is proportional to $\sin\xi/r$, where ξ is the angle measured from the slip direction in the plane perpendicular to the dislocation line and r is the distance from the dislocation line. This angular variation results in a complete modification of the usual formulas, and, in fact, all of the terms ordinarily present disappear for this case, and conversely. The scattering shows a parabolic increase from zero at small angles, a maximum, and finally a monotonic decrease with increasing scattering angle. There is a large degree of anisotropy in the scattering, depending on the direction of the incident beam relative to the slip and dislocation axes. Multiple scattering from an array of dislocations in even a thick specimen is shown to be negligible. The theory is compared with Blin and Guinier's preliminary experimental results, and it is concluded that dislocations are incapable of explaining their data, although it is expected that under suitable conditions the measurement of the scattering from dislocations should be experimentally feasible.

I. INTRODUCTION

ACCORDING to current interpretations, the process of cold-working leads to the introduction of considerable imperfection into a crystal. For the last dozen or so years it has been generally believed that much of this imperfection may be described in terms of dislocations of the lattice, and more recently, Seitz[1] has suggested that vacancies are produced as a result of the motion of dislocations during cold-work. The concentrations of dislocations involved are perhaps of the order 10^7 or 10^8 dislocation line-cm per cm^3 in a "perfect" crystal, increasing to 10^{12} lines per cm^2 in a heavily cold-worked specimen.

Associated with imperfection of these types, there is a variation in the density of the lattice and a variation in the scattering power and potential within the material. These variations will, of course, affect the electrical conductivity, measurements of which are proving an important tool in the investigation of crystalline imperfection.[2] Conductivity measurements, combined with stored energy experiments on cold-worked metals, suggest that the observed resistivity change per unit stored energy q may be too large to be explained by dislocations or perhaps even by isolated vacancies. A more advantageous ratio q is expected for clusters of vacancies.[2c]

The density variations will also result in a scattering of x-rays or thermal neutrons. The small angle scattering of x-rays (wavelength $\sim$1A) has developed in the past 15 years into an accurate and powerful technique for the investigation[3] of small particles of linear dimensions $L>10\lambda$. Although the most striking successes have dealt with biological materials,[4] Blin and Guinier[5] have recently made preliminary measure-

[1] F. Seitz, Phil. Mag. Supplement **1**, 43 (1952).

[2] For discussion of the effects of (a) dislocations, (b) isolated vacancies, and (c) clustered vacancies or cavities on the resistance, see, respectively, the following papers and references contained therein: (a) D. L. Dexter, Phys. Rev. **86**, 447 (1952); (b) D. L. Dexter, Phys. Rev. **87**, 768 (1952); J. W. Kauffman and J. S. Koehler, Phys. Rev. **88**, 149 (1952); (c) D. L. Dexter (to be published).

[3] A. Guinier, Ann. phys. **12**, 161 (1939) and many subsequent papers.

[4] The reader is referred to a series of papers in J. Chem. Phys. by W. W. Beeman and his co-workers on the scattering from protein molecules in aqueous solution, for example, Ritland, Kaesberg, and Beeman, J. Chem. Phys. **18**, 1237 (1950). See also Kaesberg, Ritland, and Beeman, Phys. Rev. **74**, 71 (1948).

[5] J. Blin and A. Guinier, Compt. rend. **233**, 1288 (1951). Professor Guinier has kindly informed the writer in a private communication that the concentration of scatterers mentioned in their paper was too large by about a factor of 10.

ments of the scattering from cold-worked metals, and it seems probable that small angle scattering will become useful also in the investigation of crystalline imperfection. It is, therefore, the purpose of this paper to compute the scattering to be expected from clustered vacancies, or cavities, and from dislocations.

II. SCATTERING FROM CAVITIES

We consider an effectively infinite medium consisting of a solid of electron density n_e and atomic density n_a. For the sake of definiteness we shall speak throughout of x-ray scattering and shall consequently be concerned with the electron density and the Thompson scattering coefficient I_e. It is clear that our conclusions will likewise apply to neutron scattering, if we replace n_e by n_a and the x-ray scattering cross section per electron by the neutron scattering cross section per atom.

The coherent scattering by the perfect infinite medium will, of course, all be in the forward direction and will be indistinguishable from the direct, unscattered beam. Thus only the deviations from the density n_e will be of interest to us. If we introduce into the medium a number N of randomly arranged identical regions of constant electron density n_e', the coherent scattered intensity will be

$$I(\epsilon)=I_eN[V(n_e'-n_e)]^2\{1-\tfrac{1}{3}(2\pi\rho\epsilon/\lambda)^2+\cdots\}, \quad (1)$$

where ϵ is the angle of deviation from the incident beam, I_e the intensity scattered by a single electron, V the volume of one of the identical regions of electron density n_e', and ρ is the radius of gyration defined by

$$\rho^2=V^{-1}\int r^2 d\tau, \quad (2)$$

where the integration is carried out over a single region. Thus the scattering is a maximum in the forward direction. It should be noted that $I(\epsilon)$ is proportional to the square of the difference between the electron densities. Thus a number N of cavities of density zero scatter exactly as N particles in vacuum of the same shape and of density n_e. The small angle approximation in Eq. (1) requires that $\sin(\epsilon/2)$ be replaceable by $\epsilon/2$ and that $\frac{1}{3}(2\pi\rho\epsilon/\lambda)^2$ be less than 1. Thus we may not treat the scattering of 1-A x-rays from isolated vacancies by this approximation, since the radiation will be scattered over large angles where $\sin(\epsilon/2)$ is not $\sim\epsilon/2$. For large cavities, however, Eq. (1) is valid out to the angle of half-maximum intensity, in so far as the model is adequate. All of the above follows directly from well-known results.

The above model is certainly applicable if the elastic distortions surrounding a cavity are sufficiently small. According to the theory[6] of elasticity of an isotropic continuum, the displacement of the medium surrounding a spherical cavity of radius a at the center of a sphere of radius R is of the form

$$\mathbf{U}(\mathbf{r})=A\mathbf{r}/r^3+B\mathbf{r}, \quad (3)$$

where

$$B=2A(1-2\nu)/R^3(1+\nu), \quad (4)$$

ν is Poisson's ratio and A is determined by the boundary condition at the surface of the cavity. The fractional change in the density of the medium associated with the distortions resulting from N cavities is then given approximately by $(\delta D/D)\sim-N\nabla\cdot\mathbf{U}$, and since the divergence of the first term in Eq. (3) vanishes, we obtain a density change

$$\frac{\delta D}{D}=-6\frac{NA}{R^3}\frac{(1-2\nu)}{(1+\nu)}. \quad (5)$$

Since the density change is roughly independent of position, the lattice surrounding the cavity does not contribute to the scattering, and the radius of gyration is determined solely by the size of the cavity. Thus the primary effect of the distortions is to change the factor $[V(n_e'-n_e)]^2$ by an amount that will be seen to be negligible, and the radius of gyration determined by experiment will correspond in the usual way to the geometrical size of the cavity.

If the magnitude of the displacement at the surface of a cavity is γ, then $A=a^3(\gamma/a)$, so that Eq. (5) becomes, on setting $\nu\sim\frac{1}{3}$, $\delta D/D\sim-\frac{3}{2}(Na^3/R^3)(\gamma/a)$. The factor Na^3/R^3 is the fractional volume of the sphere occupied by cavities and is less than or about equal to 10^{-3} for all cases in which we shall be interested. Likewise, γ/a is in all cases much less than unity. Thus we may certainly neglect the effects of distortion in the surrounding medium, and we may treat the cavities in a medium of initial density n_e as small particles of the same size and shape and of the same density n_e dispersed in vacuum.

Blin and Guinier[5] found that their experimental data on copper, heavily cold-worked at room temperature, are characterized in angular dependence by a radius of gyration of about 6.6A. This experimental figure is, of course, determined independently of any model which may be chosen to interpret the data and corresponds to the spatial extent of whatever regions may be present in which the electron density differs from that of the surrounding medium. If these regions be spherical and of a constant density, they correspond to a volume of 2.6×10^{-21} cm^3, or about 200 times the atomic volume of copper. If the density is constant but the regions are nonspherical, the volume is correspondingly less. Blin and Guinier[5] likewise made a preliminary measurement of the absolute intensity of the scattered x-radiation. From comparison with Eq. (1), therefore, the total number of scattering regions may be determined for any specific model consistent with the measured radius of gyration. For example, if one assumes the scattering regions are spherical cavities of radius of gyration 6.6A

[6] S. Timoshenko, *Theory of Elasticity* (McGraw-Hill Book Company, Inc., New York, 1934).

and electron density zero, the concentration of cavities turns out to be about 7×10^{17} per cm^3.[5] The fractional volume of the copper occupied by cavities would then be 1.8×10^{-3}. If one chooses cylindrical cavities in the shape of a pancake, of thickness $L=3$A, radius $a=9.25$A $[\rho=(a^2/2+L^2/12)^{\frac{1}{2}}=6.6\text{A}]$, the concentration becomes 7.3×10^{18} cm^{-3}, and the fractional volume occupied by the cavities is 5.9×10^{-3}. Similarly for cylindrical rods of $L=22.6$A, $a=1.5$A ($\rho=6.6$A), the concentration would be 1.8×10^{20} cm^{-3}, and the fractional volume occupied would be 2.9×10^{-2}. (The above shapes for the cylinders were not selected as reasonable physical models but were chosen as limiting cases.)

It is clear that if one employs too thick a scattering sample, a given photon may be scattered more than once. Such multiple scattering would broaden the scattering curve and would result in too small a radius of gyration, if the interpretation were made on the basis of Eq. (1). There exists an alternative procedure for obtaining the radius of gyration, however, in cases where multiple scattering is present, a method depending on the variation of the width of the scattering curve with sample thickness.[7] As will be seen below, this method is not practicable for the systems of interest here.

The condition that multiple scattering have negligible effect is given by $\mu_s t<1$, where μ_s, the linear scattering coefficient, is equal to

$$\mu_s=27r_0^2\lambda^2[V(n_e'-n_e)]^2\mathfrak{N}/(40\pi\rho^2), \tag{6}$$

where r_0 is the classical electron radius e^2/mc^2, λ the x-ray wavelength, $V(n_e-n_e')$ the deficit of electrons in each scattering region, $\mathfrak{N}$ the number density of the regions, and where t is the thickness of the scattering sample. Thus, for example, in the first case mentioned above for copper, with $\lambda=1.54$A, $V=2.6\times10^{-21}$ cm^3, $n_e=2.4\times10^{24}$ cm^{-3}, $n_e'=0$, $\mathfrak{N}=7\times10^{17}$ cm^{-3}, and $\rho=6.6$A, the condition for negligible multiple scattering is

$$t<40 \text{ cm}.$$

Since V varies as ρ^3, μ_s appears to vary as the fourth power of the radius of gyration. It must be recalled, however, that the fractional volume of a cold-worked specimen occupied by cavities cannot become greater than about 10^{-3}, since the fractional density change of cold-worked crystals is of this order of magnitude. Hence, we must add the condition that $\mathfrak{N}V\gtrsim10^{-3}$, so that μ_s varies linearly with ρ for samples in which $\mathfrak{N}V$ has reached its maximum value of $\sim10^{-3}$. Thus we see that for very small cavities $\rho\gtrsim100$A, multiple scattering need not be considered at all. Alternatively, we may say that the condition for the experimental practicability of the multiple scattering method[7] for determining particle size is given by

$$\mu_s>\mu_a, \tag{7}$$

where μ_a is the linear absorption coefficient of the bulk medium. For spherical cavities, we may solve the inequality (7) for the radius of gyration, first inserting the relation $V\mathfrak{N}\leqslant10^{-3}$. We find as a necessary condition that appreciable multiple scattering occur,

$$\rho>2\times10^4\tau_a\Delta/Z\lambda^2, \tag{8}$$

where τ_a is the mass absorption coefficient, Δ the atomic volume, and Z the atomic number of the bulk medium. In obtaining Eq. (8), we have taken the atomic weight as approximately twice the atomic number. All quantities in Eq. (8) are expressed in cgs units. It is clear that Eq. (8) cannot be satisfied and that the multiple scattering method cannot practicably be applied to cold-worked specimens by means of x-rays.

For neutron scattering the linear scattering coefficient is given by Eq. (6) with $n_e'-n_e$ replaced by $n_a'-n_a$ and with r_0^2 replaced by $\sigma_s/4\pi$, the coherent scattering cross section per unit solid angle for neutrons of wavelength λ. With $n_a'-n_a$ of the order 10^{23} cm^{-3} and σ_s of the order one barn, the condition for negligible multiple scattering becomes

$$t<2\times10^4 \text{ cm},$$

a condition that indicates the experimental impracticability of a multiple scattering measurement with neutrons.

III. SCATTERING FROM DISLOCATIONS

In this section we shall compute, on the basis of certain approximations, the coherent, diffuse, small angle scattering to be expected from dislocations in a heavily cold-worked crystal. Since the separation of dislocations in heavily cold-worked metals is believed to be of the order of 100A,[2] it perhaps seems impossible at first sight to attempt to explain with dislocations the scattering observed by Blin and Guinier[5] which was characteristic of a linear dimension of 17A (for spherical regions). Nevertheless, the peculiar distribution function for electrons surrounding a dislocation [see Eq. (9)] would be expected to reduce the "effective" characteristic length for the distorted region about a dislocation. It will be seen below that this is indeed the case but that the reduction is insufficient to explain the observed data.

We shall ignore the effects of vacancies in this section; we shall likewise ignore the scattering from the screw-type component of the dislocations, since to a first approximation, there is no electronic density change associated with this type of imperfection.[8] It is assumed that the dislocation lines do not remain straight for macroscopic distances, but rather are frequently bent, that the principal component changes from screw to edge type over distances of the order of the mean spacing of the dislocations, and that only small coherence occurs in the scattering from regions surround-

[7] D. L. Dexter and W. W. Beeman, Phys. Rev. **76**, 1782 (1949).

[8] A. J. C. Wilson, Research **2**, 541 (1949); **3**, 387 (1950).

ing neighboring edge-type sections. This assumption of random phase from one such region to the next certainly is not exactly valid; quantitatively correct results can perhaps be obtained by the calculation of corrections for coherence, when more is known about the detailed geometry of interacting dislocations. (Such corrections would be expected to reduce the scattering at very small scattering angles, as in the scattering from concentrated solutions. As will be seen below, however, the scattering has a minimum in the forward direction anyway, so it seems certain that interference corrections will not change the qualitative aspects of the scattering curves.) A further approximation, which is closely related to the others, is the neglect of end effects at the termination of an edge-type component of the dislocation line. Now, according to the theory of elasticity for a homogeneous isotropic medium, the density change at a point in a lattice is equal to[9]

$$\Delta n = n_e\left(\frac{1-2\nu}{1-\nu}\right)\frac{a}{2\pi}\frac{\sin\xi}{r} \equiv \kappa\frac{\sin\xi}{r}. \tag{9}$$

In this expression ν is the Poisson ratio (approximately equal to $\frac{1}{3}$ for most materials), a is the unit slip distance, r is the distance from the point to the dislocation line (Z axis), i.e., $r=(x^2+y^2)^{\frac{1}{2}}$, and ξ is the angle from the slip direction (X axis) measured in the XY plane, i.e., $\xi=\tan^{-1}(y/x)$. (The "extra plane" of atoms, according to this notation, in the YZ plane, and XZ is the slip plane.) It will be noted that, since the density change below the slip plane is the negative of that above, there is no excess or deficit of matter in a symmetrical region region surrounding the dislocation axis. This fact has important consequences on the shape of the x-ray scattering curve.

Following the assumptions described above, we take as our model for the elementary, coherently scattering region, the lattice surrounding a length L of edge-type dislocation, out to a distance R equal to about half the average separation between dislocations. We shall arbitrarily cut off our unit scattering region by considering only that volume enclosed in a right circular cylinder of radius R and length L; the scattering we calculate with this model will be very similar to that obtained from any cylinder of length L and of symmetrical cross section, for example, a square. The matter outside the cylinder is considered to contribute to the scattering associated with neighboring dislocations, and, in accord with the assumptions discussed above, the total scattered intensity will be taken to be the sum of the intensities scattered from the individual cylinders.

The scattered intensity from one of these cylinders is given by

$$I = I_e\int\Delta n(\mathbf{r})d\tau\int\Delta n(\mathbf{r}')d\tau'\cos k(l-l'), \tag{10}$$

where $k\equiv(4\pi/\lambda)\sin(\epsilon/2)$.[10] In the usual small angle approximation, we set k equal to $2\pi\epsilon/\lambda$ throughout the following. The lengths l and l' are defined in the customary way as follows. We consider an incident x-ray AB, having a given orientation with respect to the cylinder, and consider a particular ray BC scattered through an angle ϵ. The angle bisector of the supplementary angle ABC will be referred to as L, and the angle between L and the axis of the dislocation Z will be called θ. We now define a plane Π perpendicular to L, which passes through the geometrical center of the cylinder. The quantities l and l' are the perpendicular distances from the points $\mathbf{r}$ and $\mathbf{r}'$ to Π. The value of these definitions is a result of the fact that all of the electrons on a given plane parallel to Π scatter in phase with each other. Let the projection of L on the XY plane make an angle φ with the X axis. Then the perpendicular distance from a point to Π is given by

$$l(r,\xi,z) = r\sin\theta\cos\varphi\cos\xi + r\sin\theta\sin\varphi\sin\xi + z\cos\theta, \tag{11}$$

in terms of the location of the points and the orientation of the angle bisector. Inserting Eqs. (9) and (11) into Eq. (10), we find

$$\frac{I}{\kappa^2 I_e} = \left[\int_{-L/2}^{L/2}dz\int_0^{2\pi}d\xi\sin\xi\int_0^R dr \times\cos k\{r\sin\theta(\cos\varphi\cos\xi+\sin\varphi\sin\xi)+z\cos\theta\}\right]^2 + \left[\int_{-L/2}^{L/2}dz\int_0^{2\pi}d\xi\sin\xi\int_0^R dr \times\sin k\{r\sin\theta(\cos\varphi\cos\xi+\sin\varphi\sin\xi)+z\cos\theta\}\right]^2. \tag{12}$$

In most scattering problem the density function is an even function of ξ and the second term of this sum vanishes by symmetry. In the present case, however, the reverse is true, since the presence of $\sin\xi$ in the density makes the first term vanish. Performing the integrations indicated in Eq. (12), we obtain

$$I = \frac{I_e\kappa^2 16\pi^2\sin^2(\frac{1}{2}kL\cos\theta)\sin^2\varphi}{k^4\cos^2\theta\sin^2\theta}[1-J_0(kR\sin\theta)]^2. \tag{13}$$

Let us now define an axial ratio $\alpha\equiv L/2R$ and a reduced scattering angle $\gamma\equiv 2\pi\epsilon R/\lambda = kR$. If the dislocations are distributed with random orientation in the cold-worked specimen, the average intensity scat-

[9] J. S. Koehler, Phys. Rev. **60**, 397 (1941).

[10] See, for example, A. Guinier, *Radiocristallographie* (Dunod, Paris, 1945), p. 229.

tered from each is equal to

$$\bar{I}(\gamma,\alpha)=\frac{I_e(\pi R^2 L n_e)^2}{32\pi^2\alpha^2}\left(\frac{a}{R}\right)^2\left(\frac{1-2\nu}{1-\nu}\right)^2 F(\gamma,\alpha), \quad (14)$$

where $F(\gamma,\alpha)$ is a dimensionless quantity having a maximum of about $\frac{3}{2}$:

$$F(\gamma,\alpha)=\frac{16}{\gamma}\int_0^\gamma dx\frac{\sin^2(\alpha x)}{x^2(\gamma^2-x^2)}[1-J_0((\gamma^2-x^2)^{\frac{1}{2}})]^2. \quad (15)$$

For small scattering angles, i.e., $\gamma\ll 1$, we may expand expand Eq. (15) and obtain a quadratic dependence on scattering angle,

$$\bar{I}=\frac{I_e}{48\pi^2}\left(\frac{1-2\nu}{1-\nu}\right)^2(\pi R^2 L n_e)^2\left(\frac{a}{R}\right)^2 \times\{\gamma^2-\tfrac{1}{10}\gamma^4(1+\tfrac{2}{3}\alpha^2)+\cdots\}. \quad (16)$$

This result is in striking contrast with the shape of the usual small angle scattering curves, which have a maximum in the forward direction. The absence of scattering at very small angles in the present case is a result of the fact that no net material has been added to the lattice, but the material has merely been displaced from one side of the slip plane to the other. Thus at very small angles, where all the electrons scatter approximately in phase, the scattering takes no account of inhomogeneities in the sample.[11]

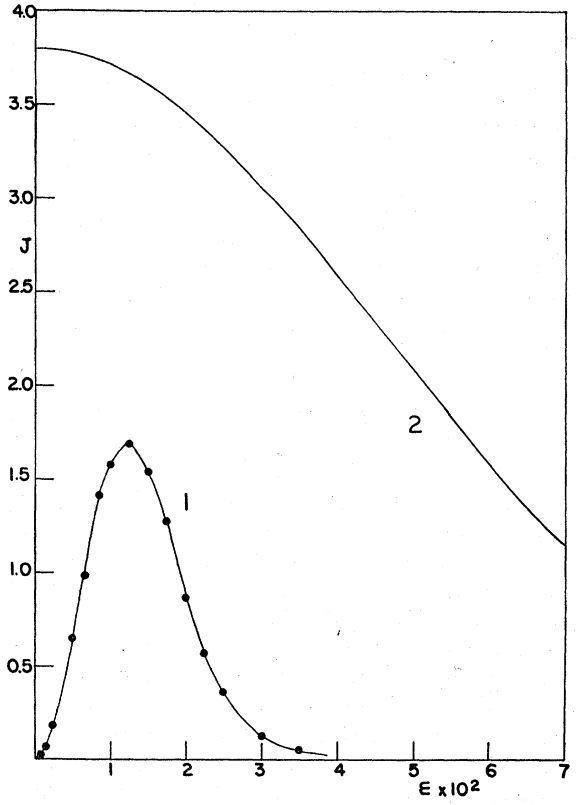

FIG. 1. Curve 1 represents the intensity scattered from an edge-type dislocation of $2R=L=98$A, as a function of scattering angle ϵ. Curve 2 is proportional to the scattered intensity from the cavities of radius of gyration 6.6A present in a volume $\pi R^2 L$, if the concentration of cavities is 7×10^{17} cm^{-3}. For both curves $\lambda=1.54$A. The ordinate scale for both curves is equal to the scattered intensity times 10^{42} $(n_e^2 I_e)^{-1}$.

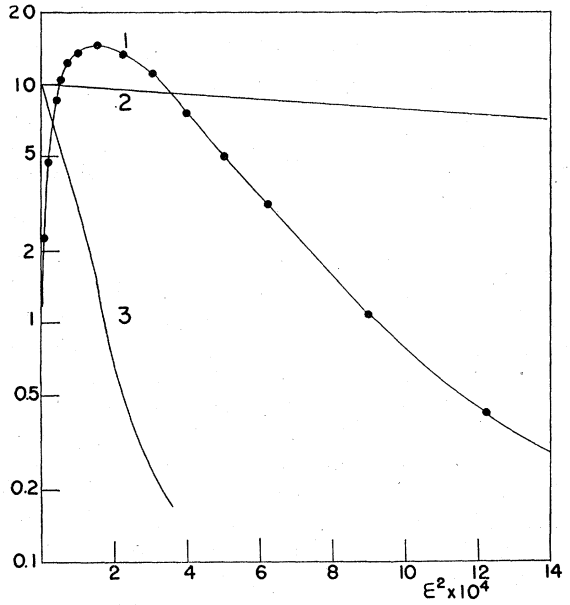

FIG. 2. These three curves represent the shape of the familiar plots of log intensity *versus* angle squared. Curve 1 is proportional to curve 1 of Fig. 1 for the scattering by dislocations; curve 2 is proportional to curve 2 of Fig. 1 for spherical cavities; and curve 3 is for solid cylinders of the same dimensions as those of the cylinders taken to approximate the effects of dislocations. The relative ordinates for the three curves are of no significance.

It is interesting to compare Eq. (16) with the intensity scattered in vacuum from a solid cylinder of the same dimensions and of electron density n_e:

$$\bar{I}=I_e(\pi R^2 L n_e)^2\{1-\tfrac{1}{6}\gamma^2(1+\tfrac{2}{3}\alpha^2)+\cdots\}. \quad (17)$$

Since in both Eqs. (16) and (17) the quantities in curly brackets have maximum values of the order unity, we see that the ratio of the peak scattered intensities is of the order one to one million for $\alpha=1$ and R equal to 50A, i.e., for dislocation concentrations of the order 10^{12} cm^{-2}.

It is also of interest to compare the absolute intensities of the scattering observed by Blin and Guinier[5] with that to be expected from edge-type dislocations. For $L=2R=100$A, $\nu\sim\frac{1}{3}$ and $a=2.55$A (for Cu), the intensity scattered from a volume $(\pi R^2 L)$ has a maxi-

[11] Professor W. W. Beeman has kindly called the attention of the writer to a paper by A. Guinier [Metallkunde **43**, 217 (1952)], who has measured the x-ray scattering from silver-rich clusters in an alloy of Al: 6 percent Ag. Since each Ag-rich cluster is surrounded by a Ag-poor shell, for this system also there is no net excess of scattering power in each scattering unit, and the scattered intensity is observed to have a minimum in the forward direction.

mum of

$$\bar{I}_{\max}\sim 1.7\times 10^{-42} n_e^2 I_e. \tag{18}$$

On the other hand, from Eq. (1), the absolute intensity scattered from the cavities (of radius of gyration 6.6A and of concentration 7×10^{17} cm^{-3}) contained within this volume would be equal to about twice this value,

$$I_{\max}\sim 3.8\times 10^{-42} n_e^2 I_e. \tag{19}$$

However, the total scattered intensity is much smaller for the dislocations than that observed, as indicated in Fig. 1. The upper curve of Fig. 1 corresponds to Eq. (19), the scattering from 7×10^{17} spherical cavities per cm^3, of radius of gyration 6.6A, as estimated by Blin and Guinier with Cu $K\alpha$ radiation. On the same absolute intensity scale, the lower curve corresponds to a numerical integration of Eqs. (14) and (15) for axial ratio equal to unity. To fit the absolute abscissa scale of Fig. 1, λ is set equal to 1.54A and R equal to 49A, so that γ becomes 200ϵ. The quantity plotted vertically in Fig. 1 is the absolute intensity scattered from a sample of volume $(\pi R^2 L)$, multiplied by $10^{42}(n_e^2 I_e)^{-1}$.

Figure 2 shows the customary curves of the log of the intensity *versus* the square of the scattering angle for (1) dislocations of $2R=L=98$A, (2) spherical cavities of radius of gyration 6.6A, and (3) solid cylinders of $2R=L=98$A. The relative heights of the three curves are of no significance. It will be noticed that curve (1) is accurately represented by a straight line for more than a full cycle on the log scale and that, if experimental data were lacking for angles smaller than that of the maximum, the data would ordinarily be interpreted as representing the scattering from homogeneous regions of radius of gyration $0.542R$, or 26.5A. Note also that this "effective" radius of gyration is 41 percent smaller than that for the solid cylinder of the same dimensions, 44.8A.

It will also be observed that dislocations of this size clearly cannot be responsible for the scattering observed[5] by Blin and Guinier. Furthermore, if one should attempt to interpret their data on the angular variation on the basis of suitably reduced cylinders of this type, namely, $2R=L=24.4$A, the absolute scattered intensity would be too small by a factor of 8.

If the scattering observed by Blin and Guinier should indeed be a result of clustered vacancies, as they tentatively suggest, it may be possible to observe the scattering from dislocations by cold-working and measuring the scattering at liquid nitrogen temperatures, where isolated vacancies are not sufficiently mobile to cluster. The extremely characteristic minimum in the scattered intensity at zero scattering angle should enable the effects of dislocations to be observed even above a background of diffuse scattering which is due to a large number of isolated vacancies. The expected counting rates ($\gtrsim 3$ sec^{-1}) with apparatus similar to that in use by Beeman and co-workers,[4] should be high enough above room background (~ 0.5 sec^{-1}) to make the experiment feasible. It is possible, however, that some geometrical mechanism not as yet understood favors the production of high concentrations of vacancies in localized regions such that clustering is possible even at extremely low temperatures. If such is the case, it may prove impossible sufficiently to reduce the background scattering due to vacancies to isolate the effects of dislocations. Certainly the variation with temperature of the scattering curves for samples cold-worked at nitrogen temperatures should be most interesting.

From the discussion given in Sec. II it is clear that multiple scattering can have an appreciable influence on the scattering from dislocations. The scattering coefficients for solid cylinders is more than a million times larger than that for our model of the dislocation [see Eqs. (16) and (17)] and even a solid cylinder would have to be hundreds of angstroms in size for multiple scattering to become experimentally significant.[7]

Another point of interest in the scattering from dislocations is the large degree of anistropy in the scattering, as can be seen from Eq. (13). For example, if x-rays are incident along the dislocation axis (Z axis), the scattered intensity in the slip plane (XZ plane) is zero, but that in the plane of extra atoms (YZ plane) is relatively large. On the other hand, the scattered intensity in either the YZ plane or the YX plane from x-rays incident along the Y axis is of the order $\epsilon^2/4$ times the latter. If we let the first subscript denote the direction of incidence of the x-ray beam and the two subscripts together indicate the plane of scattering, the anisotropy relations may be summarized as follows, for very small angles:

$$\begin{aligned} &I_{ZX}=I_{XZ}\equiv 0,\\ &I_{ZY}=I_{XY}=\tfrac{1}{4}\kappa^2\pi^2 I_e L^2 R^4 k^2,\\ &I_{YZ}=I_{YX}=\tfrac{1}{4}\epsilon^2 I_{ZY}. \end{aligned} \tag{20}$$

The anistropy of the x-ray scattering may prove useful in the investigation of kinking phenomena, where dislocations may be oriented primarily in one direction.

Another possible imperfection of interest is the collapsed vacancy disk, similar in some respects to an edge dislocation and in others to a flat cylinder of vacancies of the type discussed in Sec. II. Since there is a net absence of matter in the vicinity of the disk, we would expect that the scattering would be a maximum in the forward direction and that by a suitable choice of size and concentration of the collapsed disks, it should be possible to fit Blin and Guinier's results.