Spins and Parities of Excited States in Even-Even Nuclei. II

IGAL TALMI
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received April 9, 1953)

RECENTLY[1] evidence has been brought in favor of the rule that in even-even nuclei the low-lying levels of (excited) configurations with odd parity have odd spins, whereas the low-lying levels of configurations with even parity (a special case of which is the ground state configuration) have even spins. Such a rule can be explained theoretically by considering short-range attractive (central) forces between the nucleons, in the same way that on the basis of such a force the order of levels in the j^n configuration (in jj coupling) or the l^n configuration (in LS coupling) is predicted to be 0, 2, 4, $\cdots$.

In order to see which are the low-lying levels of excited configurations in even-even nuclei, let us consider the simple case of the l_1l_2 configuration of two protons (or two neutrons). In this configuration the energies of the triplet and singlet states are given as follows:[2]

$$\begin{aligned}E(l_1l_2\,{}^3L)&=(l_1l_2LM|V(r_{12})|l_1l_2LM)\\&\quad-(-1)^{l_1+l_2-L}(l_1l_2LM|V(r_{12})|l_2l_1LM),\\E(l_1l_2\,{}^1L)&=(l_1l_2LM|V(r_{12})|l_1l_2LM)\\&\quad+(-1)^{l_1+l_2-L}(l_1l_2LM|V(r_{12})|l_2l_1LM).\end{aligned}\tag{1}$$

In the case of the δ-potential (extreme short-range force) the interaction energy vanishes in triplet estates (as the wave function is antisymmetric in the space coordinates of the two nucleons). In this case, for $(-1)^{l_1+l_2-L}=+1$ the singlet state energy is $E(l_1l_2\,{}^1L)=2(l_1l_2LM|V(r_{12})|l_1l_2LM)$; for $(-1)^{l_1+l_2-L}=-1$, $(l_1l_2LM|V(r_{12})|l_1l_2LM)=0$ and the singlet state energy $E(l_1l_2\,{}^1L)$ vanishes *a fortiori*. (The direct and exchange integrals are equal in the δ-limit.) In the first case one obtains, using Racah's methods,

$$\begin{aligned}(l_1l_2LM|V(r_{12})|l_1l_2LM)&=\textstyle\sum_k F^k(l_1l_2LM|(\mathbf{C}_1^{(k)}\cdot\mathbf{C}_2^{(k)})|l_1l_2LM)\\&=(-1)^{l_1+l_2-L}\textstyle\sum_k F^k(l_1\|C^{(k)}\|l_1)(l_2\|C^{(k)}\|l_2)W(l_1l_2l_1l_2;Lk)\\&=(-1)^{l_1+l_2}(2l_1+1)(2l_2+1)\textstyle\sum_k F^kV(l_1l_1k;000)\\&\qquad\times V(l_2l_2k;000)W(l_1l_2l_1l_2;Lk).\end{aligned}\tag{2}$$

In the δ-limit $F^k=(2k+1)F^0$; inserting this in (2) and using a well-known formula of tensor algebra, we obtain

$$\begin{aligned}&(l_1l_2LM|V(r_{12})|l_1l_2LM)\\&=(-1)^{l_1+l_2}(2l_1+1)(2l_2+1)F^0\textstyle\sum_k(2k+1)\\&\qquad\times W(l_1l_2l_1l_2;Lk)V(l_1l_1k;000)V(l_2l_2k;000)\\&=(2l_1+1)(2l_2+1)F^0V(l_1l_2L;000)V(l_1l_2L;000)\\&\qquad=(2l_1+1)(2l_2+1)\tfrac{1}{2}C_{l_1l_2L}F^0.\end{aligned}\tag{3}$$

The result is*

$$E(l_1l_2\,{}^1L)=(2l_1+1)(2l_2+1)C_{l_1l_2L}F^0,\tag{4}$$

where F^0 and $C_{l_1l_1L}$ are defined by Condon and Shortley[3] (for equivalent nucleons put $l_1=l_2$ and divide by two). Therefore the only states with nonvanishing energy are ${}^1L$ for which $(-1)^{l_1+l_2-L}=+1$ (among these the lowest occurs usually for minimum L). The L's of these low-lying levels (for which $J=L$) are odd if l_1+l_2 is odd, i.e., in odd-parity configurations and are even if l_1+l_2 is even, i.e., in even-parity configurations. This order of levels is the same also in other cases, particularly in the configuration $l_1^{-1}l_2$; it is not expected to be appreciably changed if we consider also the interaction with the other, nonexcited, group.

The situation in jj coupling is essentially the same. In the configuration j_1j_2 of two protons (or two neutrons) the energy of a state with spin J is given in terms of the LS coupling term energies as follows

$$\begin{aligned}E(l_1l_2j_1j_2J)&=a_1E[l_1l_2\,{}^3(J+1)]+a_2E(l_1l_2\,{}^3J)\\&\quad+a_3E[l_1l_2\,{}^3(J-1)]+a_4E(l_1l_2\,{}^1J),\end{aligned}\tag{5}$$

where the a_i are positive and are simple functions of l_1l_2, j_1j_2, and J.[4] In the case of the δ-potential the only nonvanishing energies are $E(l_1l_2\,{}^1J)$ for which $(-1)^{l_1+l_2-J}=+1$ (L is equal to J in this case) and only for such J's is $E(j_1j_2J)$ different from zero. Therefore, the J values of the low-lying levels are odd if l_1+l_2 is odd, i.e., in odd-parity configurations, and are even if l_1+l_2 is even, i.e., in even-parity configurations (note that this result is independent of the relative orientation of $\mathbf{s}$ and $\mathbf{l}$ in j_1 and j_2).

The energies of these states are obtained by inserting the values of a_4 in (5) as follows:

$$\begin{aligned}&j_1=l_1+\tfrac{1}{2},\ j_2=l_2+\tfrac{1}{2},\\&\qquad E(j_1j_2J)=(j_1+j_2+J+1)(j_1+j_2-J)\tfrac{1}{2}C_{l_1l_2J}F^0;\\&j_1=l_1-\tfrac{1}{2},\ j_2=l_2-\tfrac{1}{2},\\&\qquad E(j_1j_2J)=(j_1+j_2+J+2)(j_1+j_2-J+1)\tfrac{1}{2}C_{l_1l_2J}F^0;\\&j_1=l_1\pm\tfrac{1}{2},\ j_2=l_2\mp\tfrac{1}{2},\\&\qquad E(j_1j_2J)=(J\mp j_1\pm j_2+1)(J\pm j_1\mp j_2)\tfrac{1}{2}C_{l_1l_2J}F^0.\end{aligned}\tag{6}$$

For equivalent nucleons ($l_1=l_2$, $j_1=j_2$) the first two expressions divided by two give the energies of the j^2 configuration. If we want to find out the order of levels among those with nonvanishing energy (which have the proper odd or even character) we can use these expressions, although this should not be expected to be as accurate as the general relation between the spin and the parity of the low-lying states. The lowest state will have the minimum J for $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\pm\frac{1}{2}$, and this is simply $J=|j_1-j_2|$ [it is odd if $(-1)^{l_1+l_2}=-1$ and even if $(-1)^{l_1+l_2}=+1$]. In the case $j_1=l_1\pm\frac{1}{2}$, $j_2=l_2\mp\frac{1}{2}$, the lowest state will have the maximum spin, namely $J=j_1+j_2$ [also it satisfies $(-1)^{l_1+l_2-J}=+1$]. These rules are in a sense the opposite of Nordheim rules concerning the coupling of unlike nucleons (in that case the coupling has to produce maximum triplet component but in the case considered here the coupling produces maximum singlet component as the triplet states have space-antisymmetric wave functions). This order of levels holds also in other cases and is not expected to change appreciably if the interaction with the nonexcited group is also considered. One could choose the probable configurations in all the cases of Table I in reference 1, so that the resulting spins will conform to these more definite coupling rules.

* It has been pointed out to the author that this formula is found in a paper by M. H. L. Pryce in Proc. Phys. Soc. (London) **65A**, 773 (1952). Also A. de-Shalit (private communication) has independently obtained equivalent results.
[1] M. J. Glaubman, (preceding letter), Phys. Rev. **90**, 1000 (1953).
[2] G. Racah, Phys. Rev. **61**, 186 (1941).
[3] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951), pp. 177 and 182, respectively.
[4] G. Racah, Physica **16**, 651 (1950).

Odd-Odd Spins and j-j Coupling*

R. W. KING, *Nuclear Development Associates White Plains, New York*
AND
D. C. PEASLEE, *Columbia University, New York, New York*
(Received April 15, 1953)

ODD-ODD nuclei with neutrons and protons filling equivalent shells have the unique feature of providing evidence both for and against the j-j coupling scheme in nuclei. When the spins of these nuclei are plotted against A, they show a regularly repeating pattern with a periodicity that exactly matches the ranges of the j-j subshells $p_{3/2}$, $d_{5/2}$, $d_{3/2}$, and $f_{7/2}$. At the same time, the pattern entirely fails to show the symmetry within a given subshell that would be demanded by perfect j-j coupling. If j-j coupling were ideally followed, the pattern of odd-odd spins should be entirely symmetric with respect to holes and particles within the subshells; the empirical pattern, however, is that the spins tend to increase continuously throughout the filling of each subshell.

This is illustrated in Fig. 1, where the spins of odd-odd nuclei having neutrons and protons in equivalent shells are plotted against the total number of particles in the subshell. The nuclei symmetric with respect to the half-filled shell have the same

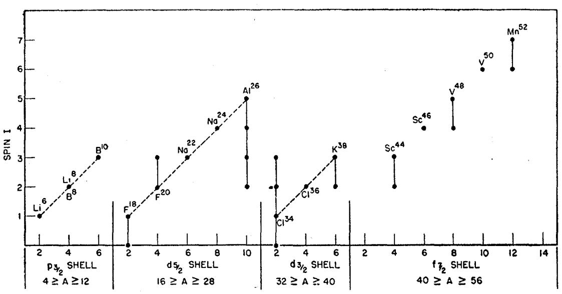

FIG. 1. Spins of ground states of odd-odd nuclei with neutrons and protons filling equivalent shells. The abscissa is the number of nucleons in the given subshell. The spin assignments are derived from the survey of β-decay data in R. W. King, Thesis, Washington University, 1952 (unpublished). Several points for the same isotope indicate possible spin values.

value of isotopic spin T; they should therefore have the same real spins I, if the equivalence of holes and particles is strictly according to the j-j scheme. The surprising fact shown by Fig. 1 is that the holes tend toward higher I values than the equivalent particles. Within the limits of present experimental uncertainties, the variation of the ground-state spin I appears roughly linear with the number of particles in the subshell.

If the variation of spin with the number of particles in a subshell is truly linear, the spins of a number of the above nuclei would be predictable. In particular, the following spins would be expected: F^{18}, $I=1$; F^{20}, $I=2$; Al^{26}, $I=5$; Cl^{34}, $I=1$; K^{38}, $I=3$.

More detailed experimental information is needed to establish the ground-state spins of many of the above-mentioned nuclei, but in some cases even qualitative information will help. For example, if Cl^{34} can be shown to undergo allowed β-decay to the ground state of S^{34}, the possible spin values for Cl^{34} would be restricted to $I=0, 1$.

For odd A nuclei this increase of I with the number of particles in the subshell is much less marked, but the tendency may still be present to some degree. The $j=3/2$ subshells provide no evidence for variations of I. With $j=5/2$ the succession of spins through the shell seems to be 1/2, 3/2, 3/2, 5/2 (with the exception of nuclei having just one particle or one hole, for which $I=j$), and the several possibilities of the $j=7/2$ subshell cannot now be surveyed for lack of data.

* Partially supported (D.C.P.) by the research program of the U. S. Atomic Energy Commission.

Fermi's Theory of Nucleon Collisions and the Zero-Point Energy of Pions

F. C. AULUCK AND D. S. KOTHARI
University of Delhi, Delhi, India
(Received April 8, 1953)

FERMI,[1] on the basis of statistical thermodynamics, has recently given a theory to account for the production of pions (and also nucleons) in collisions of high energy nucleons. The theory, and particularly the predicted distribution, of angular momentum of the pions is in rather striking agreement with experiment. The fundamental assumption of Fermi's theory is that the pions (and also nucleons) are produced in a small volume τ which is given by

$$\tau=(4\pi/3)R^3(2Mc^2/W), \tag{1}$$

where $R=\xi\hbar/\mu c$, ξ being a dimensionless number of order unity, and μ is the pion rest-mass. The factor $2Mc^2/W$ arises on account of Lorentz contraction: W is the energy of the colliding nucleons in the system in which their center of gravity is at rest (M is the nucleon rest-mass). If W' denotes the energy in the laboratory system of the incident high energy nucleon, we have

$$W'=\tfrac{1}{2}Mc^2\{(W/Mc^2)^2-2\}. \tag{2}$$

The purpose of this note is to take into account the effect of the zero-point energy of the pions.[2] This has not been done in Fermi's treatment. Because of the reduction on account of Lorentz contraction of the volume in which the pions are produced, the zero-point energy (as shown below), becomes comparable to the energy of the colliding nucleons. This results in a substantial reduction in the number of pions emitted in the process, and further, the number of sufficiently large energies of collision, becomes independent of the collision energy.

Following Fermi, if N particles are present in the final state, then the probability $P(N)$ of the final state is

$$P(N)=k(\tau/h^3)^{N-1}(dQ_{N-1}/dW), \tag{3}$$

where k is a constant and $Q_{N-1}(W)$ denotes the volume of the momentum space which contains all those states for which the energy is equal to or less than W. Assuming that there are n relativistic pions and S nonrelativistic nucleons, we write (following Fermi)

$$W=\frac{1}{2M}\left\{\sum_{i=1}^{S-1}P_i^2+\left(-\sum_{i=1}^{S-1}P_i\right)^2\right\}+SMc^2+\sum_{i=1}^{n}c|p_i|. \tag{4}$$

Let ϵ_0 be the zero-point energy of a pion. The volume of the momentum space bounded by the surface defined by Eq. (4) and

$$c|p_i|\geq\epsilon_0, \quad (i=1, 2, \cdots n)$$

is given by

$$\frac{(2\pi M)^{3(S-1)/2}(4\pi)^n}{S^{\frac{3}{2}}c^{3n}}\sum_{r=0}^{2n}\frac{a_r\epsilon_0^{2n-r}}{\Gamma(r+3S/2+n-\frac{1}{2})}\times(W-SMc^2-n\epsilon_0)^{n+r+3(S-1)/2}. \tag{5}$$

The constants a_r are given by

$$(\kappa^2+2\kappa+2)^n=\sum_{r=0}^{2n}a_r\kappa^{2n-r}. \tag{6}$$

Thus, the probability that there are n pions and S nucleons in the final state becomes

$$P(S, n)=\frac{k}{Mc^2}\frac{(\pi/2)^{(S-1)/2}}{S^{\frac{3}{2}}w^{n+S-1}}\left(\frac{4\xi^3}{3\pi}\frac{M^3}{\mu^3}\right)^{n+S-1}\times\left(w-S-\frac{n\epsilon_0}{Mc^2}\right)^{n+3S/2-5/2}\sum_{r=0}^{2n}\frac{a_r}{\Gamma(n+3(S-1)/2+r)}\times\left(w-S-\frac{n\epsilon_0}{Mc^2}\right)^r\left(\frac{\epsilon_0}{Mc^2}\right)^{2n-r}, \tag{7}$$

where $w=W/Mc^2$.

It has been shown by Fermi that for moderate energies, the probability of there being more than two nucleons (and there are the two initial nucleons) in the final state is very small. In the sequel we shall therefore confine ourselves to the case of $S=2$. For the zero-point energy ϵ_0, we take

$$\epsilon_0=hc/2R=W(\pi/2\xi)(\mu/M). \tag{8}$$

The maximum number of pions n is therefore given by

$$w\geq 2+(n\pi/2\xi)w(\mu/M),$$

or

$$n\leq\frac{w-2}{w}\frac{2\xi}{\pi}\frac{M}{\mu}<\frac{2\xi}{\pi}\frac{M}{\mu}.$$

Thus we have the relation that $n<4.3\xi$ always.

Tables I and II give the relative values of $Mc^2P(2, n)/k$ for $\xi=1$ and $\xi=2$, respectively. The last column gives the average number of pions emitted.

TABLE I. Relative probability of emission of n pions in a collision of two nucleons for $\xi=1$: wMc^2 is the energy of the colliding particles in the rest system; $\bar{n}$ is the average number of pions emitted.

$w \backslash n$	0	1	2	3	$\bar{n}$
3	25	75			0.75
5	1	40	59		1.59
10		2	74	24	2.22