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The Hyperfine Structure of the $3p\ ^2P_{\frac{1}{2}}$ State of Al^{27}†

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The hyperfine structure interval of the $3p\ ^2P_{\frac{1}{2}}$ state of Al^{27} has been measured and found to be 1506.14 $\pm$0.05 Mc/sec. From this datum and the known hyperfine structure of the $3p\ ^2P_{\frac{3}{2}}$ state a recalculation of the nuclear electric quadrupole moment, taking into consideration configuration interaction, yields the value $Q=+0.149\times10^{-24}$ cm^2.

I. INTRODUCTION

PREVIOUS work by Jackson and Kuhn[1] and by Lew[2] on the $3p\ ^2P_{\frac{1}{2}}$ state of Al^{27} yielded values for the hyperfine separation $\Delta\nu$ of 1440$\pm$30 Mc/sec and 1500$\pm$50 Mc/sec, respectively. Although the latter value was obtained by the atomic beam magnetic resonance method, it was calculated by means of the Breit-Rabi formula from only rough measurements of the so-called low frequency line ($F=3$, $m_F=-2\rightarrow F=3$, $m_F=-3$). Attempts to observe high frequency lines, i.e., those between $F=3$ and $F=2$, were unsuccessful at that time because of difficulties with the radiofrequency oscillator in the region of 1500 Mc/sec. It has been considered worth while to make another attempt to get an accurate value for $\Delta\nu$ by the magnetic resonance method both for the sake of completeness and for the sake of permitting the calculation of a more accurate value of the nuclear electric quadrupole moment. In a recent paper, Koster[3] has shown that in gallium and, by extension, in the other elements of the same column of the periodic table, the hyperfine structure of the $^2P_{\frac{1}{2}}$ state is less perturbed by higher configurations than that of the $^2P_{\frac{3}{2}}$ state. Consequently, a more accurate value of $\langle r^{-3}\rangle$, necessary for the calculation of the nuclear quadrupole moment, is obtainable from the magnetic dipole interaction constant of the $^2P_{\frac{1}{2}}$ state than from that of the $^2P_{\frac{3}{2}}$ state.

II. APPARATUS

The atomic beam apparatus used in this experiment[4] is very similar to the one described by Davis, Nagle, and Zacharias.[5] The two inhomogeneous deflecting magnets have their gradients parallel to each other, and the ribbon which carries the radiofrequency power is oriented so that the magnetic component of the rf field has an appreciable strength only in the direction perpendicular to that of the static field. The method of generation and detection of the aluminum beam is identical with that described by Lew[2] in connection with the measurement of the metastable state of Al, except that a thorium oxide crucible is used instead of one of aluminum oxide. The thorium oxide crucible has been found to have a much longer useful life. The rf power required for the transitions is supplied by a power oscillator using a 2C39 tube with coaxial cavity tuning.[6] Frequencies are measured with a General Radio Type 720-A Heterodyne Frequency Meter in conjunction with a General Radio Type 1110-A Interpolating Frequency Standard. The accuracy of the frequency measuring system is about 1 part in 10^5.

III. RESULTS

The energy levels of an Al^{27} atom with spin 5/2 in the $^2P_{\frac{1}{2}}$ state in the presence of an external field are shown in Fig. 1. All the transitions that are allowed by the selection rules and by the geometrical arrangement

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[1] D. A. Jackson and H. Kuhn, Proc. Roy. Soc. (London) **A164**, 48 (1938).
[2] H. Lew, Phys. Rev. **76**, 1086 (1949).
[3] G. F. Koster, Phys. Rev. **86**, 148 (1952).
[4] A description of the apparatus will be given in a forthcoming paper by Lew.
[5] Davis, Nagle, and Zacharias, Phys. Rev. **76**, 1068 (1949).
[6] Airborne Instruments Laboratory, Inc., Mineola, New York, Type 124-A Power Oscillator.

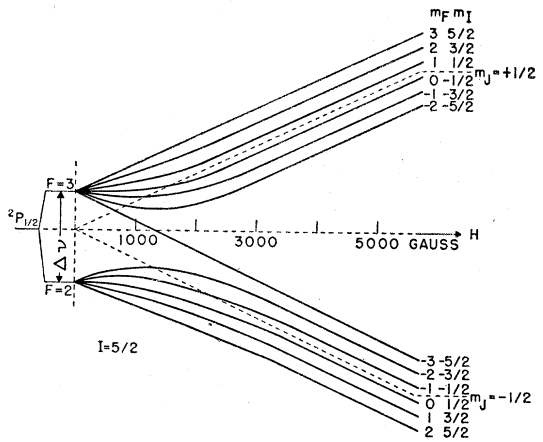

FIG. 1. Energy levels of Al^{27} in the $3p\ ^2P_{\frac{1}{2}}$ state in an external magnetic field.

of the magnetic fields have been observed and are shown in Fig. 2. The observed frequencies have been plotted as a function of the external magnetic field. On extrapolating the observations to zero field, we find for the hyperfine separation:

$$\Delta\nu(F=3-F=2)=3a_{\frac{1}{2}}=1506.14\pm0.05 \text{ Mc/sec},$$
$$a_{\frac{1}{2}}=502.05\pm0.02 \text{ Mc/sec}.$$

IV. CALCULATION OF THE NUCLEAR ELECTRIC QUADRUPOLE MOMENT

In calculating the nuclear electric quadrupole moment of Al^{27}, the primary observed quantity used is the electric quadrupole interaction constant b found by Lew[2] in his study of the $^2P_{\frac{3}{2}}$ state,

$$b=18.76\pm0.25 \text{ Mc/sec}.$$

In addition it is necessary to know the quantity $\langle r^{-3}\rangle$ which is the average of the separation between the $3p$ electron and the nucleus calculated for the state $m_J=J$. For hyperfine structure arising from a single p electron, Casimir's relation between the nuclear quadrupole moment and the interaction constant is

$$Q=\frac{5hb}{2e^2\langle r^{-3}\rangle R}, \tag{1}$$

where R is a relativistic correction factor. The quantity $\langle r^{-3}\rangle$ may be evaluated from the magnetic dipole interaction constants $a_{\frac{1}{2}}$ or $a_{\frac{3}{2}}$ of the $^2P_{\frac{1}{2}}$ or $^2P_{\frac{3}{2}}$ states using the well-known relation

$$a_J=-\frac{\mu_0{}^2g_I}{h}\frac{2L(L+1)}{J(J+1)}\langle r^{-3}\rangle F_J, \tag{2}$$

where F_J is another relativity correction factor. Using his observed value of $a_{\frac{3}{2}}=94.25$ Mc/sec, Lew found for the quadrupole moment the value $Q=0.155\times10^{-24}$ cm² (with relativity correction). If we use the $a_{\frac{1}{2}}$ found in the present work, we get $Q=0.147\times10^{-24}$ cm². It is obvious that the disagreement between these values of Q arises from the disagreement between the observed ratio $(a_{\frac{1}{2}}/a_{\frac{3}{2}})$ and the ratio as required by Eq. (2). Experimentally we find

$$(a_{\frac{1}{2}}/a_{\frac{3}{2}})=(502.05/94.25)=5.327, \tag{3a}$$

while from Eq. (2) we should have

$$(a_{\frac{1}{2}}'/a_{\frac{3}{2}}')=5(F_{\frac{1}{2}}/F_{\frac{3}{2}})=5.049, \tag{3b}$$

where we have used relativistic correction factors corresponding to $Z_i=Z-3=10$. As in the cases of Tl and Ga,[3,7] the disagreement is undoubtedly due to a perturbation of the ground configuration by higher configurations. In the case of Ga, Koster has shown that the perturbing configuration is mainly $4s4p5s$. It is to be expected that in Al the main perturbing configuration is correspondingly $3s3p4s$. Since the form of Koster's relations should not depend on the principal quantum number n, his Eq. (16) should hold for Al, but with values for his parameters α_0, α_1, α_2, $s(0)$, $\sigma(0)$ appropriate to Al. In simplified form, these relations may be written

$$a_{\frac{1}{2}}=a_{\frac{1}{2}}'+\epsilon, \quad a_{\frac{3}{2}}=a_{\frac{3}{2}}'-(1-\gamma)\epsilon, \tag{4}$$

where the primed quantities are the magnetic dipole interaction constants in the absence of configuration interaction, involving $\langle r^{-3}\rangle$ for the p electron[8] in accordance with Eq. (2). ϵ and $(1-\gamma)\epsilon$ are correction terms which take into consideration the degree of admixture between configurations as well as the coupling of the $3s3p4s$ states with the nucleus. In Ga, where the observed ratio $(a_{\frac{1}{2}}/a_{\frac{3}{2}})$ exceeds by 30 percent the ratio

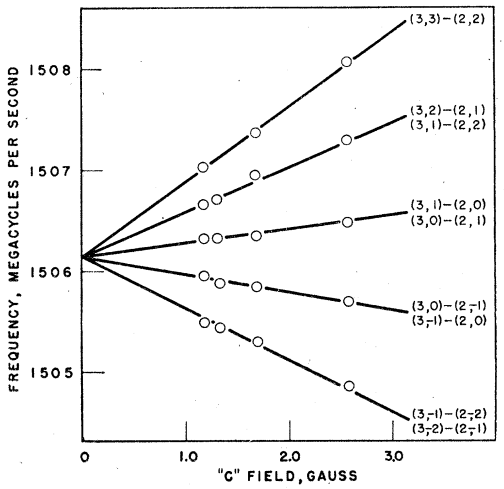

FIG. 2. Observed transitions between levels $F=3$ and $F=2$ of the $3p\ ^2P_{\frac{1}{2}}$ state of Al^{27}.

[7] E. Fermi and E. Segrè, Z. Physik 82, 729 (1933).

[8] It may be noted from Koster's equation (16) that $\langle r^{-3}\rangle$ is in very good approximation the same with or without configuration interaction. The difference between the constants a and a' is mainly due to the influence of the s electrons in the $3s3p4s$ state.

of 5.41 that is expected from Eq. (2), γ can be shown from Koster's figures to be less than one percent. In the case of Al, where the two ratios differ by only five percent, it can be estimated[9] that γ is much smaller than one percent.

Neglecting γ altogether one can evaluate ϵ and $a_{\frac{1}{2}}'$ from Eqs. (3a), (3b), and (4) and obtain

$$\epsilon = 4.30 \text{ Mc/sec}, \quad a_{\frac{1}{2}}' = 497.75 \text{ Mc/sec}.$$

Using Eq. (2), we get

$$\langle r^{-3} \rangle = 8.95 \times 10^{24} \text{ cm}^{-3}.$$

The quadrupole moment of Al then becomes

$$Q = (0.149 \pm 0.002) 10^{-24} \text{ cm}^2.$$

This we shall take as our final value of the quadrupole moment[10] because even if we had not neglected γ but had set it as high as ten percent, a figure which can be safely excluded, the last digit in Q would not be changed.

V. DISCUSSION OF $\langle r^{-3} \rangle$

The value of $\langle r^{-3} \rangle$ for the $3p$ electron given above is probably the most reliable value that can be determined by the various methods at our disposal both because of the fairly reliable theoretical foundations on which it is based and the fairly accurate experimental data. It is of interest to compare this value with that obtainable by other frequently used methods. In Table I are shown values for $\langle r^{-3} \rangle$ calculated in five different ways without relativistic or Sternheimer correction.

In column I is shown the value found above.

In column II, the value is that calculated by the use of Biermann's[11] eigenfunctions and the relation

$$\langle r^{-3} \rangle = \int_0^\infty \frac{1}{r^3} \psi_{3p}^2 r^2 dr \Big/ \int_0^\infty \psi_{3p}^2 r^2 dr.$$

[9] The estimate has been made using Biermann's Al eigenfunctions (reference 11) for the calculation of $\sigma(0)$ and $s(0)$ and assuming that the degree of admixture in Al is about one-fifth that in Ga.

[10] Without Sternheimer correction for induced quadrupole moment [Phys. Rev. **84**, 244 (1951); **86**, 316 (1952)]. This correction factor is 1.005 for Al.

[11] L. Biermann and H. Harting, Z. Astrophys. **22**, 81 (1943); L. Biermann and K. Lübeck, Z. Astrophys. **25**, 325 (1948).

TABLE I. Values of $\langle r^{-3} \rangle$ for the $3p$ electron of the configuration $3s^2 3p$ of Al in units of 10^{24} cm^{-3}, without relativistic or Sternheimer corrections.

I from hfs	II from wave functions	III from fine structure splitting (a) $Z_i = 10.0$	(b) $Z_i = 9.41$	(c) $Z_i = 10.84$
8.95	9.94	8.63	9.18	7.96

In columns III (a), (b), and (c) we give values for $\langle r^{-3} \rangle$ as calculated from the fine structure splitting of the ground configuration according to the relation

$$\delta = (3\mu_0^2/hc) \langle r^{-3} \rangle Z_i = 112.04 \text{ cm}^{-1},$$

in which Z_i has been evaluated in three different ways. Under (a), Z_i has been evaluated, as Koster has done, from the observed effective quantum numbers n^* and the fine structure splittings of the configurations $3s^2 np$, $n = 5, 6, 7$. Under (b) the value of Z_i has been obtained in the same way for $n = 3$ using the method of Crawford and Schawlow[12] for evaluating the Fermi-Segrè correction dn^*/dn. Under (c) the value of Z_i has been calculated by the use of Biermann's eigenfunction and potential for the $3p$ electron in the relation

$$Z_i = -\int_0^\infty \frac{1}{r} \frac{dV}{dr} \psi_{3p}^2 r^2 dr \Big/ \int_0^\infty \frac{1}{r^3} \psi_{3p}^2 r^2 dr.$$

It is interesting to note that the values of $\langle r^{-3} \rangle$ calculated from the Hartree wave functions of Biermann are only ten percent off from our value, while those using Z_i from semi-empirical formulas agree with ours remarkably well in view of the uncertainties in the evaluation of Z_i.

We are very grateful to Dr. P. Brix for many helpful discussions.

[12] M. F. Crawford and A. L. Schawlow, Phys. Rev. **76**, 1310 (1949).