X-RAY EMISSIVITY AS A FUNCTION OF CATHODE POTENTIAL.

BY DAVID L. WEBSTER.

IN the PHYSICAL REVIEW of June, 1916, I published an account of some experiments on X-rays of definite wave-lengths, characteristic and general, emitted by a rhodium target at definite potentials. One of the results of these experiments was to prove that as the potential was raised the general radiation of any given wave-length would appear first when the potential reached the value which would give a single cathode electron one quantum of that wave-length. This result was similar to that previously observed by Duane and Hunt,[1] and Hull,[2] for the case of tungsten. Furthermore, it appeared that above this potential the intensity of such rays would increase most rapidly at first and then more slowly, the graph soon becoming linear.

Evidently it should be of interest in connection with the problem of the mechanism of X-ray emission to find the emissivity of atoms struck by cathode electrons of a single velocity, as a function of that velocity. These data, however, do not give this emissivity directly, since some of electrons penetrate into the metal, and are slowed down by minor collisions before giving up the quantum of energy that appears necessary to produce any X-rays.

The ideal method of obtaining the true emissivity would be the use of an extremely thin target, which the electrons that did not lose quanta would penetrate with very little loss of speed. As such an experiment presents considerable difficulties in getting strong enough rays for spectrum measurements without melting the target, it seems desirable at present to get the information from known facts as to the loss of speed of electrons in a metal. Fortunately we are enabled to do this by the fact that the chance of any electron's losing a quantum is very small,[3] so that when the potential is increased by a small amount dV almost all the electrons are reduced to the original V without losing a quantum.

[1] Duane & Hunt, PHYS. REV., Aug., 1915.

[2] Hull, PHYS. REV., Jan., 1916.

[3] This point may be deduced from the remarkably low efficiency of an X-ray tube, as shown by Beatty's measurements. See Kaye, "X-Rays" Longmans Green & Co., 1914, p. 106; or Beatty, Proc. Roy. Soc., Nov., 1913.

After this reduction, which occurs in a very thin surface layer, they will behave as though they were striking the target from the potential V. Consequently the increment of intensity by the addition of dV is simply the intensity from this surface layer, and the problem before us is that of finding the chance of emission in one atom in terms of the thickness of this layer and the increment of intensity.

Experimental Laws and Assumptions.—A. As a starting point for this calculation I shall take the law first deduced theoretically by Sir J. J. Thomson[1] and confirmed by the experiments of Whiddington,[2] that the kinetic energy of a cathode particle at a depth x (measured along its path) is given by the formula

$$e^2V^2 - e^2V_x^2 = ax \tag{1}$$

where V is the potential on the tube, V_x the potential that would give the energy it has at the depth x, and a is a constant of the material of the target. This law does not, of course, apply to an electron which has already lost an X-ray quantum, but since such electrons form a very small percentage of the whole, it seems safe to apply it to all as a first approximation.

B. From the results of my experiments quoted above, it appears that the intensity of radiation from rhodium can be represented within the limits of experimental error by the formula

$$I(V, \nu) = k(V - Q) + l(1 - e^{-q(V-Q)}) \tag{2}$$

where $I(V, \nu)d\nu$ is the energy radiated per unit time and per unit of cathode ray current in the frequency interval $d\nu$, V the cathode ray potential, Q the quantum potential $h\nu/e$, and k, l, and q are functions of ν. For example, in the curve for "General radiation .645 to .661 Å." in Fig. 1 of the paper quoted, $kd\nu = 0.0331$ ionization unit per watt, $ld\nu = 0.055$ ionization unit per milliampere, and $q = 0.62$ kilovolt^{-1}, while in the curve for ".571 to .587 Å.," the numbers are 0.0465, 0.065 and 0.77 respectively. The ionization unit used here may be considered as a unit of rate of emission of energy, whose mechanical equivalent varies with frequency but not otherwise. The exact form of the functions k, l and q is not known, since the measured ionizations give comparisons of actual intensity only at a constant frequency and also since the range of frequencies tested was small. Doubtless k increases continuously with ν, and the l term is always comparatively small except near $V = Q$. The spectrum curves for tungsten at constant potentials

[1] "Conduction of Electricity Through Gases," 1906, p. 383.

[2] Camb. Phil. Soc., Proc., Dec., 1911; Proc. Roy. Soc., Apr., 1912.

published by Hull,[1] and Hull and Rice,[2] as well as curves for molybdenum which he has shown me indicate the same general type of law for these materials; though these curves are adapted primarily to the study of I as a function of ν rather than V and do not give any test of the details of equation (2).

Whatever may be said as to the generality of equation (2) as an exact statement, one fact made certain by the work of Duane and Hunt, and

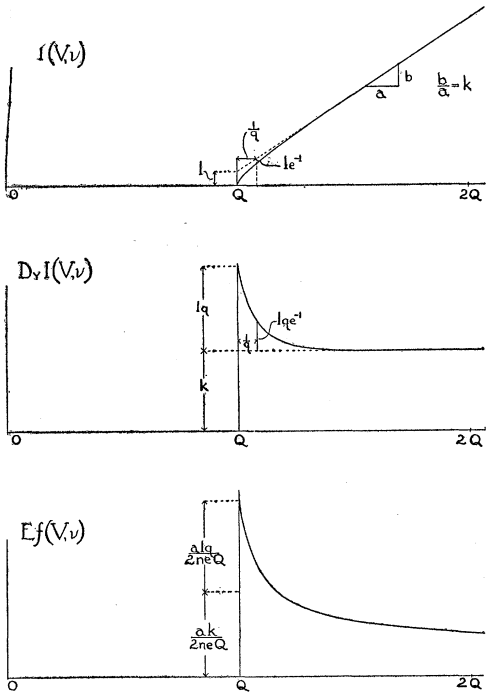

Fig. 1.

Hull, as well as my own, is that the slope of the I, V graph jumps from zero to a finite value at $V = Q$, and another is that in my experiments on rhodium at least the slope is greater at a voltage within one or two per cent. above this potential than at any higher one.[3] These facts, especially the former, are the really essential features of assumption B.

C. Since no X-rays of a given frequency are started by an electron having less than a whole quantum, the energy emitted at each start is

[1] A. W. Hull, Amer. J. of Röntgenology, Dec., 1915.

[2] Hull and Rice, Proc. Nat. Acad., Mar., 1916.

[3] The slight rounding off at the bottoms of these curves is due, as noted in the other paper, to the width of the slit and consequent lack of homogeneity of the rays. It disappears as the slit is narrowed down.

most probably equal to or greater than $h\nu$; and it is evidently equal to or less than the energy of an electron eV_x. These facts, together with the known tendency of radiant energy to become collected in quanta, as shown in the photo-electric effect, lead to the assumption C, that the energy emitted is $h\nu$ itself.

D. It will be assumed that the number of atoms penetrated by an electron in a given distance is independent of V_x and that the probability of starting X-rays of frequency ν to $\nu + d\nu$ in any one of them is $f(V_x, \nu)d\nu$. The problem is that of finding how this function depends on V_x.

Calculation of $f(V, \nu)$.—From the above assumption it is evident that if the potential on the tube is increased from V to $V + dV$, the number of atoms penetrated by an electron in losing energy down to the original V can be expressed as

$$ndx = n\frac{e^2}{a}\{(V + dV)^2 - V^2\} = \frac{2ne^2}{a} VdV \tag{3}$$

where n is the number of atoms per unit distance. Therefore if E is the energy emitted by an atom radiating with frequency ν, if excited at all by this electron, the total energy started by an average electron from these atoms with frequencies between ν and $\nu + d\nu$ is

$$\frac{2ne^2}{a} Ef(V, \nu) VdvdV. \tag{4}$$

Dividing by e one obtains the rate of emission per unit of cathode ray current, from these atoms, which, as we have pointed out above, is the increase of $I(V, \nu)d\nu$ with the increase of potential dV. Therefore

$$Ef(V, \nu) = \frac{a}{2neV} D_V I(V, \nu) = \frac{a}{2ne} \frac{k + lqe^{-q(V-Q)}}{V} \tag{5}$$

The presence of the unknown function E causes some ambiguity in this result, but as we have seen, E is most probably equal to $h\nu$. On the other side of the equation are k, l and q, functions of ν requiring further investigation. For the cases quoted above, k and lq are nearly equal, and q is such that the exponential term disappears at a potential about 20 per cent. above Q. Graphs of I, $D_V I$, and f (assuming $E = h\nu$), for wave-length 0.579 Å. are shown in the accompanying figure.

Conclusions.—From equation (5) it appears that the probability of starting a quantum of X-rays from an atom penetrated by a cathode particle is zero for all energies below the quantum value, at which it rises discontinuously to a maximum. A further increase of energy of the cathode particle decreases this probability. One might expect by analogy with collisions such as those to which Thomson's law (1) relates,

that the chance of losing energy would be less at very high velocities than at low ones, as indicated by (5). Nevertheless the present case is different, in that (5) represents not an average loss of energy to an atom, but a probability of starting a definite frequency of radiation that is emitted only in definite quanta.

In ordinary mechanical systems, and in the systems assumed in dispersion theory, the frequency of oscillation is determined entirely by the constants of the oscillator. It is therefore reasonable to expect that the frequency of the X-ray oscillators must be determined similarly, and that the existence of the short wave limit of the constant potential spectrum means that for some unknown reason no oscillation can occur unless the cathode particle imparts *at least* a quantum of energy to the oscillator. That is, the collision must be within certain limits of obliquity. Obviously these limits will become narrower as the energy of the colliding electron is reduced toward the quantum value. This assumption would therefore make the chance of radiation approach zero, rather than the maximum given by equation (5). Notwithstanding this, the existence of the discontinuous rise in f, followed by a slow decline, is a consequence of the part of the intensity-voltage law of which we are most certain.

An assumption that fits the facts much better is that radiation occurs when the energy imparted to the oscillator is not greater than a quantum, but almost exactly equal to a quantum. In this case, using the equations of Thomson's theory of the slowing of cathode particles, one may readily show that if the force between the oscillator and the cathode electron is an inverse square repulsion only, the chance of transferring energy within a definite small fraction above or below the quantum is inversely proportional to the energy of the cathode electron whenever the latter exceeds the quantum. The same result would follow if the collision were between two hard spheres, though not for some other laws of repulsion such as an inverse cube. While the assumptions and the data are now too uncertain to justify a long mathematical theory on this basis the similarity of this result to equation (5) makes it reasonable to expect that some such assumptions may later be found correct. This means that quantum collisions obey laws that differ from those we are accustomed to even more than one might expect from the mere existence of a short wave limit of the constant potential spectrum.

Summary.—At any rate the above calculations prove the following points:

A. That if radiation occurs in quanta, the probability of its occurrence with any one frequency in any one atom struck by a cathode electron

rises from zero to a finite value when the energy of the cathode electron is increased past the quantum.

B. That for the case of rhodium at least the probability decreases with any further increase of energy of the cathode electron.

C. That these laws of quantum collisions are radically different from those of collisions which merely slow down the electron slightly without producing X-rays.

Jefferson Physical Laboratory,
Cambridge, Mass.