

of energy in excess of the accepted value. This accounts for the additional energy term.

4. When a wave train travels down a tube, there is a region of higher density (where the wave forms) followed by a region of lower density. The density variation seems to be a basic feature of a traveling wave and does not depend on the amplitude.

Two additional conclusions, first given by Schoch, should be mentioned:

5. One may separate the relation between the energy and the intensity into two parts. One is related to the stored energy, while the other is related to the oscillatory energy.

6. As long as the total volume is held fixed, the net stored energy is zero. This does not mean that the energy may not have a space distribution.

One would expect the effect described in this paper to be unimportant in liquids. Equation (4) is for an ideal gas; however, corresponding forms exist in liquids.¹⁷ Combining the expression for water² with Eqs. (10a) and (15a), one may see that Andrejew's terms are not important. One might expect similar results for other liquids. These calculations, however, have not been made.

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¹⁷ See, for example, P. J. Westervelt, *J. Acoust. Soc. Am.* **22**, 319 (1950).

Transport Phenomena in a Completely Ionized Gas*

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The coefficients of electrical and thermal conductivity have been computed for completely ionized gases with a wide variety of mean ionic charges. The effect of mutual electron encounters is considered as a problem of diffusion in velocity space, taking into account a term which previously had been neglected. The appropriate integro-differential equations are then solved numerically. The resultant conductivities are very close to the less extensive results obtained with the higher approximations on the Chapman-Cowling method, provided the Debye shielding distance is used as the cutoff in summing the effects of two-body encounters.

I. INTRODUCTION

A PREVIOUS paper by Cohen, Spitzer, and Routly,¹ referred to hereafter as CSR, presented a new approach to the problem of transport phenomena in a completely ionized gas. In effect, the influence of mutual electron encounters on the velocity distribution function for electrons was considered as a problem of diffusion in velocity space. In particular, the electrical conductivity of an electron-proton gas was computed in this way. However, the results were not exact, since one term in the diffusion equation was neglected. In the present paper, a solution of the complete diffusion equation is obtained, and the results are extended to completely ionized gases with different mean nuclear charges. Computations are carried out for the thermal as well as the electrical conductivity.

In the first section below the general principles are explained and justified. Subsequent sections outline the derivation of the equations, the method of solution, and the results obtained.

II. GENERAL PRINCIPLES

The velocity distribution function $f_r(v)$ for particles of type r is determined by the familiar Boltzmann equation, basic in all studies of this sort,

$$\frac{\partial f_r}{\partial t} + \sum_i v_{ri} \frac{\partial f_r}{\partial x_i} + \sum_i F_{ri} \frac{\partial f_r}{\partial v_{ri}} = \sum_s \left(\frac{\partial_s f_r}{\partial t} \right), \quad (1)$$

where the notation in CSR has been followed. The complexity of the problem arises entirely from the term $(\partial_s f_r / \partial t)_s$, which gives the change in f_r produced by encounters of r particles with particles of type s .

To visualize the physical situation more accurately, let us follow a single electron as it moves through the gas. The random electrical fields encountered by the electron will produce deflections and changes in velocity. To some extent these fields can be described in terms of separate two-body encounters; let b be the impact parameter for such an encounter—the distance of closest approach between the two particles in the absence of any mutual force. The situation is characterized by the values of the following four distances: d , the mean distance from an electron to its nearest neighbor; b_0 , the value of the collision parameter for

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¹ Cohen, Spitzer, and Routly, *Phys. Rev.* **80**, 230 (1950).

TABLE I. Values of the velocity distribution function $D(x)$ when an electrical field is present.

x	$ZD(x)/A$				
	$Z=1$	$Z=2$	$Z=4$	$Z=16$	$Z=\infty$
0.10	0.0008093	0.0001340	0.0001000
0.11	0.001300	0.0002262	0.0001464
0.12	0.001970	0.0003630	0.0002074
0.13	0.002847	0.0005582	0.0002856
0.14	0.003955	0.0008262	0.0003842
0.15	0.005317	0.001183	0.0005062
0.16	0.006955	0.001645	0.0006554
0.17	0.008886	0.002228	0.0008352
0.18	0.01113	0.002952	0.001050
0.19	0.01370	0.003832	0.001303
0.20	0.01660	0.004884	0.002163	0.001645	0.001600
0.22	0.02347	0.007576	0.003373	0.002432	0.002343
0.24	0.03180	0.01116	0.005044	0.003477	0.003318
0.26	0.04165	0.01575	0.007280	0.004833	0.004570
0.28	0.05304	0.02146	0.01018	0.006560	0.006147
0.30	0.06601	0.02840	0.01386	0.008721	0.008100
0.32	0.08057	0.03666	0.01842	0.01139	0.01049
0.34	0.09672	0.04632	0.02398	0.01463	0.01336
0.36	0.1145	0.05746	0.03063	0.01853	0.01680
0.38	0.1338	0.07012	0.03849	0.02317	0.02085
0.40	0.1548	0.08440	0.04764	0.02866	0.02560
0.44	0.2015	0.1180	0.07028	0.04249	0.03748
0.48	0.2545	0.1586	0.1352	0.06082	0.05308
0.52	0.3137	0.2066	0.1789	0.08443	0.07312
0.56	0.3792	0.2620	0.2309	0.1142	0.09834
0.60	0.4508	0.3254	0.2917	0.1511	0.1296
0.64	0.5285	0.3968	0.3618	0.1958	0.1678
0.68	0.6123	0.4764	0.4416	0.2494	0.2138
0.72	0.7023	0.5646	0.5324	0.3127	0.2687
0.76	0.7983	0.6612	0.6336	0.3868	0.3336
0.80	0.9005	0.7668	0.8704	0.4722	0.4096
0.88	1.123	1.005	1.156	0.6819	0.5997
0.96	1.371	1.281	1.494	0.9499	0.8493
1.04	1.645	1.596	1.889	1.287	1.170
1.12	1.945	1.952	2.344	1.702	1.574
1.20	2.273	2.352	2.864	2.204	2.074
1.28	2.630	2.796	3.455	2.804	2.684
1.36	3.017	3.290	4.120	3.510	3.421
1.44	3.435	3.836	4.868	4.331	4.300
1.52	3.887	4.440	5.700	5.281	5.338
1.60	4.375	5.096	6.604	6.366	6.554
1.76	5.465	6.604	10.06	8.996	9.595
1.92	6.728	8.406	12.96	12.34	13.59
2.08	8.190	10.54	16.46	16.54	18.72
2.24	9.880	13.05	20.64	21.74	25.18
2.40	11.83	16.00	25.60	28.10	33.18
2.56	14.06	19.40	31.4	35.80	42.95
2.72	16.62	23.3	38.2	45.0	54.74
2.88	19.53	27.7	46.0	56.0	68.80
3.04	22.74	32.7	54.6	68.8	85.41
3.20	26.00	38.5	54.6	83.9	104.9

which an electron is deflected 90° in an encounter with a stationary positive ion; h , the Debye shielding distance; and λ , the mean free path for a net deflection of 90° . It is readily verified that for virtually all situations of interest,

$$b_0 \ll d \ll h \ll \lambda. \quad (2)$$

It is clear that encounters for which $b \ll d$ can be described adequately in terms of successive two-body encounters, since usually an encounter with one particle will be effectively over before another particle approaches to a distance less than d . These successive encounters may be divided into two classes. Those with $b \leq b_0$ produce large deflections, and will be termed "close" encounters. Those with $b_0 < b < d$ produce relatively small deflections, and will be called "distant" encounters. As shown in CSR and elsewhere, the cumulative effect of many distant encounters outweighs the effect of the less frequent close encounters, in the special case of inverse-square forces between the particles.

Encounters for which $d \leq b < h$ cannot be regarded as independent, since several such encounters will be taking place at the same time. More correctly, the deflection of a particular electron caused by such "en-

counters" must be attributed to statistical fluctuations of the electron density in a sphere of radius b . As shown in CSR, however, the mean square change of electron velocity produced by such fluctuations is correctly given if the formulas derived for successive two-body encounters are applied for $b > d$.

Particles passing at a distance large compared to h produce a negligible effect. From the standpoint of the Debye shielding theory, the effective field of a charge in a plasma varies as e^{-hr}/r , where h is given by

$$h^2 = \frac{kT}{4\pi n_e e^2 (1+Z)}. \quad (3)$$

If one considers rather the statistical fluctuations in electron density, Pines and Bohm² have shown that collective phenomena in a plasma reduce markedly the statistical fluctuations in electron density with wavelengths large compared to h , thus justifying the neglect of encounters such that $b > h$. There is some interaction between a single electron and the organized oscillations of the plasma—see Eq. (59a) of Pines and Bohm. However, comparison of their Eqs. (59a) and (59b) shows that for thermal electrons, with mean kinetic energies of the order kT , the rate of energy loss due to this process is less by a factor $1/\ln(h/b_0)$ than the energy loss due to random encounters such that $b < h$. The generation of plasma oscillations by a single thermal electron may therefore be neglected, together with a number of other terms of the same order.³ Hence, we may neglect all interactions between electrons for which the distance of closest approach exceeds h .

Since λ is much greater than h , it is evident that many small deflections will be experienced by a particle traversing its mean free path. It is also clear that these deflections are essentially independent of each other. Inasmuch as collective phenomena (oscillations) have been neglected, the random electrical fields encountered by an electron in one region will be completely independent from the fields in a similar region separated by a distance appreciably greater than h . Hence, the successive changes in velocity represent a Markoff process, and the change of the velocity distribution function may be found from the Fokker-Planck equation.⁴ This equation neglects the close encounters; the relative error introduced is again of the order $1/\ln(h/b_0)$.

III. DERIVATION OF EQUATIONS

Equations (10), (23), and (24) of CSR give the basic equations of the problem on the following assumptions: (a) the Fokker-Planck equation may be used to give $\partial f_e / \partial t$; (b) a steady state is established; (c) the velocity

² D. Pines and D. Bohm, Phys. Rev. **85**, 338 (1952).

³ The various terms of relative magnitude $1/\ln(d/b_0)$ have been called "nondominant" by Chandrasekhar, and are usually neglected in the computation of diffusion coefficients—see S. Chandrasekhar, *Principles of Stellar Dynamics* (University of Chicago Press, Chicago, 1939).

⁴ See S. Chandrasekhar, Revs. Modern Phys. **15**, 1 (1943).

distribution function may be expressed as the sum of a Maxwellian function $f^{(0)}$ plus a small term $f^{(1)}$ whose square may be neglected. The values of the diffusion coefficients have now been recomputed, using a straightforward and conceptually very simple method. A new integro-differential equation has then been obtained for $D(lv)$, the function which gives the dependence of the ratio $f^{(1)}/f^{(0)}$ on the velocity v ; $1.5/l^2$ is the mean square electron velocity.

The electron-ion interaction is relatively simple to consider. Equation (28) of CSR must be modified to include a factor Z , the mean ionic charge, defined by

$$Z \equiv \sum n_i Z_i^2 / n_e, \quad (4)$$

summed over all positive ions, each of charge $Z_i e$ and of particle density n_i . We find

$$K(ff_i) = [3ZL f^{(0)} D(lv) \cos \theta] / 2v^3, \quad (5)$$

where all the symbols have the same meaning as in CSR, except that to avoid confusion with the current density, j has been replaced by l .

The quantity $K(ff)$, giving the contribution of electron-electron interactions to $\partial_e f / \partial t$, is much more complicated. To evaluate the diffusion coefficients needed, the values of Δ_ξ , Δ_η , and Δ_ζ , the changes of velocity in a single collision, along coordinates parallel and perpendicular, respectively, to the original velocity, were first determined. In this computation, the velocity changes were first taken in a frame of reference moving with the center of gravity of the two particles. If the orbit lies in the xy plane, and the x axis is taken to be in the direction in which the original particle is moving, the velocity changes become very simple. The components of the vector change in velocity along the ξ , η , and ζ axes may then be found by successive rotations of the coordinate axes.

Next the values of the velocity changes, together with their products and squares, are averaged over all collisions, in accordance with Eq. (8) of CSR. The final values for $\langle \Delta_\xi \rangle$, $\langle \Delta_\xi^2 \rangle$, and $\langle \Delta_\eta^2 \rangle$ are the same as given in Eqs. (31) through (35) of CSR. For the two remaining coefficients we find

$$\langle \Delta_\eta \rangle = -\frac{4Ll^2}{\pi^{\frac{1}{2}}} \sin \theta \{ I_0(\infty) - I_0(x) + I_3(x)/x^2 \}, \quad (6)$$

$$\langle \Delta_\xi \Delta_\eta \rangle = -\frac{2Ll}{\pi^{\frac{1}{2}}} \sin \theta \{ 0.4x(I_0(\infty) - I_0(x)) + I_3(x)/x^2 - 0.6I_5(x)/x^4 \}, \quad (7)$$

where L and $I_n(x)$ are defined in Eqs. (29) and (37) of CSR.

If these values of the diffusion coefficients are substituted into $K(ff)$, Eq. (1) yields, after considerable algebra, the equation

$$D''(x) + P(x)D'(x) + Q(x)D(x) = R(x) + S(x), \quad (8)$$

exactly as in Eq. (41) of CSR; x is defined as lv . While $P(x)$ is unchanged from the form given in Eq. (42) of CSR, $Q(x)$ and $S(x)$ become

$$Q(x) = \frac{1}{x^2} - 2 \frac{Z + \Phi(x) - 2x^3 \Phi'(x)}{\Lambda}, \quad (9)$$

$$S(x) = \frac{16}{3\pi^{\frac{1}{2}} \Lambda} \{ xI_3(x) - 1.2xI_5(x) - x^4 I_0(x)(1 - 1.2x^2) \}, \quad (10)$$

where $\Phi(x)$ is the error function, while $\Lambda(x)$ is defined in Eq. (46) of CSR. If an electrical field is present,

$$R(x) = -(2x^4 A / Z \Lambda) \{ Z - 1 + 1.2x^2 \}, \quad (11)$$

where A is again given by Eq. (40) of CSR. If a temperature gradient is present, we have

$$R(x) = -(x^4 B / \Lambda) \{ 2.5 - x^2 \}, \quad (12)$$

where

$$B = \frac{2k^2 T |\nabla T|}{\pi n_e e^4 \ln(qC^2)}; \quad (13)$$

qC^2 is given in Eq. (65), CSR.

TABLE II. Values of the velocity distribution function $D(x)$ when a temperature gradient is present.

x	$Z=1$	$Z=2$	$ZD(x)/B$ $Z=4$	$Z=16$	$Z=\infty$
0.10	0.0005906	0.0002306	0.0001245
0.11	0.0009023	0.0003630	0.0001821
0.12	0.001309	0.0005460	0.0002577
0.13	0.001821	0.0007914	0.0003546
0.14	0.002448	0.001110	0.0004764
0.15	0.003197	0.001515	0.0006271
0.16	0.004074	0.002016	0.0008108
0.17	0.005082	0.002624	0.001032
0.18	0.006225	0.003348	0.001295
0.19	0.007504	0.004200	0.001605
0.20	0.008922	0.005188	0.003095	0.002146	0.001968
0.22	0.01217	0.007601	0.004668	0.003156	0.002872
0.24	0.01596	0.01064	0.006744	0.004487	0.004052
0.26	0.02027	0.01434	0.009400	0.006199	0.005558
0.28	0.02508	0.01874	0.01270	0.008356	0.007442
0.30	0.03035	0.02384	0.01673	0.01103	0.009760
0.32	0.03607	0.02966	0.02152	0.01428	0.01257
0.34	0.04218	0.03616	0.02713	0.01818	0.01593
0.36	0.04865	0.04338	0.03360	0.02281	0.01991
0.38	0.05545	0.05126	0.04092	0.02824	0.02456
0.40	0.06254	0.05978	0.04916	0.03453	0.02995
0.44	0.07746	0.07866	0.06840	0.04995	0.04322
0.48	0.09302	0.09966	0.09116	0.06955	0.06024
0.52	0.1090	0.1224	0.1174	0.09370	0.08151
0.56	0.1250	0.1466	0.1468	0.1226	0.1075
0.60	0.1407	0.1717	0.1790	0.1565	0.1387
0.64	0.1558	0.1973	0.2134	0.1952	0.1754
0.68	0.1700	0.2228	0.2497	0.2386	0.2178
0.72	0.1829	0.2476	0.2871	0.2864	0.2663
0.76	0.1944	0.2714	0.3249	0.3384	0.3207
0.80	0.2036	0.2934	0.3624	0.3940	0.3809
0.88	0.2151	0.3296	0.4324	0.5135	0.5174
0.96	0.2139	0.3506	0.4896	0.6384	0.6703
1.04	0.1965	0.3502	0.5244	0.7584	0.8297
1.12	0.1587	0.3210	0.5252	0.8565	0.9800
1.20	0.0957	0.2546	0.4796	0.9096	1.099
1.28	+0.0021	+0.1414	0.3717	0.8858	1.156
1.36	-0.1289	-0.0302	+0.1834	0.7474	1.113
1.44	-0.3041	-0.2734	-0.1067	+0.4525	0.9167
1.52	-0.5339	-0.6040	-0.5240	-0.1736	+0.5060
1.60	-0.8268	-1.040	-1.098	-0.7999	-0.1966
1.76	-1.657	-2.320	-2.861	-3.313	-2.867
1.92	-2.921	-4.330	-5.752	-7.691	-8.061
2.08	-4.774	-7.366	-10.25	-14.79	-17.09
2.24	-7.448	-11.81	-16.98	-25.78	-31.69
2.40	-11.23	-18.18	-26.78	-42.20	-54.08
2.56	-16.5	-27.1	-40.6	-66.0	-87.05
2.72	-23.5	-39.3	-59.9	-99.4	-134.1
2.88	-33.2	-55.9	-86.0	-145.	-199.3
3.04	-46.0	-77.8	-121.	-207.	-287.9
3.20	-62.0	-106.	-166.	-290.	-405.8

From the principle of conservation of momentum the integral $I_0(\infty)$ can be evaluated in simple form. In the case of an electrical field, Eq. (39) of CSR becomes modified if Z differs from unity, and we have

$$I_0(\infty) = 3\pi^{1/2}A/8Z. \tag{14}$$

If a temperature gradient is considered, then we have instead

$$I_0(\infty) = 0. \tag{15}$$

IV. SOLUTION OF THE EQUATIONS

The method of numerical solution followed is, in principle, identical with that employed in CSR. For values of Z different from unity the functions $Q_{01}(x)$ and $Q_{02}(x)$ differed from those used previously, with somewhat different forms for the solutions $U(x)$ and $V(x)$. No changes in $P_{01}(x)$ or $P_{02}(x)$ were required. In each specific range the substitutions given below were the same for both electrical and thermal conductivity.

In the range $x \leq 0.80$, we have

$$Q_{01}(x) = (-3Z\pi^{1/2}/2x^3) - 2/x^2, \tag{16}$$

$$U_1 = (Z/x)^{1/2}I_3(Z^{1/2}\alpha/x^{1/2}), \tag{17}$$

$$V_1 = (Z/x)^{1/2}K_3(Z^{1/2}\alpha/x^{1/2}), \tag{18}$$

where α^2 is again equal to $6\pi^{1/2}$. For the range $0.80 \leq x < 3.20$, we have

$$Q_{02} = -2(Z+1), \tag{19}$$

$$U_2 = D_Z(i2^{1/2}x) \exp(x^2/2), \tag{20}$$

$$V_2 = D_{-1-Z}(2^{1/2}x) \exp(x^2/2). \tag{21}$$

In Eqs. (20) and (21) $D_n(y)$ is the parabolic cylinder function, which, for any integral value of n , may be expressed in terms of the error function $\Phi(x)$ by means of the recursion formula.⁵

As before, two independent numerical solutions were computed, each satisfying the boundary condition for small x . A linear combination of these was taken to satisfy the boundary condition at $x = 3.2$. The resulting values of the velocity distribution function are given in Table I for electrical conductivity and in Table II for thermal conductivity. For $Z = 4$ the integration was started at $x = 0.20$, and no values were computed for smaller x .

As Z becomes very large, the mutual electron inter-

TABLE III. Values of transport coefficients.

	$Z=1$	$Z=2$	$Z=4$	$Z=16$	$Z=\infty$
γ_E	0.5816	0.6833	0.7849	0.9225	1.000
γ_T	0.2727	0.4137	0.5714	0.8279	1.000
δ_E	0.4652	0.5787	0.7043	0.8870	1.000
δ_T	0.2252	0.3563	0.5133	0.7907	1.000
ϵ	0.4189	0.4100	0.4007	0.3959	0.4000

⁵ See section 16.7, E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, Cambridge, 1950).

actions become unimportant, and the following simple formulas for a so-called Lorentz gas are applicable.

$$D(x) = x^4A/Z \text{ for electrical conductivity,} \tag{22}$$

$$D(x) = x^4B(2.5 - x^2)/2Z \text{ for thermal conductivity.} \tag{23}$$

Hence, for ready comparison with the Lorentz gas, Table I gives values of $ZD(x)/A$, and Table II gives $ZD(x)/B$.

Also listed in Tables I and II are values of the velocity distribution function for Z equal to 16 and ∞ . The latter values were found directly from Eqs. (22) and (23). Those for $Z = 16$ were determined from an asymptotic series for $D(x)$ in increasing powers of $1/Z$, which we may write

$$D(x) = \sum_{n=1}^{\infty} \frac{d_n(x)}{Z^n}. \tag{24}$$

If Eq. (24) is substituted into Eq. (8) and coefficients of each power of Z are separately set equal to zero, each d_n is given as a function of d_{n-1} , the first and second derivatives of d_{n-1} , and certain integrals over d_{n-1} . The quantity d_1 is simply the value of $D(x)$ for a Lorentz gas, given by Eqs. (22) or (23); successive functions d_n were found by straightforward computation up to $n = 4$.

V. RESULTS

In the presence of a weak electrical field \mathbf{E} and a small temperature gradient ∇T , the current density \mathbf{j} and the rate of flow of heat \mathbf{Q} per unit area are given by

$$\mathbf{j} = \sigma \mathbf{E} + \alpha \nabla T, \tag{25}$$

$$\mathbf{Q} = -\beta \mathbf{E} - K \nabla T. \tag{26}$$

In terms of the velocity distribution function $D(x)$, j , summed over all electrons—see Eq. (61) of CSR—is given by

$$j = -2\pi^{-1/2}en_eC(2/3)^{1/2}I_3(\infty), \tag{27}$$

while for the heat flow Q we have

$$Q = \pi^{-1/2}mn_eC^3(2/3)^{5/2}I_5(\infty), \tag{28}$$

where e and m are the electron charge and mass, and C is the root mean square electron velocity. From the numerical values found for the integrals $I_3(\infty)$ and $I_5(\infty)$, values of the coefficients σ , α , β , and K may be determined.

It is convenient to express these transport coefficients in terms of their values in a Lorentz gas. In the case of an electrical field, we define

$$\gamma_E = ZI_3(\infty)/3A, \tag{29}$$

$$\delta_E = ZI_5(\infty)/12A. \tag{30}$$

When $Z = 1$, γ_E is identical with the quantity γ introduced in CSR. In the corresponding case where a

temperature gradient is present, we write

$$\gamma_T = -4ZI_3(\infty)/9B, \quad (31)$$

$$\delta_T = -ZI_5(\infty)/15B. \quad (32)$$

From Eqs. (22) and (23) above the definition of $I_n(\infty)$ —see Eq. (37) of CSR—it is readily verified that for a Lorentz gas all four of these coefficients are unity. On elimination of A and B by means of Eq. (40) of CSR and Eq. (13) above, we obtain, after some substitutions,

$$\sigma = \frac{2mC^3}{e^2Z \ln(qC^2)} \left(\frac{2}{3\pi}\right)^{\frac{3}{2}} \gamma_E, \quad (33)$$

$$\alpha = \frac{3mkC^3}{e^2Z \ln(qC^2)} \left(\frac{2}{3\pi}\right)^{\frac{3}{2}} \gamma_T, \quad (34)$$

$$\beta = \frac{8m^2C^5}{3e^2Z \ln(qC^2)} \left(\frac{2}{3\pi}\right)^{\frac{3}{2}} \delta_E, \quad (35)$$

$$K = \frac{20m^2kC^5}{3e^4Z \ln(qC^2)} \left(\frac{2}{3\pi}\right)^{\frac{3}{2}} \delta_T. \quad (36)$$

Values of the four transport coefficients are given in Table III for various values of Z .

The quantity qC^2 , which is essentially h/b_0 , is given in Eq. (65) of CSR, while values of $\ln(qC^2)$ are tabulated in Table IV of the same paper. The electron charge e has here been taken in electrostatic units throughout. To obtain the conductivity in practical units (mho) the value found from Eq. (33) must be divided by 9×10^{11} .

It should be noted that if a temperature gradient is present in a steady state, but no steady current is flowing, the electrostatic field E will build up to such a value that j vanishes. This field then reduces the flow of heat, and K' , the effective coefficient of heat conductivity is readily shown to be

$$K' = \epsilon K, \quad (37)$$

where

$$\epsilon = 1 - 3\delta_E\gamma_T/(5\delta_T\gamma_E). \quad (38)$$

Values of ϵ are also given in Table III.

It remains to compare these results with those of previous workers. When this work was undertaken, the best available results for the electrical conductivity of a completely ionized gas were those of Chapman and Cowling⁶ and of Cowling,⁷ who had obtained first and second approximations, respectively, for the conductivity. In terms of the present notation, the Chapman-

⁶ S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, 1939).

⁷ T. G. Cowling, Proc. Roy. Soc. (London) A183, 453 (1945).

TABLE IV. Comparison with results obtained by Landshoff for $Z=1$.

n	1	2	3	4	∞
γ_E	0.2945	0.5693	0.5743	0.5777	0.5816

Cowling method utilizes the expansion

$$D(x) = \frac{3\pi^{\frac{3}{2}}x}{4} \sum_{j=0}^{n-1} (-1)^j \frac{\Delta_{0j}^{(n)}}{\Delta^{(n)}} L_j^{(3/2)}(x^2), \quad (39)$$

where $L_j^{(3/2)}$ is a Laguerre polynomial, and the ratios $\Delta_{0j}^{(n)}/\Delta^{(n)}$ are determined from encounter theory and the Boltzmann equation; n is the order of the approximation used. Since the value of σ found by Cowling⁷ with $n=2$ was about twice the value obtained by Chapman and Cowling⁶ with $n=1$, it appeared that the present treatment, equivalent to letting $n=\infty$ in Eq. (39), might give a markedly different value.

More recently, this same problem has been considered by Landshoff,⁸ using the Chapman-Cowling method, but with values of n up to 4. From the values of $\Delta_{0j}^{(n)}/\Delta^{(n)}$ which he gives for $Z=1$, the constant γ_E has been computed, and is given in Table IV, together with the value found in the present work ($n=\infty$).

In view of the large difference between the first and second approximation, it is rather remarkable how close to the truth is the second approximation for γ_E . For thermal conductivity the convergence is somewhat less rapid, with the fourth approximation in close agreement with the value for $n=\infty$. Evidently the present results agree with Cowling's⁷ second approximation for the electrical conductivity, provided that for the cutoff in the integration over the impact parameter b , we take the Debye shielding distance h rather than the electronic separation d taken by Cowling. The value 0.490 obtained for γ_E in CSR, in disagreement with Cowling's value 0.578, was the consequence of the neglect of the $\langle \Delta_\epsilon \Delta_n \rangle$ term in $K(ff)$; inclusion of this term removes virtually all the disagreement in γ_E .

It should be emphasized that the present theory considers only those terms in $\partial f_e/\partial t$ which are of order $\ln(h/b_0)$, and a variety of terms of order unity have been neglected, including, for example, the interaction between a high speed electron and its wake of plasma oscillation, an effect explored by Pines and Bohm.² Thus, the relative accuracy of the present theory does not exceed $1/\ln(h/b_0)$, or some 5 to 10 percent for most conditions of astrophysical interest. In view of the lack of observational data in this field, development of a more refined theory does not seem worth the very considerable effort required.

⁸ R. Landshoff, Phys. Rev. 82, 442 (1951).