of powdered tellurium metal. Under the conditions of slow passage and with sweep amplitudes small compared to the natural line width of the nuclear induction signals, the output voltage of the spectrometer as registered by the dc milliammeter is the derivative of the slow passage signal. Under these conditions the area under the trace is zero. The aspect of the signals in metallic tellurium indicated that the experimental conditions of the metal were not those of slow passage; in addition, it was not possible to detect an absorption mode with the lowest available half-amplitude of the rf field of about 0.01 gauss. Both facts indicate that the longitudinal relaxation time in the metal is comparatively long. No observable chemical shift was detected between the resonant frequency of Te^{125} in the pulverized metal and in the solutions.

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Atomic Excitation and Ionization Accompanying Orbital Electron Capture by Nuclei*

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The process of atomic excitation and ionization accompanying orbital electron capture by nuclei is treated on the basis of the general theory of β-decay using a configuration space representation for both nucleons and leptons. Quantitative expressions are obtained (a) for the total number of double holes produced in the K-shell due to K-capture accompanied by excitation or ejection of the other K-electron [Eq. (15)], and (b) for the total probability of electron ejection in orbital electron capture together with the ejected electron momentum spectrum [Eqs. (17)–(18)]. A discussion is given of the possibilities of experimental verification of the theory developed.

I. INTRODUCTION

In the customary physical description of orbital electron capture, one views the process as involving the transformation of one of the atomic orbital electrons (most probably from the K-shell) into an emitted neutrino with the simultaneous transformation of one of the nuclear protons into a neutron. In this description, no other particles are considered to be emitted, and the other electrons in the atom are presumed to remain in their original orbits, adjusted from the charge of the parent nucleus Z_i to that of the daughter $Z_f=Z_i-1$. Actually, however, a certain nonvanishing probability exists (a) for photon emission as a result of the charge acceleration involved in the orbital electron capture[1] and (b) for the excitation of one (or more) of the noncaptured orbital electrons into unoccupied atomic bound[2] states and into unbound states as a consequence of the nuclear charge alteration as conditioned by the electron-electron Coulomb interaction. In a certain proportion of the orbital electron captures then, the emitted neutrino is accompanied by another particle, *viz.*, a photon or a previously bound electron with which the neutrino must share the available energy. The energy release in orbital electron capture, which is otherwise not directly obtainable, may then be found by measuring the maximum energy carried off by the photon or by the ejected electron. Such a measurement has indeed been carried out recently for the case of photons associated with orbital electron capture in $_{26}Fe^{55}$,[3,4] and the energy release in the transition accurately determined.

In the present paper we shall calculate (a) the total number of double holes produced in the K-shell due to the transformation of one of the K-electrons into the emitted neutrino and the excitation of the other K-electron into an unoccupied bound[2] or into an unbound (ejected) state, and (b) the total probability of (other) electron ejection in orbital electron capture together with the ejected electron momentum distribution. Our work is closely related to the work of Migdal[5] and of Feinberg[6] on the ejection of atomic orbital electrons during nuclear negatron decay.[7]

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[1] P. Morrisson and L. Schiff [Phys. Rev. 58, 24 (1940)] give the theory.

[2] The binding is virtual in all cases of interest (see reference 11).

[3] Bell, Jauch, and Cassidy, Science **115**, 12 (1952).

[4] D. Maeder and P. Preiswerk, Phys. Rev. **84**, 595 (1951); see also the work on $_{18}A^{37}$ by Anderson, Wheeler, and Watson, Phys. Rev. **87**, 668 (1952).

[5] A. Migdal, J. Phys. (U.S.S.R.) **IV**, 449 (1941).

[6] E. L. Feinberg, J. Phys. (U.S.S.R.) **IV**, 423 (1941).

[7] See also the careful discussion of atomic electron excitation and ejection in the negatron transition $_2He^6 \rightarrow {_3Li^6}$, by A. Winther,

II. CALCULATIONS

Specification of Ψ_{initial}, Ψ_{final}; $H^{(0)}$, $H^{(1)}$

We consider for the time being only orbital electron capture from the K-shell and the possibility of excitation of the other K-electron. Neglecting the influence of the L, M, $\cdots$ electrons on the K-electrons, we have for the (effectively two-electron) wave function of the parent atom in the initial state

$$\Psi_{\text{ini}}=\Phi_{E_i,Z_i}(\cdots,\mathbf{x}_n,s_n,q_n,\cdots)\left(\frac{1-P_{12}}{\sqrt{2}}\right)\times[u_{\text{ini}}(\mathbf{r}_1,\mathbf{r}_2)v_+(s_1)v_-(s_2)w_e(q_1)w_e(q_2)],\quad(1)$$

and for the (effectively one-electron) wave function of the daughter atom plus emitted neutrino

$$\Psi_{\text{fin}}=\Phi_{E_f,Z_f}(\cdots,\mathbf{x}_n,s_n,q_n,\cdots)\left(\frac{1-P_{12}}{\sqrt{2}}\right)\times[u_{\text{fin}}(\mathbf{r}_1,\mathbf{r}_2)v_{\sigma_\nu}(s_1)v_{\sigma_e}(s_2)w_\nu(q_1)w_e(q_2)].\quad(2)$$

In Eqs. (1) and (2), the Φ's are wave functions of the parent and daughter nuclei, the $\mathbf{x}_n$, s_n, q_n, being nucleon space, spin and charge coordinates; P_{12} is the lepton coordinate permutation operator; $v_{\sigma_\nu}(s_l)=v_+(s_l)$ or $v_-(s_l)$, $v_{\sigma_e}(s_l)=v_+(s_l)$ or $v_-(s_l)$, are the lepton spin wave functions; $w_e(q_l)$, $w_\nu(q_l)$ are lepton charge wave functions ($w_e(1)=1$; $w_e(0)=0$; $w_\nu(1)=0$, $w_\nu(0)=1$); $u_{\text{ini}}(\mathbf{r}_1,\mathbf{r}_2)$ and $u_{\text{fin}}(\mathbf{r}_1,\mathbf{r}_2)$ are the lepton space wave functions. Explicitly we have

$$u_{\text{fin}}(\mathbf{r}_1,\mathbf{r}_2)=V^{-\frac{1}{2}}\exp(i\mathbf{k}_\nu\cdot\mathbf{r}_1)\chi_f(\mathbf{r}_2),\quad(3)$$

where $u_{\text{fin}}(\mathbf{r}_1,\mathbf{r}_2)$ can be written as a product of single lepton space wave functions since the two leptons have no electromagnetic interaction with each other in the final state (one of the two leptons is a neutrino, the other an electron); $\chi_f(\mathbf{r}_2)$ is the space wave function of the other (noncaptured) electron after the K-capture. Thus, for example, if we are calculating the probability that the other electron is still in the K-shell (about the daughter nucleus) after the capture, we would put (since $Z_f=Z_i-1$)

$$\chi_f(\mathbf{r}_2)=(\pi a^3)^{-\frac{1}{2}}(Z_i-1)^{\frac{3}{2}}\exp[-r_2(Z_i-1)/a],\quad(4)$$

whereas, if we are calculating the probability that the other electron is ejected from the atom as a consequence of the capture, we would write

$$\chi_f(\mathbf{r}_2)=V^{-\frac{1}{2}}\exp(i\mathbf{k}_e\cdot\mathbf{r}_2)F_{\mathbf{k}_e,Z_i-1}(\mathbf{r}_2),\quad(5)$$

$F_{\mathbf{k}_e,Z_i-1}(\mathbf{r}_2)$ being the (properly normalized) Coulomb field confluent hypergeometric function. Further,

$$u_{\text{ini}}(\mathbf{r}_1,\mathbf{r}_2)=N(\pi a^3)^{-1}Z_i^3\exp[-Z_i(r_1+r_2)/a]\times\exp[\gamma_2(r_1+r_2)/a]\exp(\gamma_1r_{12}/a),\quad(6)$$

with $N^2\cong1-(35/8)(\gamma_1/Z_i)-6\gamma_2/Z_i$. In Eq. (6) we use a space wave function for the two leptons in the initial state (both are electrons) which describes the effect of their mutual Coulomb interaction (a) on the electrostatic shielding of the Coulomb attraction of the parent nucleus (factor $\exp[\gamma_2(r_1+r_2)/a]$), and (b) on their spatial correlation (factor $\exp[\gamma_1r_{12}/a]$). The analytic forms chosen for these shielding and correlation factors make u_{ini} a reasonably good fit to Hylleraas' variational nonrelativistic wave function for the problem of two electrons in the ground state about a nucleus of charge Z_i,[8] providing that the numerical values for γ_1 and γ_2 are chosen as 0.38 and 0.12, respectively. In spite of the smallness of γ_1 and γ_2 compared to Z_i, it is essential to keep the factors containing these quantities since the approximate orthogonality of u_{fin} and u_{ini} makes the transition matrix element, $M_{i\to f}$, depend sensitively on γ_1 and γ_2 (see Eqs. (14)–(16) below—according to Eq. (16), $M_{i\to f}\sim[(Z_i-\gamma_1-\gamma_2)-(Z_i-1)]=1-\gamma_1-\gamma_2$; the here neglected largely shielding effect of the L, M, $\cdots$ electrons on the K electrons is expected, at least in first order, to affect (Z_i-1) and $(Z_i-\gamma_1-\gamma_2)$ equally and so to cancel out in their difference).

The initial and final state wave functions Ψ_{ini} and Ψ_{fin} are eigenfunctions of the "zero order" Hamiltonian $H^{(0)}$,

$$H^{(0)}=H_{\text{nucleons}}(\cdots,\mathbf{x}_n,s_n,q_n,\cdots)+\sum_{l=1}^{2}H_{\text{leptons};\,l}(\mathbf{x}_l,s_l,q_l)+\frac{e^2q_1q_2}{r_{12}}+\sum_{n=1}^{A}\sum_{l=1}^{2}\left[\frac{(-e^2)q_nq_l}{|\mathbf{x}_n-\mathbf{r}_l|}\right],\quad(7)$$

to within the errors made in using the approximate form for u_{ini} in Eq. (6), instead of the rigorous solution of

$$\left\{\sum_{l=1}^{2}(H_{\text{leptons};\,l})+\frac{e^2}{r_{12}}-\sum_{n=1}^{A}\frac{e^2q_n}{|\mathbf{x}_n-\mathbf{r}_1|}-\sum_{n=1}^{A}\frac{e^2q_n}{|\mathbf{x}_n-\mathbf{r}_2|}\right\}u_{\text{ini}}=\epsilon_{\text{ini}}u_{\text{ini}}.$$

Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 2 (1952). Very recently also, a thorough discussion of the "Effects of radioactive disintegrations on the inner electrons of the atom" has been given by J. S. Levinger (Bull. Am. Phys. Soc. **27**, No. 5, 23 (1952)). We wish to thank Dr. Levinger for a helpful discussion and letter and for the communication of a copy of his paper before publication. In general, agreement exists between his results and ours on the probability of excitation and ejection of atomic orbital electrons in $\beta^{\pm}$ decay and in K-capture.

[8] See, for example, H. Bethe, *Handbuch der Physik* (J. Springer, Berlin, 1933), Vol. XXIV/1, p. 362

The wave functions Ψ_{ini} and Ψ_{fin} are also eigenfunctions of all operators commuting with $H^{(0)}$ such as the total nuclear charge operator

$$Z_{\text{op}} = \sum_{n=1}^{A} q_n$$

(with eigenvalues Z_i, $Z_f = Z_i - 1$). We note that the last term in $H^{(0)}$ [Eq. (7)] (nucleon-lepton Coulomb interaction) is very well approximated by

$$(-e^2) Z_{\text{op}} \sum_{l=1}^{2} \frac{q_l}{|\mathbf{r}_l|},$$

and that the nonrelativistic (Schrödinger) approximation to the free particle Dirac H_{lepton} is used when the leptons are in the electron state; also the relatively small effect of daughter atom recoil on Ψ_{fin} is neglected.

Finally, in calculating the transition probability for K-orbital electron capture with excitation of the other K-electron, we need to specify the nucleon-lepton β-decay interaction Hamiltonian, $H^{(1)}$. This takes the form, in our configuration space representation for both nucleons and leptons,

$$H^{(1)} = g \sum_{n=1}^{A} \sum_{l=1}^{2} Q_n \Omega_n^{(\mu)} Q_l \Omega_l^{(\mu)} \delta(\mathbf{x}_n - \mathbf{r}_l) + (\text{Hermitian conjugate}). \quad (8)$$

The Q_l are lepton charge transformation operators [$Q_1 w_e(q_1) = w_\nu(q_1)$; $Q_1 w_\nu(q_1) = 0$; etc.]. The Q_n are the analogous and usual nucleon charge transformation operators. The $\Omega_n^{(\mu)}$ and $\Omega_l^{(\mu)}$ are the customary Dirac covariant operators appropriate to the form of β-decay coupling employed (e.g., $\Omega_l^{(\mu)} = \sigma_l^{(1)}$, $\sigma_l^{(2)}$, $\sigma_l^{(3)}$, $\beta_l \alpha_l^{(1)}$, $\beta_l \alpha_l^{(2)}$, $\beta_l \alpha_l^{(3)}$ for tensor coupling), a summation being implied over the repeated superscript (μ).

The transition probability per unit time, $P_{i\to f}$, from the initial to the final state is then given by the standard formula involving the square of the matrix element of $H^{(1)}$ taken between Ψ_{ini} and Ψ_{fin}:

$$P_{i\to f} = \frac{2\pi}{\hbar} \left| \int \Psi^*_{\text{fin}} H^{(1)} \Psi_{\text{ini}} \right|^2 \rho_{\text{fin}}. \quad (9)$$

III. CALCULATION

Evaluation of Matrix Element and $P_{i\to f}$

In order to calculate the total probability of the noncaptured K electron *not* remaining in the K-shell as a result of the K-capture, we calculate the complementary probability of its remaining in the K-shell. We have then, from Eqs. (1), (2), (4), (8),

$$\begin{aligned} M_{i\to f} &= \int \Psi^*_{\text{fin}} H^{(1)} \Psi_{\text{ini}} \\ &= \int \Phi^*_{E_f, Z_i-1} \left\{ \left[\frac{1-P_{12}}{\sqrt{2}} \right] [V^{-\frac{1}{2}} \exp(i\mathbf{k}_\nu \cdot \mathbf{r}_1) \right. \\ &\quad \times (\pi a^3)^{-\frac{1}{2}} (Z_i - 1)^{\frac{3}{2}} \exp[-(r_2/a)(Z_i - 1)] \\ &\quad \left. \times v_{\sigma_\nu}(s_1) v_{\sigma_e}(s_2) w_\nu(q_1) w_e(q_2)] \right\}^* \\ &\quad \cdot \left\{ g \sum_{n=1}^{A} \sum_{l=1}^{2} Q_n \Omega_n^{(\mu)} Q_l \Omega_l^{(\mu)} \delta(\mathbf{x}_n - \mathbf{r}_l) \right\} \\ &\quad \cdot \Phi_{E_i, Z_i} \left\{ \left[\frac{1-P_{12}}{\sqrt{2}} \right] [N (\pi a^3)^{-1} Z_i^3 \exp(-Z_i r_1/a) \right. \\ &\quad \times \exp(-Z_i r_2/a) \exp[(\gamma_2/a)(r_1 + r_2)] \\ &\quad \left. \times \exp(\gamma_1 r_{12}/a) v_+(s_1) v_-(s_2) w_e(q_1) w_e(q_2)] \right\}. \quad (10) \end{aligned}$$

Neglecting, x_n compared to r_2 in $\exp[(\gamma_1/a)|\mathbf{x}_n - \mathbf{r}_2|]$ (x_n nuclear, r_2 atomic dimensions, respectively), we can write

$$\begin{aligned} M_{i\to f} &= \int \{(\pi a^3)^{-\frac{1}{2}} (Z_i - 1)^{\frac{3}{2}} \exp[-(r_2/a)(Z_i - 1)]\} \\ &\quad \times \{N (\pi a^3)^{-\frac{1}{2}} Z_i^{\frac{3}{2}} \exp[-(r_2/a)(Z_i - \gamma_1 - \gamma_2)]\} d\mathbf{r}_2 \\ &\quad \cdot \int v_{\sigma_\nu}^*(s_1) v_{\sigma_e}^*(s_2) \Omega_1^{(\mu)} (1 - P_{12}) v_+(s_1) v_-(s_2) \\ &\quad \cdot g \int \Phi^*_{E_f, Z_i - 1} \sum_{n=1}^{A} Q_n \Omega_n^{(\mu)} V^{-\frac{1}{2}} \\ &\quad \times \exp(-i\mathbf{k}_\nu \cdot \mathbf{x}_n) (\pi a^3)^{-\frac{1}{2}} Z_i^{\frac{3}{2}} \\ &\quad \times \exp[-(x_n/a)(Z_i - \gamma_2)] \Phi_{E_i, Z_i} \\ &= N (Z_i - 1)^{\frac{3}{2}} Z_i^{\frac{3}{2}} [Z_i - \tfrac{1}{2}(1+\gamma)]^{-3} \\ &\quad \cdot \mathcal{S}^{(\mu)} \cdot [\text{M.E.}^{\text{nucl.}\,(\mu)}]_{if}, \quad (11) \end{aligned}$$

where $\gamma \equiv \gamma_1 + \gamma_2 \approx 0.5$; $\mathcal{S}^{(\mu)}$, the lepton spin term, involves the integral over s_1, s_2; and $[\text{M.E.}^{\text{nucl.}\,(\mu)}]_{if}$ involves the integral over $\mathbf{x}_n$, s_n, q_n $[\text{M.E.}^{\text{nucl.}\,(\mu)}]_{if}$ is just the nuclear matrix element of the usual theory of K-capture where, in effect, the system considered to represent the parent atom is the nucleus plus one K-shell electron which is subsequently transformed into a neutrino.

The total probability per unit time for K capture is similarly

$$1/\tau_K = 2\pi\hbar^{-1}\sum_f |M_{i\to f}|^2\rho_f$$

$$= 2\pi\hbar^{-1}\sum_f \left|\int \chi_f^*(\mathbf{r}_2)\{N(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}} \times\exp[-(r_2/a)(Z_i-\gamma)]\}d\mathbf{r}_2\right|^2$$

$$\cdot \sum_{\sigma_\nu,\,\sigma_e,\,M_f} |\mathcal{S}^{(\mu)}|^2|\mathrm{M.E.}^{\mathrm{nucl.}\,(\mu)}]_{if}|^2 \times 4\pi[p_\nu]_f^2V(8\pi^3\hbar^3c)^{-1}$$

$$= \sum_f\{[p_\nu]_f^2/(p_\nu^{(0)})^2\}\left|\int\cdots d\mathbf{r}_2\right|^2 \times\{2\pi\hbar^{-1}2\sum_{\sigma_\nu,\,M_f}|(\sigma_\nu|\Omega^{(\mu)}|+)|^2|[\mathrm{M.E.}^{\mathrm{nucl.}\,(\mu)}]_{if}|^2 \cdot 4\pi(p_\nu^{(0)})^2V(8\pi^3\hbar^3c)^{-1}\}, \quad (12)$$

where

$$c[p_\nu]_f = [(\mathfrak{M}_i-\mathfrak{M}_f)c^2 - B_{(Z_i-1)}] - B_{(Z_i-1)} - \epsilon_f \equiv cp_\nu^{(0)} - (B_{(Z_i-1)}+\epsilon_f),$$

and the sum over f runs over every energetically possible final state for the noncaptured electron. Here $\mathfrak{M}_i$ and $\mathfrak{M}_f$ are atomic masses in ground-state electronic configurations, $B_{(Z_i-1)}$ is the K-shell electron binding energy about a nucleus of charge Z_i-1, and ϵ_f is the energy (not inclusive of the rest energy) of the non-captured K-shell electron in its final state.[9] Thus $cp_\nu^{(0)}$ is the neutrino energy in the conventional "one-electron" type calculation; in this type of calculation, the total K-capture probability per unit time $(1/\tau_K)_{\text{one-elec.}}$ is just the expression in the curly brackets in the third form of Eq. (12).

In order to perform the sum over f, we introduce the definition

$$K_{\mathrm{op}} \equiv 1-(cp_\nu^{(0)})^{-1}\left[B_{(Z_i-1)}+\left\{\frac{-\hbar^2}{2m}\nabla_{r_2}{}^2-\frac{(Z_i-1)e^2}{r_2}\right\}\right],$$

and note that

$$\tfrac{1}{2}(K_{\mathrm{op}}+|K_{\mathrm{op}}|)\chi_f(\mathbf{r}_2) = ([p_\nu]_f/p_\nu^{(0)})\chi_f(\mathbf{r}_2) \quad\text{or}\quad =0,$$

depending on whether $[p_\nu]_f>0$ or <0, i.e. depending on whether the final state f is energetically possible or not. Equation (12) now becomes

$$\frac{1}{\tau_K} = \sum_{\text{all } f}\left|\int[\tfrac{1}{2}(K_{\mathrm{op}}+|K_{\mathrm{op}}|)\chi_f(\mathbf{r}_2)]^*\{N(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}} \times\exp[-(Z_i-\gamma)r_2/a]\}d\mathbf{r}_2\right|^2(1/\tau_K)_{\text{one-elec.}}$$

$$= \int|\tfrac{1}{2}(K_{\mathrm{op}}+|K_{\mathrm{op}}|)\{N(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}} \times\exp[-(Z_i-\gamma)r_2/a]\}|^2 d\mathbf{r}_2(1/\tau_K)_{\text{one-elec.}}, \quad (13a)$$

the second equality following from the completeness of the χ_f since the sum over f now runs over all final states without exception. Neglecting $1-\gamma$ compared to Z_i-1 in $\exp\{-(r_2/a)[(Z_i-1)+(1-\gamma)]\}$ and remembering that $\exp[-(r_2/a)(Z_i-1)]$ is an eigenfunction of K_{op} with eigenvalue 1, we obtain from Eq. (13a)

$$1/\tau_K \cong N^2Z_i^3(Z_i-\gamma)^{-3}(1/\tau_K)_{\text{one-elec.}}, \quad (13b)$$

with a relative error of order $[(1-\gamma)/(Z_i-1)]\cdot[B_{(Z_i-1)}/cp_\nu^{(0)}]$ if $B_{(Z_i-1)}/cp_\nu^{(0)}\ll 1$ and of order $(1-\gamma)/(Z_i-1)$ if $B_{(Z_i-1)}/cp_\nu^{(0)}\gtrsim 1$. The ratio of $1/\tau_K$ to $(1/\tau_K)_{\text{one-elec.}}$ in Eq. (13b) is just the ratio of the electronic charge densities at the nucleus, calculated in the present, and, in the "one-electron type," formulations.

Thus, the probability, per K-capture, P_{remain}, for the noncaptured electron to remain in the K-shell is, from Eqs. (11), (12), (13b),

$$P_{\text{remain}} = \frac{N^2(Z_i-1)^3Z_i^3}{[Z_i-\frac{1}{2}(1+\gamma)]^6}\left(\frac{1}{\tau_K}\right)_{\text{one-elec.}} \div\frac{N^2Z_i^3}{(Z_i-\gamma)^3}\left(\frac{1}{\tau_K}\right)_{\text{one-elec.}}$$

$$\cong 1-(3/4)(1-\gamma)^2/Z_i^2, \quad (14)$$

neglecting higher order terms in $1/Z_i$.[10] The probability per K-capture for the production of a double hole in the K-shell is, then,

$$1-P_{\text{remain}} \cong (3/4Z_i^2)(1-\gamma)^2 \cong 3/16Z_i^2. \quad (15)$$

The sensitivity of this result [as well as of others below, e.g., Eq. (17)] to the somewhat uncertain value of γ should be especially noted. *Note added in proof:*— In addition, in the actual many-electron atom, the probability per K-capture for the production of a double hole in the K-shell is equal to $1-\{$probability per K-

[9] The present formulation makes it obvious that in the analogous case of negatron (or positron) emission from β-unstable nuclei, the available transition energy arising both from the difference in nuclear mass and from the difference in atomic binding (as between parent and daughter atoms) is shared statistically between the emitted negatron, the emitted neutrino, and any excited or ejected orbital electron. The necessity of inclusion of this difference of atomic binding energy into the transition energy so that it is shared statistically between the emitted negatron and the emitted neutrino, has recently been emphasized by H. M. Schwartz [Phys. Rev. **86**, 195 (1952)]. See also R. Serber and H. S. Snyder, Phys. Rev. **87**, 152 (1952); and, Freedman, Wagner, and Engelkemeir, Phys. Rev. **88**, 1155 (1952).

[10] The quantitative validity of the deviations of P_{remain} from 1 as given in Eq. (14) holds only when Eq. (13b) for $1/\tau_K$ is quantitatively valid, i.e., in the usual case: $B_{(Z_i-1)}/cp_\nu^{(0)}\ll 1$. If $B_{(Z_i-1)}/cp_\nu^{(0)}\gg 1$, the only possible value for ϵ_f is $-B_{(Z_i-1)}$, and $1/\tau_K = N^2(Z_i-1)^3Z_i^3[Z_i-\frac{1}{2}(1+\gamma)]^{-6}(1/\tau_K)_{\text{one-elec.}}$, yielding, as required, $P_{\text{remain}}=1$.

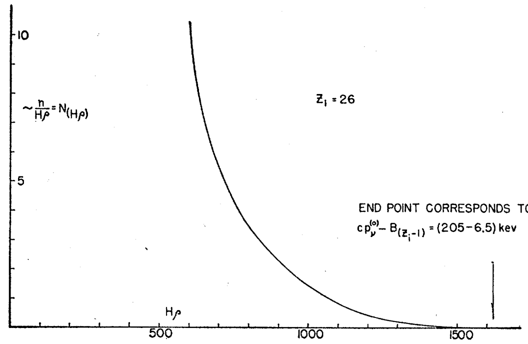

FIG. 1. Predicted shape of the momentum distribution of ejected orbital electrons [Eq. (18a, b)] with energies above 30 kev, accompanying K-capture for $Z_i=26$ and with $cp_\nu^{(0)}=205$ kev. Note that the end point would be difficult to determine from any experimental data unless this data is treated as suggested in the text.

capture for the production of a single hole in the K-shell$\}-\{$probability per K-capture for the production of zero holes in the K-shell$\}\equiv 1-p^{(1)}-p^{(0)}\equiv p^{(2)}$; a treatment entirely analogous to the above, but using atomic many-electron wave functions built up out of an appropriately antisymmetrized superposition of products of hydrogenic one-electron orbital wave functions, indicates that $1-p^{(1)}\cong 1-P_{\text{remain}}$ of Eqs. (14), (15), while

$$p^{(0)}\cong \sum_{n=2}^{\infty} [(\Delta Z)_n/Z_i]^2 C_n \eta_n.$$

In the expression for $p^{(0)}$, the successive terms in the series give the contributions of the L, M, $\cdots$ shells, the η_n being 1, $\frac{1}{2}$, or 0 depending on whether the ns orbit in the corresponding shell is filled, half-filled, or unfilled; the C_n are given by $C_2=0.31$, $C_3=0.06$, $\cdots$, while $(\Delta Z)_n$ is the difference between the effective nuclear charges associated with an hydrogenic ns orbit in the parent atom and an hydrogenic ls orbit in the daughter atom (in fact, the nth term in the series for $p^{(0)}$ is just the square of an integral analogous to the integral over $\mathbf{r}_2$ in Eq. (11) with the first factor there replaced by a daughter atom hydrogenic ls orbital wave function and the second factor by a parent atom hydrogenic ns orbital wave function). The value of $p^{(0)}$ can be roughly estimated as about one-third of $1-p^{(1)}$, so that $p^{(2)}\cong[(3/4Z_i^2)(1-\gamma)^2](2/3)$; the importance of this correction has been emphasized to us by Levinger (reference 7) who, however, estimates the correction factor to $p^{(2)}$ as $\frac{1}{2}$ instead of our 2/3. In light of the uncertainty regarding the exact numerical value of this correction factor, we have generally omitted it in the text below.

We now calculate the total probability in K-orbital electron capture that the other K-electron is ejected, and we also obtain the momentum distribution of the ejected K-electrons. In a manner wholly analogous to the above, the matrix element $M_{i\to f}$ for K-capture with the other K-electron ejected with a wave number $\mathbf{k}_e$ is [see Eqs. (5), (10), (11)],

$$\begin{aligned}
M_{i\to f}=&\int [V^{-\frac{1}{2}}\exp(i\mathbf{k}_e\cdot\mathbf{r}_2)F_{\mathbf{k}_e,Z_i-1}(\mathbf{r}_2)]^*\\
&\times\{N(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}}\exp[-(r_2/a)(Z_i-\gamma)]\}d\mathbf{r}_2\\
&\qquad\cdot \mathcal{S}^{(\mu)}\cdot[\text{M.E.}^{\text{nucl.}\,(\mu)}]_{if}\\
=&NV^{-\frac{1}{2}}(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}}\{[2\pi(Z_i-1)/(137\beta_e)]\\
&\times[1-\exp(-2\pi(Z_i-1)/137\beta_e)]^{-1}\}^{\frac{1}{2}}\\
&\cdot\{8\pi/a[(Z_i-\gamma)^2/a^2+k_e^2]^2\}\\
&\cdot\left[\frac{Z_i-\gamma-ik_ea}{Z_i-\gamma+ik_ea}\right]^{(Z_i-1)/i137\beta_e}[(Z_i-\gamma)-(Z_i-1)]\\
&\qquad\cdot \mathcal{S}^{(\mu)}\cdot[\text{M.E.}^{\text{nucl.}\,(\mu)}]_{if},
\end{aligned}\tag{16}$$

the integral over $\mathbf{r}_2$ being evaluated from formulas given in A. Sommerfeld's *Wellenmechanik* (this integral with $\gamma=0$ has also been previously evaluated by Migdal[5] and by Feinberg[6] in their problem of orbital electron ejection associated with nuclear negatron decay). The total probability per K-capture, P_{ejec}, for ejection of the other K-electron with any (energetically possible) wave number $k_e=(mc/\hbar)p_e$ is then from Eqs. (16), (12), (13b),

$$\begin{aligned}
P_{\text{ejec}}=&\left[\frac{N^2Z_i^3}{(Z_i-\gamma)^3}\right]^{-1}\int_0^{(p_e)_{\max}}\frac{N^2}{\pi}\frac{Z_i^3(137)^3}{(Z_i-\gamma)^8}64\pi^2(1-\gamma)^2\\
&\times\left\{\frac{2\pi(Z_i-1)}{137\beta_e}\left(1-\exp\left[\frac{-2\pi(Z_i-1)}{137\beta_e}\right]\right)^{-1}\right\}\\
&\cdot\exp\left[-4\tan^{-1}\left(\frac{137p_e}{Z_i-\gamma}\right)\frac{(Z_i-1)}{137\beta_e}\right]\\
&\times\{1+[137/(Z_i-\gamma)]^2p_e^2\}^{-4}\\
&\cdot[1-(cp_\nu^{(0)})^{-1}(B_{(Z_i-1)}+mc^2p_e^2/2)]^2\\
&\qquad\times(2\pi^2)^{-1}p_e^2dp_e
\end{aligned}\tag{17a}$$

or remembering that

$$(p_e)_{\max}=[(Z_i-1)/137][(cp_\nu^{(0)}/B_{(Z_i-1)})-1]^{\frac{1}{2}},$$

and neglecting the difference between $Z_i-\gamma$ and Z_i-1 and between β_e and p_e,

$$P_{\text{ejec}}\cong[(1-\gamma)^2/(Z_i-\gamma)^2]0.3\cong 0.08/Z_i^2. \tag{17b}$$ [11]

The approximate equalities in Eq. (17b) are valid for the usual case: $cp_\nu^{(0)}/B_{(Z_i-1)}\gg 1$. On the other hand, near the energetic threshhold for other K-electron ejection: $[cp_\nu^{(0)}/B_{(Z_i-1)}]-1\ll 1$, the integral in Eq.

[11] From Eqs. (15), (17b) one sees that the probability per K-capture, of excitation of the other K electron into a previously unoccupied orbit of the actual many electron atom is $\cong(0.75-0.30)(1-\gamma)^2/Z_i^2$. The corresponding excited state of the daughter atom then possesses two holes in the K-shell (as indeed it also does after other-K-electron ejection), and, in all cases of interest is energetically capable of spontaneous ionization. In the readjustment of this atomic excited state to the ground state one will thus in general have even more Auger electrons emitted (with kinetic energy $\lesssim B_{(Z_i-1)}$) than in the readjustment of the atomic excited state following a K-capture with the other K-electron remaining in the K-shell.

(17a) gives

$$P_{\mathrm{ejec}}\cong[(1-\gamma)^2/(Z_i-\gamma)^2](32/3)\times e^{-4}(cp_\nu^{(0)}/B_{(Z_i-1)}-1)^3. \quad (17c)$$

Equation 17(a) shows that the ejected electron momentum distribution $D_{\mathrm{ejec}}(p_e)dp_e$ is of the form (see Fig. 1)

$$D_{\mathrm{ejec}}(p_e)=D_{\beta^-}(p_e)\exp\left\{\frac{-4(Z_i-1)}{137\beta_e}\tan^{-1}\left(\frac{137p_e}{Z_i-\gamma}\right)\right\}\times\left[1+\left(\frac{137}{Z_i-\gamma}\right)^2p_e^2\right]^{-4}, \quad (18a)$$

where

$$D_{\beta^-}(p_e)=\mathrm{const}\times p_e^2\left[\frac{cp_\nu^{(0)}}{mc^2}-\frac{B_{(Z_i-1)}}{mc^2}-\frac{p_e^2}{2}\right]^2\times\left[\frac{2\pi(Z_i-1)}{137\beta_e}\left(1-\exp\left[\frac{-2\pi(Z_i-1)}{137\beta_e}\right]\right)^{-1}\right] \quad (18b)$$

is the type of momentum distribution found in an allowed negatron transition of a beta-unstable nucleus with available kinetic energy $cp_\nu^{(0)}-B_{(Z_i-1)}$. It is, therefore, seen that the momentum distribution of the ejected K-electrons is very different from the momentum distribution of negatrons from beta-unstable nuclei, since $\exp\{-4[(Z_i-1)/137\beta_e]\tan^{-1}[137p_e/(Z_i-\gamma)]\}\times\{1+[137/(Z_i-\gamma)]^2p_e^2\}^{-4}$ is a very rapidly varying function of p_e—in fact for $(2cp_\nu^{(0)}/mc^2)^{\frac{1}{2}}\gg p_e\gg Z_i/137$, we have

$$D_{\mathrm{ejec}}(p_e)\cong[(1-\gamma)^2/Z_i^2]64(Z_i/137p_e)^7(137/Z_i)\times[\exp(2\pi Z_i/137\beta_e)-1]^{-1}. \quad (18c)$$

It is also apparent from the form of Eqs. (18a), (18b) that the data in any future observation of ejected electrons associated with K-capture may be treated in a manner analogous to the treatment of nuclear β-momentum distributions via the F-K plot. Thus, one can graph

$$\{N(p_e)\}^{\frac{1}{2}}\div\left\{\exp\left[-\frac{4(Z_i-1)}{137\beta_e}\right]\tan^{-1}\left(\frac{137p_e}{Z_i-\gamma}\right)\right\}\times\left[1+\left(\frac{137p_e}{Z_i-\gamma}\right)^2\right]^{-4}\left\{\frac{2\pi(Z_i-1)}{137\beta_e}\times\left[1-\exp\left(-\frac{2\pi(Z_i-1)}{137\beta_e}\right)\right]^{-1}p_e^2\right\}^{\frac{1}{2}}$$

vs $1+p_e^2/2$ and a straight line should result. The extrapolated end point of the plot

$$1+cp_\nu^{(0)}/mc^2-\tfrac{1}{2}(Z_i-1)^2/(137)^2=1+(\mathfrak{M}_i-\mathfrak{M}_f)/m-\tfrac{1}{2}(Z_i-1)^2/(137)^2-\tfrac{1}{2}(Z_i-1)^2/(137)^2$$

will then give the K-capture transition energy

$$[(\mathfrak{M}_i-\mathfrak{M}_f)/m-\tfrac{1}{2}(Z_i-1)^2/(137)^2]mc^2.$$

IV. DISCUSSION

It is to be noted that our Eq. (15) for the total number of double holes per K-capture and Eq. (17) for the number of K-electrons ejected per K-capture involve the shielding and correlation parameters γ_1 and γ_2 (through their sum $\gamma=\gamma_1+\gamma_2$) in the initial wave function of the two K-electrons, bearing out the remark made earlier regarding the importance for our problem of a reasonably accurate choice of $u_{\mathrm{ini}}(\mathbf{r}_1,\mathbf{r}_2)$. Physically, our process involves the effect, on one of the two Coulomb interacting electrons, arising from the transformation of the other into a neutrino (with simultaneous passage of a nuclear proton into a neutron); it is then obvious that a proper theory of the process necessitates a reasonably accurate description of the electron-electron interaction. The present point of view also indicates that, if one considers the possibility of L-electron ejection accompanying K-capture, the appropriate (effectively two-atomic electron) initial lepton space wave function is

$$u_{\mathrm{ini}}\sim\exp[-Z_ir_1/a]\exp[-(Z_i-2)r_2/a]\times[1-(r_2/2a)(Z_i-2)],$$

while the appropriate (effectively one-ejected atomic electron) final lepton space wave function becomes

$$u_{\mathrm{fin}}\sim\exp(i\mathbf{k}_\nu\cdot\mathbf{r}_1)\exp(i\mathbf{k}_e\cdot\mathbf{r}_2)F_{\mathbf{k}_e;Z_i-2}(\mathbf{r}_2).$$

Here, it is to be noted that the shielding parameters for the K and L electrons in u_{ini} are taken as 0 and 2, respectively, and that the correlation factor is neglected. This is roughly justified by examination of variational calculations in He and Li.[12] It is also to be noted that in u_{fin} the effective nuclear charge seen by the ejected (originally L) electron after the K-capture is taken as Z_i-2. The justification of this last choice involves the idea that, at values of $\mathbf{r}_2$ where the ejected L-electron wave function in u_{fin} makes the most important contribution to $M_{i\to f}$, the electrostatic potential which is due to the daughter nuclear charge of Z_i-1 plus the electrostatic potential due to the uncaptured K-electron very roughly look like the electrostatic potential of a point charge of Z_i-2. Hence, it is not too inaccurate to use, as a first approximation for the Coulomb field correction factor to the plane wave describing the ejected electron, a confluent hypergeometric function appropriate to an effective nuclear charge of Z_i-2, $[F_{\mathbf{k}_e;Z_i-2}(\mathbf{r}_2)]$. It then follows that the probability of L-electron ejection accompanying K-capture is relatively small, in our approximation zero, because of the orthogonality of the factors depending on $\mathbf{r}_2$ in u_{ini} and u_{fin}; these factors will enter into the expression for $M_{i\to f}$ in a manner analogous to that in Eq. (16). Physically, the L-electron is shielded from the Coulomb force of the nucleus by the two K-electrons in such a way that, in the simultaneous transformation of one of the nuclear protons into a neutron together with that of one of the K-shell electrons into a neutrino, the L-electron experiences comparatively little change in its electrostatic environment and so has small incentive to leave its original orbit. On the other hand it is obvious that this conclusion is extremely sensitive

[12] See reference 8, p. 365.

to any departures from orthogonality of u_{ini} and u_{fin}, so that its quantitative validation will require a rather accurate (and at present unavailable) calculation of the form of the ejected L-electron wave function.

In any case, one can show that the momentum spectrum, of whatever ejected L, M, etc., electrons may be associated with K-capture, must have approximately the same dependence on p_e, for $(mcp_e)^2/2m \gg B_{(Z_i-1)}$, as the momentum spectrum of the ejected K-electrons [Eqs. (17), (18)]. This can be seen by evaluating integrals analogous to that in Eq. (16) with $(u_{\text{ini}})_{K,\, L \text{ or } M \text{ or } \cdots}$ replacing $(u_{\text{ini}})_{K,\cdot K}$.

The possibility of electron ejection (from the K, L, $\cdots$ shells) accompanying orbital L-capture should also be mentioned. Discussing for simplicity the case of allowed transitions, one sees, first of all, that only capture from the $2s$ orbits need be considered, since the nuclear matrix element involves u_{ini} at points within the nucleus where the (nonrelativistic) $2p$ orbital wave functions effectively vanish. On the other hand, the normalizing factor in the ns orbital wave functions is proportional to $n^{-\frac{3}{2}}(Z_i-\gamma_{Z_i;ns})^{\frac{3}{2}}$, so that, for example, the contribution of L-capture relative to K-capture, to the K-electron ejection, is smaller approximately in the ratio

$$\frac{2(Z_i-\gamma_{Z_i;2s})^3}{2^3(Z_i-\gamma_{Z_i;1s})^3}\left(\frac{1-(1/2^2)B_{(Z_i-1)}/(\mathfrak{M}_i-\mathfrak{M}_f)c^2}{1-B_{(Z_i-1)}/(\mathfrak{M}_i-\mathfrak{M}_f)c^2}\right)^2.$$

The momentum distribution is again essentially given by Eqs. (17), (18). Analogous arguments can be given for the various forbidden transitions.

It now remains to discuss the related problem, previously treated by Migdal[5] and by Feinberg,[6] of orbital electron ejection associated with negatron (or positron) emission from a beta-unstable nucleus. In the perturbation scheme used in the present paper, we would have, for example, in the case of negatron emission with K-shell electron ejection

$$u_{\text{ini}}=V^{-\frac{1}{2}}\exp(-i\mathbf{k}_\nu\cdot\mathbf{r}_1)(\pi a^3)^{-\frac{1}{2}}Z_i^{\frac{3}{2}}\exp(-Z_ir_2/a),$$
$$u_{\text{fin}}=V^{-\frac{1}{2}}\exp(i\mathbf{k}_\beta\cdot\mathbf{r}_1)F_{\mathbf{k}_\beta,Z_i+1}(\mathbf{r}_1)V^{-\frac{1}{2}}\times\exp(i\mathbf{k}_e\cdot\mathbf{r}_2)F_{\mathbf{k}_e,Z_i+1}(\mathbf{r}_2),$$

as the appropriate forms of the (effectively one-atomic electron) lepton space wave functions. The shielding and correlation factors analogous to the γ_1, γ_2 above are quite negligible in this u_{fin} because of the relatively large velocity of the negatron (shielding and relativistic corrections in the electron part of u_{ini} are also neglected). The ejected electron spectrum for a given $\beta^{\mp}$ momentum is then given by Eqs. (18) with γ replaced by 0, (Z_i-1) replaced by $(Z_i\pm1)$, and $cp_\nu^{(0)}/mc^2$ redefined as $(\mathfrak{M}_i-\mathfrak{M}_f)/m-[(1+p_\beta^2)^{\frac{1}{2}}-1]-(1\mp1)$. This result has already been obtained by Migdal[5] and by Feinberg[6] using an argument in which the Coulomb interaction between the β^- and the ejected K-shell electron, as well as the β-decay nucleon-lepton interaction, are treated as perturbations in the sense of the time-dependent perturbation theory. We may also add that we have calculated the probability for orbital electron ejection associated with K-capture by this last method, with the further refinements of (a) use of the Möller rather than the Coulomb interaction between the two electrons, and (b) consideration of virtual intermediate states of negative as well as positive energy for them. The results obtained agree with our work above, in the nonrelativistic approximation, i.e., to within errors $\approx(mcp_e)^2/2m(mc^2)$.

An interesting difference between L, $M\cdots$ orbital electron ejection associated with K-capture (as treated above) and L, $M\cdots$ orbital electron ejection associated with $\beta^{\mp}$ emission must now be emphasized. We have seen that in the K-capture case relatively little L, $M\cdots$ orbital ejection occurs as a consequence of the near orthogonality of the bound and unbound wave functions of the electron which is to be ejected (effective charge $\approx Z_i-2$ for an L electron, see above). In the $\beta^{\mp}$ case, however, the unbound wave function of any ejected L-electron must be taken as approximately appropriate to an effective nuclear charge $\approx[(Z_i\pm1)-2]$ so that nothing like near orthogonality obtains. It even follows, as pointed out by Migdal,[5,7] that the total probability of electron ejection from the L or the M shell in $\beta^{\mp}$ emission is actually larger than that from the K-shell. We may add the remark that, as in the second paragraph of this section, the momentum distribution of electrons ejected from the L, $M\cdots$ shells and from the K-shell should be roughly the same for $(mcp_e)^2/2m\gg B_{(Z_i\pm1)}$.

Certain comments must be made in conclusion. It will be seen from Eqs. (11)–(18) above that in the nonrelativistic approximation for the leptons in the electron state, the probability, per nuclear disintegration, of electron excitation or ejection accompanying K-capture (and also $\beta^{\mp}$ emission) is independent of the form of the nucleon-lepton beta-interaction and of the degree of forbiddenness of the K or $\beta^{\mp}$-transition. In addition, the calculated mean life for K-capture results in a slightly different expression when one takes into account all possible "other" K-electron excitation and ejection processes than if one considers the "one-electron" model. Thus Eq. (13b) shows that

$$(1/\tau_K)\div(1/\tau_K)_{\text{one-elec.}}=N^2Z_i^3/(Z_i-\gamma)^3\approx1-1/Z_i.$$

This necessitates a small adjustment, compared to the conventional procedure, in estimating nuclear matrix elements for K-capture from the observed values of τ_K and $\mathfrak{M}_i-\mathfrak{M}_f$. On the other hand, the conventionally calculated mean lives for $\beta^{\mp}$ emission are unaffected (in our approximation) by the possibility of orbital electron excitation and ejection, since, in this case, the electrostatic shielding of the nuclear charge by the $\beta^{\mp}$, and the $\beta^{\mp}$-orbital electron correlation effects are negligible.

V. SUGGESTED EXPERIMENTAL VERIFICATION OF THE THEORY

In our opinion no certain experimental observation of orbital electron ejection associated with orbital electron capture by nuclei has been reported up to the present. However, Bruner's[13] magnetic spectrometer work on negative electrons accompanying β^+ emission in $_{21}Sc^{44}$ should be briefly discussed. Bruner's value of 0.04 for the total number of electrons with energy greater than 30 kev, per β^+ disintegration, is much higher than the value[14] $2\times(2\times10^{-5})$ predicted by Migdal,[5] by Feinberg,[6] and by Eqs. (17)–(18) above with $\gamma=0$ and $(137/Z_i)p_e>(137/21)(2\times30/511)^{\frac{1}{2}}=2.3$. The shape of Bruner's electron momentum distribution is also quite different from that predicted above, he having observed relatively many more high energy electrons than would be expected from Eq. (18). Altogether, no satisfactory theoretical explanation of Bruner's results seems available. Admitting the experimental difficulties, magnetic spectrometer work still appears to offer the best means of checking the shape of the ejected electron momentum spectrum, at least in the case of K-capture or of β^+ emission where there is no superimposed spectrum of β^- electrons. For example, for reasonably successful detection (4 times background) of ejected electrons with energies, say, $\gtrsim30$ kev, working with a typical spectrometer-counter arrangement (effective solid angle $\cong10^{-3}\times4\pi$ steradians: effective resolution $\cong10^{-2}$, counter background $\cong0.1$ count/sec), one would need a source strength of [see Eq. (18)]

$$4\times0.1\times10^3\times10^2\times\left\{(1-\gamma)^2\times2\times10^{-5}\left[\left(\frac{Z_i}{21}\right)^4\times\exp\left[-\left(\frac{2\pi}{2.3}\right)\left(\frac{Z_i}{21}-1\right)\right]\right]\right\}^{-1}\text{ disintegrations/sec,}$$

or

$$50(1-\gamma)^{-2}(21/Z_i)^4\exp[(2\pi/2.3)(Z_i/21-1)]$$

millicuries of K-capturing material.[15] In addition, to avoid excessive "thick source" difficulties, this quantity of radioactive material on a spectrometer source should have a surface density of less than 10^{-1} mg/cm² (i.e., a specific activity≈curie/mg). The above expression for the necessary amount of K-capture material indicates that for every K-capture there must be less than

$$(1/100)\{(Z_i/21)^4\exp[(-2\pi/2.3)(Z_i/21-1)](1-\gamma)^2\times2\times10^{-5}\}$$

extraneous β^- disintegrations if no more than 1 percent impurity contribution to the electron counting rate is to be tolerated. Such requirements on the specific activity and radiochemical purity are obviously severe but perhaps may be attained.

One method of detection of the ejected electrons accompanying K-capture which does not require as strong sources as the spectrometer method (but requires comparable purity criteria) is the magnetic cloud-chamber technique. An experiment of this type has recently been performed[16] in which an upper limit, consistent with Eqs. (17), (18), is set on the number of ejected electrons with energies $\geqq30$ kev accompanying K-capture in $_{26}Fe^{55}$. On the other hand, magnetic spectrometer observation of the ejected electrons accompanying K-capture, in coincidence with the immediately afterward emitted K x-ray photons or the corresponding Auger electrons, while clearly not subject to stringent purity requirements nevertheless poses other difficult problems.

Another prediction of the theory here developed which may become accessible to experimental verification involves the expected number of double holes in the K-shell per K-capture: $3/16Z_i^2$ [Eq. (15)]. This quantity might be observed by comparison of the relative numbers, of K x-ray photon proportional counter pulses of the normal energy, and, of pulses of twice this energy, since, the filling of any double hole will result in the emission of two photons practically coincident in time. Thus, with a 4π solid angle proportional counter, the ratio r of the total number of double energy pulses per sec to the number of single energy pulses per sec, N_{single}, is expected to be

$$r=(3/16Z_i^2)+N_{\text{single}}\tau,$$

where τ is the effective resolving time of the proportional counter and the second term represents the contribution of accidental double energy pulses (arising from x-ray photons emitted by different atoms). For example, considering $_{26}Fe^{55}$ and taking $\tau\cong10^{-6}$ sec, N_{single} must be $\leqq30$ sec^{-1}, if the accidental double energy counting rate is to be $\leqq10$ percent of the real rate. Such a low value for N_{single} places stringent requirements on the allowable background rate.

A final possibility for verification of the ideas of electron ejection accompanying β^- emission involves the detection of the number of single K-shell holes due to K-electron excitation and ejection; this number is $2\times(3/4Z_i^2)$ per β^- emission [see Migdal,[5] Feinberg[6] and our Eqs. (14) and (15) with $\gamma=0$]. These single holes in the K-shell may be observed by the detection of the subsequent x-ray photons. Such photons have been reported by Novey[17] in RaE, where $2\times(3/4Z_i^2)\cong2\times10^{-4}$. Novey finds roughly one K x-ray photon per 10^4 β^- decays, which seems of the correct order of magnitude. *Note added in proof:*—Similarly, J. J. Howland, Jr., and W. Rubinson at Brookhaven have recently observed 5×10^{-4} x-ray per β^- emission in $_{16}S^{35}$ (private communication from J. Levinger, reference 7) and this is again in order of magnitude agreement with the theoretical expectation of (see Eq. (15), et seq.) $2\times(\frac{3}{4}Z_i^2)\times(2/3)\times$fluorescence yield$=2\times[\frac{3}{4}\cdot(16)^2]\times2/3\times10^{-1}=4\times10^{-4}$.

[13] J. A. Bruner, Phys. Rev. **84**, 282 (1951).

[14] The additional factor of 2 outside the parenthesis is inserted since there are in this case two K-electrons to be ejected. Inclusion of the contribution of L, M, and N shell ejection with energy greater than 30 kev will raise the theoretically expected number from $2\times(2\times10^{-5})$ to a value some twenty percent larger $(\approx2\times(2\times10^{-5})\{1^{-3}+2^{-3}+3^{-3}+4^{-3}\})$. Moreover, as pointed out above, the ejected electron momentum distribution for $(mcp_e)^2/2m\gg B(Z_i-1)$ is roughly the same from all shells.

[15] Note that this expression has its minimum at $Z_i=33$.

[16] F. T. Porter and H. P. Hotz, following paper [Phys. Rev. **89**, 938 (1953)]

[17] T. B. Novey, Phys. Rev. **86**, 619 (1952).