In the individual-particle model the expectation value of M_σ in a closed shell (± 1) nucleus is the sum of several terms. Each term is the product of a spatial integral and a matrix element over ordinary and isotopic spin. If the function $f(r_{uv})$ is nowhere negative, the spatial integrals seem to be non-negative. The sign of the integrals has been established for the cases in which $f(r_{uv})$ is a delta-function, a Coulomb potential, or a constant independent of the distance r_{uv} between nucleons u and v. For this reason M_σ tends to put the calculated magnetic moments outside the Schmidt lines; but nearly all the measured magnetic moments are inside. There are at least two ways to reverse such an unwelcome conclusion. Perhaps better wave functions should be used. Perhaps $f(r_{uv})$ is positive when the nucleons are close together and negative when they are far apart, like the nuclear interaction proposed by Lévy.[7] The M_σ exchange moment seems to have the same order of magnitude in light and in heavy nuclei.

The M_σ of four closed shell (± 1) nuclei were calculated. A delta-function was used for $f(r_{uv})$. The radial wave functions $R(r)$ were of the oscillator type

$$R(r)=P(r)\exp(-\tfrac{1}{2}\nu^2r^2),$$

in which $P(r)$ is a polynomial in r and ν is related to the nuclear radius. The results are listed in Table I. The values of $1/\nu$ are

TABLE I. Calculated and measured magnetic moments.

Nucleus	$1/\nu$	M_σ	M_l	M_x	ΔM
N^{15}	1.5	−0.18	0.1	−0.1	−0.02
O^{17}	1.5	−0.34	?	?	0.02
F^{19}	1.5	0.59	0.0	0.6	−0.16
K^{39}	2.0	−0.20	0.4	0.2	0.27

stated in units of 10^{-13} cm. The $1/\nu$ for H^3 was taken to be 1.6×10^{-13} cm. All magnetic moments are expressed in nuclear magnetons. The ΔM are the deviations of the measured magnetic moments from the Schmidt lines. The ΔM may be compared with the calculated M_x. The values of M_l are those calculated by Spruch.[8]

The preceding results are taken from a doctoral thesis submitted to the Graduate School of Cornell University. Professor Philip Morrison suggested the investigation. Prepublication copies of the papers by Ross and Osborne and Foldy were available.

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Superconducting Silicides and Germanides

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WHILE investigating the occurrence of superconductivity among the silicides and germanides of Groups IV, V, and VI transition metals, we have recently observed that the compound V_3Si becomes superconducting at about 17°K, apparently the highest temperature at which the phenomenon has so far been observed.[1] This compound and twenty-nine other silicides and germanides were prepared by sintering compressed pellets consisting of appropriate mixtures of the powdered elements for several hours in an atmosphere of purified helium at 1500°C (silicides) or 1000°C (germanides). Additional specimens which were prepared by melting the compressed pellets in an argon arc furnace gave essentially the same x-ray and superconducting results as those prepared by sintering. The presence of superconductivity was detected by ballistic measurement of the magnetic induction of the specimens in a magnetic field of a few oersteds; the transition temperatures quoted are those for the mid-point of the transition extrapolated to zero field.

The following compounds became superconducting at the temperatures given in parenthesis: V_3Si (17.0°K), V_3Ge (6.0°K), Mo_3Si (1.30°K), Mo_3Ge (1.43°K), $MoSi_{0.7}$ (1.34°K), $MoGe_{0.7}$ (1.20°K), $WSi_{0.7}$ (2.84°K), $ThSi_2$ (3.16°K). On the other hand, compounds which did not show superconductivity at temperatures just below 1.2°K were Ti_5Si_3, Ti_5Ge_3, TiSi, $TiSi_2$, $TiGe_2$, Zr_4Si, Zr_2Si, Zr_3Si_2, Zr_4Si_3, Zr_6Si_5, ZrSi, $ZrSi_2$, VSi_2, $NbSi_{0.6}$, $NbSi_2$, $TaSi_2$, Cr_3Si, Cr_3Si_2, CrSi, $CrSi_2$, WSi_2, $MoSi_2$. It will be noted that in the isomorphous series V_3Si, V_3Ge, Mo_3Si, Mo_3Ge, and Cr_3Si, which have a cubic structure with atomic positions similar to those in β-tungsten, only the chromium compound remained normal down to 1.2°K.

The transition temperature and breadth of transition of V_3Si were found to be rather sensitive to variations in impurity content of the specimens. The purest samples were prepared from vanadium supplied by The Electro Metallurgical Company in which the main impurities were about 0.1 percent of iron and manganese. In these samples, the transition temperatures ranged from about 16.5° to 17°K, the sharpest transition being that of an arc furnace specimen which passed from a completely normal to a completely superconducting state between 17.1° and 16.8°K. On the other hand, both sintered and arc furnace specimens of V_3Si prepared from vanadium containing about 1 percent of iron as its major impurity showed superconducting transitions close to 14.5°K with breadths of more than 1°K. This appreciable drop in transition temperature in the presence of 1 percent Fe suggests that even for our purest samples, containing about 0.1 percent Fe, the transition temperatures probably lie a few tenths of a degree below the correct value for spectroscopically pure V_3Si.

Finally, in an effort to produce superconductivity above 17°K, we replaced a portion of the vanadium or silicon in V_3Si by neighboring elements in the periodic system. The effect of replacing one-tenth of the vanadium by either Ti, Zr, Nb, Mo, Cr, or Ru, or one-tenth of the silicon by either B, C, Al, or Ge, was to depress the transition temperature by amounts ranging from a few tenths to more than ten degrees below that of control specimens of pure V_3Si. Carbon and boron produced the smallest effect, but it must be remarked that although the whole series of specimens was prepared by arc furnace melting, completely homogeneous solid solutions were not formed in all cases.

A detailed account of this work will be published later.

[1] Although Aschermann, Friederich, Justi, and Kramer [Physik. Z. **42**, 349 (1941)], reported superconductivity in niobium nitride at temperatures above 17°K, the more recent work of F. H. Horn and W. T. Ziegler [J. Am. Chem. Soc. **69**, 2762 (1947)] and H. Rögener [Z. Physik **132**, 446 (1952)] indicates that the transition point for this compound is approximately 15°K.

The Tensor Force Interaction between a Shell Closed Except for a Single Vacancy and an External Nucleon

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THE following results, here quoted without proof, are taken from the author's Ph.D. thesis.[1] They have been derived by an extension of the tensor operator methods introduced by Racah.[2]

General expressions for the tensor force interaction between two separate two-nucleon single-particle configurations have been obtained by L. W. Longdon, of Southampton University, and will soon be published. They are not discussed here. We are concerned with the interaction between two configurations both of which contain a group of $4(2l_1+1)-1$ equivalent nucleons of orbital momentum l_1 (almost closed shell) and both of which

contain an external nucleon, but this is of orbital momentum l_2 in the one configuration and l_2' in the other. The results are given for all of the neutral, symmetric, and charged isotopic spin dependencies.

The direct term is simply related to the corresponding one for the mixing of the two separate two-nucleon configurations l_1l_2 and l_1l_2', obtained from the above by replacing the almost closed shell by a single nucleon of the same orbital momentum, which can be found using Longdon's results. The relationship is as follows: The corresponding direct terms are identical for the neutral isotopic spin dependence and are of opposite sign for the other two cases.

On the other hand, the expression for the exchange term, which as here given is to be added to the direct, reduces to a very simple result. The isotopic spin factors for the three cases are as follows:

neutral	$\delta(T,0)\cdot 2$,
symmetric	$\delta(T,1)\cdot\frac{2}{3}$,
charged	$\delta(T,1)\{\delta(M_T,1)+\delta(M_T,T)\}$,

and the spin, orbit, and radial factor is

$$2\sqrt{5}\delta(S,1)(-1)^{l_1+\frac{1}{2}(l_2+l_2')-J}W(L1L'1,J2)\times\tfrac{1}{2}(2l_1+1)\{(2l_2+1)(2l_2'+1)Cl_1l_2LCl_1l_2'L'\}^{\frac{1}{2}}\times\left[\left\{\frac{C_{LL'2}}{3(2L+1)(2L'+1)}\right\}^{\frac{1}{2}}\{(2L'+1)A^L+(2L+1)B^{L'}\}-\sum_{K=0}^{\infty}\{5(2L+1)(2L'+1)C_{LK1}C_{L'K1}\}^{\frac{1}{2}}W(LL'11,2K)C^K\right].$$

In this expression $W(abcd,ef)$ is a Racah coefficient,[2] and the functions C_{abc} have been tabulated by Shortley and Fried.[3] Also, if one writes the general radial exchange integral,

$$\left.\begin{matrix}A^K\\B^K\\C^K\end{matrix}\right\}=\frac{1}{a_0^6}\int_0^\infty\int_0^\infty u_{n_1l_1}(r_1/a_0)u_{n_2l_2}(r_2/a_0)I^K(r_1,r_2)f(r_1,r_2)\,u_{n_2'l_2'}(r_1/a_0)u_{n_1l_1}(r_2/a_0)r_1^2r_2^2dr_1dr_2,$$

in which $u_{nl}(r/a_0)$ is the general normalized radial wave function (a_0 is the well parameter) and in which $I^K(r_1,r_2)$ is defined in terms of the radial distance dependence $J(r_{12})$ of the tensor force by the expansion

$$J(r_{12})/r_{12}^2=\sum_{K=0}^{\infty}I^K(r_1,r_2)P_K(\cos\theta_{12}),$$

then in the radial integral A^K we have $f(r_1,r_2)=r_1^2$; in B^K it is equal to r_2^2, and in C^K to r_1r_2.

We wish to draw attention to the similarity of these results to those for central forces as derived for the atomic case by Racah. We have also been able to obtain corresponding expressions with a two-particle spin-orbit interaction, but these are more complicated than those for the other forces and will be reported separately. A full account of the derivation of these results is contained in the writer's thesis.

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Approximate Wave Functions for Unbound Relativistic Particles in a Coulomb Field

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THE exact relativistic wave functions describing unbound charged particles moving in a Coulomb field are well known.[1] However, they have not proved convenient for actual calculations. This is because they depend on three coordinates; and, in the case of the Dirac equation, the spinor dependence is complicated.[2] For low energy particles, the nonrelativistic Schrödinger equation may be used. As is well known, this equation with a Coulomb potential is separable in parabolic coordinates, and the wave function is fairly simple, depending on only one coordinate. On the other hand, for high energies and low charge, the first Born approximation may be used; in fact, almost all calculations concerning relativistic Coulomb particles have been done in this way. Since the Born approximation is valid only for[3] $|n|\ll 1$, there is a considerable range of charge and energy values not covered by either approximation.

For the Dirac equation, this gap was partially filled by Sommerfeld and Maue.[1] The iterated Dirac equation for a charged particle moving in a Coulomb field is

$$\nabla^2\psi+\left\{\left(E-\frac{zZ}{r}\right)^2-\mu^2\right\}\psi=\left(zZe^2\gamma_4\boldsymbol{\gamma}\cdot\operatorname{grad}\frac{1}{r}\right)\psi,$$

and the solution is[4]

$$\psi(\mathbf{r})=\frac{N}{(2\pi)^{\frac{3}{2}}}e^{i\mathbf{k}\cdot\mathbf{r}}\left\{1-\frac{\gamma_4\beta}{2r}\boldsymbol{\gamma}\cdot\operatorname{grad}_k+O(n^2\beta^2)\right\}L_n(\rho)\Gamma; \quad (1)$$

also,

$$\varphi(\boldsymbol{\kappa})=-\frac{N}{2\pi^2}\operatorname*{limit}_{\epsilon\to 0}\left[\left\{\frac{\partial}{\partial\epsilon}+\frac{\beta}{2}\gamma_4\boldsymbol{\gamma}\cdot(\operatorname{grad}_k+\operatorname{grad}_\kappa)+O(n^2\beta^2)\right\}\frac{[\kappa^2+(\epsilon-ik)^2]^n}{[\epsilon^2+|\boldsymbol{\kappa}-\mathbf{k}|^2]^{1+n}}\right]\Gamma, \quad (2)$$

where E, ze, $\mathbf{k}$, μ are, respectively, the energy, charge, momentum, and rest mass of the particle; Ze/r is the Coulomb potential; γ_4, $\boldsymbol{\gamma}$ are the usual Dirac matrices; $N=2\pi in/(1-e^{-2\pi in})$ is a normalizing factor;

$$n=Zze^2/i\beta;\quad \rho=i(kr-\mathbf{k}\cdot\mathbf{r});\quad |\mathbf{k}|=k=\beta E;$$

$$L_n(\rho)=\sum_{j=0}^{\infty}(-)^j\binom{n}{j}\frac{\rho^j}{j!}; \quad (3)$$

Γ is a spinor satisfying $\{i\boldsymbol{\gamma}\cdot\mathbf{k}-\gamma_4E+\mu\}\Gamma=0$; and

$$\phi(\kappa)=(2\pi)^{-\frac{3}{2}}\int d\tau\,\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})\psi(\mathbf{r})$$

is the Fourier transform of $\psi(\mathbf{r})$.

The Sommerfeld-Maue approximation is valid for all energies, and is subject only to the condition $|n^2\beta^2|\ll 1$. This is to be distinguished from the first and second Born approximations, which are subject to the more restrictive conditions $|n|\ll 1$ and $|n^2|\ll 1$, respectively.

The first Born approximation is obtained from (1) and (2) by putting $n=0$. One finds

$$\psi(\mathbf{r})=\frac{1}{(2\pi)^{\frac{3}{2}}}e^{i\mathbf{k}\cdot\mathbf{r}}\Gamma,\quad \varphi(\boldsymbol{\kappa})=-\frac{1}{2\pi^2}\operatorname*{limit}_{\epsilon\to 0}\frac{\partial}{\partial\epsilon}\left\{\frac{1}{\epsilon^2+|\boldsymbol{\kappa}-\mathbf{k}|^2}\right\}\Gamma. \quad (4)$$

The second Born approximation is obtained from (1) and (2) by expanding to first order in n:

$$\psi(\mathbf{r})=\frac{N}{(2\pi)^{\frac{3}{2}}}e^{i\mathbf{k}\cdot\mathbf{r}}\left(1-\frac{\gamma_4\beta}{2r}\boldsymbol{\gamma}\cdot\operatorname{grad}_k\right)\{1+nf(\rho)\}\Gamma, \quad (5)$$

where

$$f(\rho)=\operatorname{Ci}|\rho|+i\operatorname{Si}|\rho|-\ln|\rho|-C=\int_0^1\frac{e^{\rho t}-1}{t}dt; \quad (6)$$

Ci and Si are the cosine- and sine-integral functions, and C is Euler's constant.

The Fourier transform of (5) is

$$\varphi(\boldsymbol{\kappa})=-\frac{N}{2\pi^2}\operatorname*{limit}_{\epsilon\to 0}\left[\left\{\frac{\partial}{\partial\epsilon}-\frac{\beta}{2}\gamma_4\boldsymbol{\gamma}\cdot(\operatorname{grad}_k+\operatorname{grad}_\kappa)\right\}\frac{1}{\epsilon^2+|\boldsymbol{\kappa}-\mathbf{k}|^2}\times\left\{1+n\ln\frac{\kappa^2+(\epsilon-ik)^2}{\epsilon^2+|\boldsymbol{\kappa}-\mathbf{k}|^2}\right\}\right]\Gamma. \quad (7)$$

For the Klein-Gordon equation with a Coulomb potential,

$$\nabla^2\psi+\left\{\left(E-\frac{zZe^2}{r}\right)^2-\mu^2\right\}\psi=0, \quad (8)$$