Measurement of Magnetostriction in Single Crystals

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A simplified procedure is given for determining the 5 magnetostriction constants of a single crystal of a ferromagnetic cubic crystal. The crystal is cut as a disk parallel to a (110) plane, and strain gauges are cemented to the surfaces to measure strains in [001] and [$1\bar{1}1$] directions. A magnetic field sufficient for saturation is oriented in 10° steps at various angles to the [001] direction, and magnetostriction is measured over a 90° range for each gauge. Each of the 18 data is then multiplied by suitable numbers, obtained by inversion of the strain matrix, to give the constants $h_1 \cdots h_5$. The method is applied to a crystal of a 78 percent nickel-iron alloy to determine the magnetostriction associated with spontaneous magnetization in the [111] direction: $\lambda_{111}=2h_2/3+2h_5/9$, a quantity important in the "Permalloy problem." The constants are also determined for a single crystal of nickel.

INTRODUCTION

The magnetostriction at saturation of single crystals of cubic ferromagnetic materials, such as iron and nickel and their alloys, is often described[1,2] by a 2-constant expression, but is more accurately described by the following expression in 5 constants[3]

$$\begin{aligned}\lambda = & h_1(\alpha_1^2\beta_1^2+\alpha_2^2\beta_2^2+\alpha_3^2\beta_3^2-1/3) \\ & +2h_2(\alpha_1\alpha_2\beta_1\beta_2+\alpha_2\alpha_3\beta_2\beta_3+\alpha_3\alpha_1\beta_3\beta_1)+h_3s \\ & +h_4(\alpha_1^4\beta_1^2+\alpha_2^4\beta_2^2+\alpha_3^4\beta_3^2+2s/3-1/3) \\ & +2h_5(\alpha_1\alpha_2\alpha_3^2\beta_1\beta_2+\alpha_2\alpha_3\alpha_1^2\beta_2\beta_3+\alpha_3\alpha_1\alpha_2^2\beta_3\beta_1), \quad (1)\end{aligned}$$

where

$$s=\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_3^2\alpha_1^2.$$

Here λ is the fractional change in length measured in the crystallographic direction having the direction cosines β_1, β_2, and β_3 with respect to the crystal axes, when the crystal is magnetically saturated in the direction having direction cosines α_1, α_2, and α_3. This equation gives the change in dimensions of a portion of the crystal, which is originally nonmagnetic, when it is spontaneously magnetized as a single domain in the specified direction. It also represents the changes in dimensions of a crystal composed of many domains, if they are originally oriented so that there are equal numbers in each of the crystallographically equivalent directions of easy magnetization ([111] directions in nickel, [100] in iron).

In previous work the constants have been evaluated only in nickel.[1] The present paper describes a new simplified procedure for determining these constants, and evaluates them for a crystal of nickel, and for a crystal composed of 78 percent nickel and 22 percent iron. This composition was chosen because (1) it is nearly that for the highest magnetic permeability of any of the binary iron-nickel alloys (Permalloys), and (2) the permeability is known to have a close connection to the magnetostriction. More particularly it is known from recent work[4] that the saturation magnetostriction in the [111] direction (λ_{111}) goes through zero at about the composition at which the permeability is a maximum. For a careful test of this coincidence, the composition for $\lambda_{111}=0$ should be known with some accuracy.

In order to determine λ_{111}, it is not sufficient to measure the change in length which occurs when a crystal is magnetized to saturation in the [111] direction, because the result of such a measurement depends on the initial domain distribution. The usual procedure in determining the saturation magnetostriction, so that it is independent of the initial distribution, is to measure the change in length when saturated in that direction (λ_l), and then the change when saturated at right angles to that direction (λ_t), the difference between these changes being independent of the initial domain distribution $\lambda=\frac{2}{3}(\lambda_l-\lambda_t)$. However, λ_{111} cannot be determined accurately in this manner, for evaluation of Eq. (1) shows that, although

$$\lambda_{111}=2h_2/3+2h_5/9, \quad (2)$$

nevertheless the difference in the magnetostrictions measured when a crystal is saturated first in the [111] direction and then in the perpendicular [$2\bar{1}\bar{1}$] direction is $\lambda_{111}-\lambda_{2\bar{1}\bar{1}}=h_2+2h_5/9$. Similarly, by saturating first in the [110] direction and then the [$1\bar{1}0$] direction, at right angles, we find that

$$\lambda_{110}-\lambda_{1\bar{1}0}=h_2.$$

However, λ_{111} can be evaluated by measuring the change in length first in the [111] direction and then in the [$11\bar{1}$] direction, in the ($1\bar{1}0$) plane, the difference being $(2h_2/3+h_3/3+2h_5/9)-(-2h_2/9+h_3/3+2h_5/27)=(4/3)(2h_2/3+2h_5/9=(4/3)\lambda_{111}$. In our experiments we have determined the 5 constants in the way to be described and then combined h_2 and h_5 according to Eq. (2).

[1] R. Becker and W. Döring, *Ferromagnetismus* (Julius Springer, Berlin, 1939).
[2] R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., New York, 1951).
[3] This applies to crystals having [100] as the direction of easy magnetization (e.g., iron). When [111] is the easy direction, the term $h_3/3$ must be subtracted.
[4] R. M. Bozorth and J. G. Walker, Phys. Rev. **88**, 1209 (1952).

TABLE I. Matrix elements for calculation of magnetostriction constants: $h_n = \sum_{i=1}^{18} \delta_{in}\lambda_i$. For data for quenched 78 percent nickel, given in last column, derived h's are given in Table III.

i	θ	Gauge direction	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{i5}	$\lambda_i(10)^6$
1	10°	[100]	0.3407	0	0	−0.3847	0	−0.48
2	20	[100]	1.1794	0	0	−1.3352	0	−1.83
3	30	[100]	2.0546	0	0	−2.3392	0	−3.58
4	40	[100]	2.4508	0	0	−2.8230	0	−6.10
5	50	[100]	2.0527	0	0	−2.4330	0	−8.92
6	60	[100]	0.9084	0	0	−1.2164	0	−11.50
7	70	[100]	−0.5766	0	0	0.3850	0	−13.28
8	80	[100]	−1.8134	0	0	1.7255	0	−14.35
9	90	[100]	−2.2924	0	0	2.2456	0	−14.47
10	10	[111]	3.9283	2.7614	−3.9283	−5.8924	−2.9159	0.38
11	20	[111]	4.3965	3.1713	−4.3965	−6.5947	−3.3247	0.87
12	30	[111]	2.5990	1.8917	−2.5990	−3.8986	−1.5557	1.24
13	40	[111]	0.3242	0.1363	−0.3242	−0.4863	1.0708	1.50
14	50	[111]	−0.9972	−0.9712	0.9972	1.4958	2.9206	1.59
15	60	[111]	−1.0053	−0.9598	1.0053	1.5080	2.8593	1.57
16	70	[111]	−0.5485	−0.2253	0.5485	0.8227	0.8153	1.40
17	80	[111]	−1.1835	+0.2015	1.1835	1.7752	−2.1742	1.14
18	90	[111]	−4.2853	−0.7800	4.2853	6.4279	−4.4587	0.81

EXPERIMENTAL PROCEDURE

Changes in length caused by magnetization are measured using the strain-gauge method described by Goldman.[5] For our measurements the gauge is cemented to a crystal disk in one position, and the disk is magnetized successively in a number of directions. Thus the β's of Eq. (1) are fixed and the α's varied, measurements of λ being made for a number of α's sufficient to determine the h constants with reasonable accuracy.

The choice of the crystal plane and the gauge position are important. If the crystal is cut parallel to the (100) plane, Eq. (1) shows that some of the terms become zero, so that evaluation of all of the constants is impossible. As a result of calculation it was decided to cut the crystal disk parallel to a (110) plane and place the gauge first parallel to [001], then parallel to $[1\bar{1}1]$ (respectively, 0° and 55° from the [001] direction). A magnetic field, sufficient to saturate the crystal, is applied to the [001] direction and at various angles θ to this direction up to $\theta=90°$ in steps of 10°. The observed values of magnetostriction, measured in the direction of the gauge when the field is reduced to zero, are designated λ_θ. In order to eliminate the effect of initial domain distribution, the data are then given in terms of $\lambda_\theta-\lambda_0$. The h-constants, adjusted for least square fit, are then derived from these 18 data as described in the next section.

It is important to align the gauge accurately so that it will measure the change in length in the specified direction. This is accomplished by observing the directions of the gauge wires under the microscope and aligning them with a scratch on the crystal surface which has previously been drawn parallel to the required crystallographic direction, as determined by x-rays. The accuracy of the gauge placement is estimated to be a few tenths of a degree.

[5] J. E. Goldman, Phys. Rev. **72**, 529 (1947).

THEORY AND RESULTS

Consider a cubic magnetic crystal cut parallel to a (110) plane, and a strain gauge placed on it so that it measures the change in length parallel to the [001] direction. Let a magnetic field, sufficient to produce saturation in the plane of the crystal, be applied at an angle θ from the [001] direction. Let the fractional change in length resulting from application of the field at angle θ be $\Delta l/l=\lambda_\theta$. If we measure λ_θ for each value of θ from $\theta=0$ to $\theta=90°$, in 10° steps and substract from each λ_θ the value of λ_θ for $\theta=0$, we have 9 data which may be represented by $\lambda_i=\lambda_\theta-\lambda_0=\lambda_1, \lambda_2, \cdots, \lambda_9$. According to Eq. (1), we then can write 9 equations ($i=1, \cdots, 9$)

$$\lambda_i=a_{i1}h_1+a_{i2}h_2+\cdots+a_{i5}h_5,$$

the a's being the trigonometric coefficients of the h's.

Because there are experimental errors involved, these equations will not be mutually consistent. If we know the correct values of the h's, the observed value of one

TABLE II. Matrix elements for calculation of magnetostriction as dependent on angle θ. Calculated values of λ_i for quenched 78 percent nickel are compared with observation in Fig. 1.

i	θ	Gauge direction	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}
1	10	[100]	−0.03015	0	0.02947	−0.03975	0
2	20	[100]	−0.11698	0	0.10671	−0.14913	0
3	30	[100]	−0.25000	0	0.20312	−0.30208	0
4	40	[100]	−0.41318	0	0.28514	−0.46554	0
5	50	[100]	−0.58682	0	0.32856	−0.61025	0
6	60	[100]	−0.75000	0	0.32812	−0.71875	0
7	70	[100]	−0.88302	0	0.29823	−0.78750	0
8	80	[100]	−0.96985	0	0.26439	−0.82283	0
9	90	[100]	−1.00000	0	0.25000	−0.83333	0
10	10	[111]	0	0.17128	0.02947	0	0.01218
11	20	[111]	0	0.34201	0.10671	0	0.05215
12	30	[111]	0	0.49158	0.20312	0	0.11353
13	40	[111]	0	0.60197	0.28514	0	0.17673
14	50	[111]	0	0.65985	0.32855	0	0.21703
15	60	[111]	0	0.65825	0.32812	0	0.21559
16	70	[111]	0	0.59735	0.29823	0	0.16821
17	80	[111]	0	0.48451	0.26439	0	0.08793
18	90	[111]	0	0.33333	0.25000	0	0

λ_i will differ from the correct value, calculated from the correct values of the h's, by an amount (the "error"),

$$\epsilon_i = a_{i1}h_1 + a_{i1}h_2 + \cdots + h_{i5}h_5 - \lambda_i,$$

and the sum of the squares of the errors in the 9 observations will be a minimum when

$$\frac{\partial}{\partial h_1}\left(\sum_{i=1}^{9} \epsilon_i^2\right) = 0 = 2\sum_{i=1}^{9} (a_{i1}h_1 + a_{i2}h_2 + \cdots + a_{i5}h_5 - \lambda_i)a_{i1},$$

or

$$\sum_{i=1}^{9} (a_{i1}h_1 + \cdots + a_{i5}h_5) = \sum_{i=1}^{9} a_{i1}\lambda_i.$$

Similarly four other equations are obtained by differentiating with respect to $h_2, \cdots h_5$. These 5 equations

TABLE III. Values of the five magnetostriction constants for nickel and for 78 percent nickel crystals. Estimated probable errors are given in parentheses below the values of the constants.

	78 percent Ni-Fe (quenched)	78 percent Ni-Fe (slowly cooled)	Nickel
h_1	13.7×10^{-6}	20.9×10^{-6}	-68.8×10^{-6}
	(1.0)	(0.7)	(3.8)
h_2	2.6	2.8	−36.5
	(0.5)	(0.3)	(1.9)
h_3	−0.3	1.7	−2.8
	(0.8)	(0.5)	(3.1)
h_4	1.1	−1.4	−7.5
	(1.4)	(1.0)	(5.2)
h_5	−0.1	−0.2	7.7
	(0.8)	(0.5)	(3.1)

are then solved for h_1 to h_5:

$$h_1 = \delta_{11}\lambda_1 + \delta_{12}\lambda_2 + \cdots + \delta_{19}\lambda_9,$$
$$h_2 = \delta_{21}\lambda_1 + \delta_{22}\lambda_2 + \cdots + \delta_{29}\lambda_9,$$
$$\cdots$$
$$h_5 = \delta_{51}\lambda_1 + \delta_{52}\lambda_2 + \cdots + \delta_{59}\lambda_9.$$

For the case considered it is found that these equations are not all independent. Consequently, a second gauge is placed to measure the magnetostriction in the [111] direction, and 9 additional data are recorded in a similar manner, with θ still representing the angle between the [001] direction and the direction of the field, and measured in such a sense that λ is parallel to the field when $\theta \approx 55°$. In practice the second gauge is cemented on the other side of the crystal. This set of data may be represented by λ_{10}, λ_{11}, $\cdots\lambda_{18}$. The above equations are then changed to

$$h_1 = \delta_{11}\lambda_1 + \delta_{12}\lambda_2 + \cdots + \delta_{1.18}\lambda_{18},$$

and so on.

The values of the δ's, obtained by solving these equations, are given in Table I. The values of h_1 to h_5

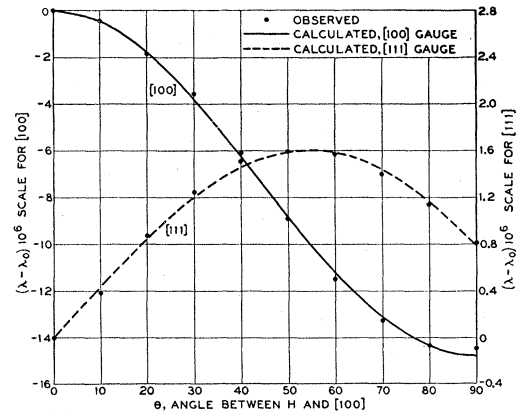

FIG. 1. Comparison of observed values of $\lambda_i = \lambda - \lambda_0$ with those calculated from $h_1, \cdots, h_5$ for least square fit. λ's were measured parallel to [100] and [111] directions.

are evaluated from the data, λ_1 to λ_{18}, and the coefficients, δ_{11} and $\delta_{18.5}$, using the summation

$$h_n = \sum_{i=1}^{18} \delta_{in}\lambda_i.$$

The observed λ_i's for the 78 percent nickel crystal are shown in the last column and the derived values of the h's are given in Table III.

It is desirable to compare the observed values of λ with those calculated from the constants h. The calculated λ's may be obtained from the summation

$$\lambda_i = \sum_{n=1}^{5} a_{in}h_n.$$

Values of the a's are given in Table II. Using the values of the h's given in Table III for quenched 78 percent nickel, we find the agreement between the calculated and observed variation of λ_i with θ as illustrated for this crystal in Fig. 1.

Similar data were taken for a single crystal of nickel, with the results shown in Table III. The results for 78 percent nickel are also repeated here, and the probable errors for all of the constants. The results for nickel diverge widely from those derived by Becker and Döring[1] from the data of Masiyama.[5]

The values of the constant h_3 for the 78 percent nickel alloy and for nickel indicate a small volume contraction associated with domain orientation. These are the first determinations of this kind. However, the values of the constants are small and in most cases do not exceed the experimental error. In slowly cooled 78 percent nickel the error is low enough so that the probability of a finite value of h_3 is 95 percent. In all of the specimens the values of h_4 and h_5 are not certainly different from zero.

[5] Y. Masiyama, Science Repts. Tôhoku Imp. Univ. **17**, 945 (1928).

TABLE IV. Longitudinal and transverse magnetostriction in (100) and (110) planes, as dependent on direction of measurement of change of length as shown in second column.

Plane	Direction	λ_{long}	λ_{trans}	$\lambda_{\text{long}}-\lambda_{\text{trans}}$
(100)	[001]	$2h_1/3+2h_4/3$	$-h_1/3-h_4/3$	h_1+h_4
	[011]	$h_1/6+h_2/2+h_3/4+h_4/12$	$h_1/6-h_2/2+h_3/4+h_4/12$	h_2
(110)	[001]	$2h_1/3+2h_4/3$	$-h_1/3+h_3/4-h_4/6$	$h_1-h_3/4+5h_4/6$
	$[1\bar{1}0]$	$h_1/6+h_2/2+h_3/4+h_4/12$	$-h_1/3-h_4/3$	$h_1/2+h_2/2+h_3/4+5h_4/12$
	$[1\bar{1}1]$	$2h_2/3+h_3/3+2h_5/9$	$-h_2/3+h_3/4$	$h_2+h_3/12+2h_5/9$
(111)	$[1\bar{1}0]$	$h_1/6+h_2/2+h_3/4+h_4/12$	$-h_1/6-h_2/6+h_3/4-5h_4/36-h_5/9$	$h_1/3+2h_2/3+2h_4/9+h_5/9$
	$[2\bar{1}\bar{1}]$	$h_1/6+h_2/2+h_3/4+5h_4/36+h_5/9$	$-h_1/6-h_2/6+h_3/4-h_4/12$	$h_1/3+2h_2/3+2h_4/9+h_5/9$

SPECIAL CASES

As it is in the experiments described above, the magnetostriction is often measured in a given direction in a plane, when the field giving rise to this magnetostriction is applied in any arbitrary direction in the plane. The magnetostriction then depends on the specified directions in the way described in the Appendix. These expressions were computed because they are important in determining the way in which the various constants enter into the data determined with different crystal and gauge orientations. The expressions in cases 3 and 5 were used in deriving the h's from our data.

Often the saturating field is applied either parallel or perpendicular to the direction of measurement of λ, the effect being then denoted longitudinal (λ_l) or transverse (λ_t). These quantities are shown for some special cases in Table IV.

One case should be mentioned, which at first led to some difficulty in interpreting the results of measurement. It will be noted that λ_l and λ_t for the [011] direction in the (100) plane both contain the terms $h_1/6+h_4/12$. If $h_1+h_4/2$ is large compared with h_2, the longitudinal and transverse effects will both be of the same sign and approximately the same magnitude. In one crystal of Mishima alloy (Fe_2NiAl) this was observed to be the case, as shown in Fig. 2. In all of the other special cases of Table IV, the terms in h_1, h_2, h_4, and h_5 occur with opposite signs in the longitudinal and transverse directions, or one of them is zero.

We are greatly indebted to Miss C. L. Froelich for carrying out the matrix inversions, and to Miss B. B. Cetlin and Miss M. Goertz for checking many of the formulas.

APPENDIX

The dependence of magnetostriction on direction of magnetization to saturation is given below for some of the special cases most frequently used. These are all derived from Eq. (1). Here λ is the fractional change in length in the specified direction, and λ_0 is the same quantity for $\theta=0$, where θ is the angle in the given plane between the direction of the applied field and the [001] direction (except in cases 6 and 7, when it is the $[1\bar{1}0]$ direction). The sense of θ is such that the direction of saturation is parallel to the direction in which λ is measured, when θ has some value $\lessgtr 90°$.

(1) Plane (100), $\lambda \| [001]$:

$$\lambda=h_1(1+3\cos 2\theta)/6+h_3(1-\cos 4\theta)/8 +h_4(3+12\cos 2\theta+\cos 4\theta)/8,$$

$$\lambda_0=2h_1/3+2h_4/3,$$

$$\lambda-\lambda_0=h_1(-1+\cos 2\theta)/2+h_3(1-\cos 4\theta)/8 +h_4(-13+12\cos 2\theta+\cos 4\theta)/24.$$

(2) Plane (100), $\lambda \| [011]$:

$$\lambda=h_1/6+h_2(\sin 2\theta)/2+h_3(1-\cos 4\theta)/8 +h_4(3+\cos 4\theta)/24,$$

$$\lambda_0=h_1/6+h_4/6,$$

$$\lambda-\lambda_0=h_2(\sin 2\theta)/2+h_3(1-\cos 4\theta)/8 +h_4(-1+\cos 4\theta)/24.$$

(3) Plane (110), $\lambda \| [001]$:

$$\lambda=h_1(1+3\cos 2\theta)/6+h_3(7-4\cos 2\theta-3\cos 4\theta)/32 +h_4(9+20\cos 2\theta+3\cos 4\theta)/48,$$

$$\lambda_0=2h_1/3+2h_4/3,$$

$$\lambda-\lambda_0=h_1(-1+\cos 2\theta)/2+h_3(7-4\cos 2\theta-3\cos 4\theta)/32 +h_4(-23+20\cos 2\theta+3\cos 4\theta)/48.$$

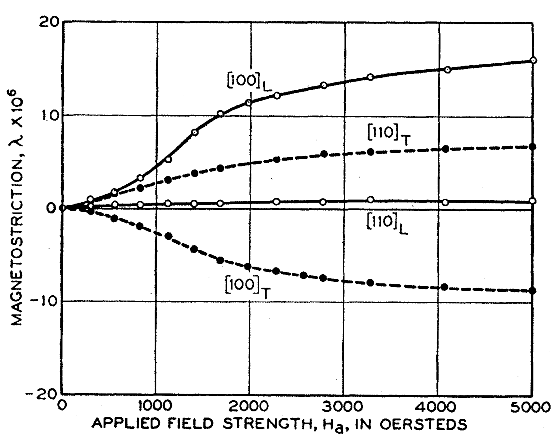

FIG. 2. Magnetostriction *vs* field strength for Fe_2NiAl (Mishima alloy), showing λ_l (longitudinal) with same sign as λ_t (transverse).

(4) Plane (110), $\lambda \| [1\bar{1}0]$:

$$\lambda = h_1(-1-3\cos 2\theta)/12 + h_2(1-\cos 2\theta)/4 + h_3(7-4\cos 2\theta - 3\cos 4\theta)/32 + h_4(-9-20\cos 2\theta - 3\cos 4\theta)/96 + h_5(1-\cos 4\theta)/16,$$

$$\lambda_0 = -h_1/3 - h_4/3,$$

$$\lambda - \lambda_0 = h_1(1-\cos 2\theta)/4 + h_2(1-\cos 2\theta)/4 + h_3(7-4\cos 2\theta - 3\cos 4\theta)/32 + h_4(23-20\cos 2\theta - 3\cos 4\theta)/96 + h_5(1-\cos 4\theta)/16.$$

(5) Plane (110), $\lambda \| [111]$:

$$\lambda = h_2(1-\cos 2\theta + 2\sqrt{2}\sin 2\theta)/6 + h_3(7-4\cos 2\theta - 3\cos 4\theta)/32 + h_5(1+2\sqrt{2}\sin 2\theta - \sqrt{2}\sin 4\theta - \cos 4\theta)/24,$$

$$\lambda_0 = 0,$$

$$\lambda - \lambda_0 = \lambda.$$

(6) Plane (111), $\lambda \| [1\bar{1}0]$:

$$\lambda = h_1(\cos 2\theta)/6 + h_2(1+2\cos 2\theta)/6 + h_3/4 + h_4(4\cos 2\theta - \cos 4\theta)/36 + h_5(\cos 2\theta - \cos 4\theta)/18,$$

$$\lambda_0 = h_1/6 + h_2/2 + h_3/4 + h_4/12,$$

$$\lambda - \lambda_0 = h_1(-1+\cos 2\theta)/6 + h_2(-1+\cos 2\theta)/3 + h_4(-3+4\cos 2\theta - \cos 4\theta)/36 + h_5(\cos 2\theta - \cos 4\theta)/18.$$

(7) Plane (111), $\lambda \| [2\bar{1}\bar{1}]$:

$$\lambda = h_1(\cos 2\theta + \sqrt{3}\sin 2\theta)/12 + h_2(1+\cos 2\theta + \sqrt{3}\sin 2\theta)/6 + h_3/4 + h_4(4\cos 2\theta + 4\sqrt{3}\sin 2\theta - \cos 4\theta + \sqrt{3}\sin 4\theta)/72 + h_5(\cos 2\theta + \sqrt{3}\sin 2\theta - \cos 4\theta + \sqrt{3}\sin 4\theta)/36,$$

$$\lambda_0 = h_1/12 + h_2/3 + h_3/4 + h_4/24,$$

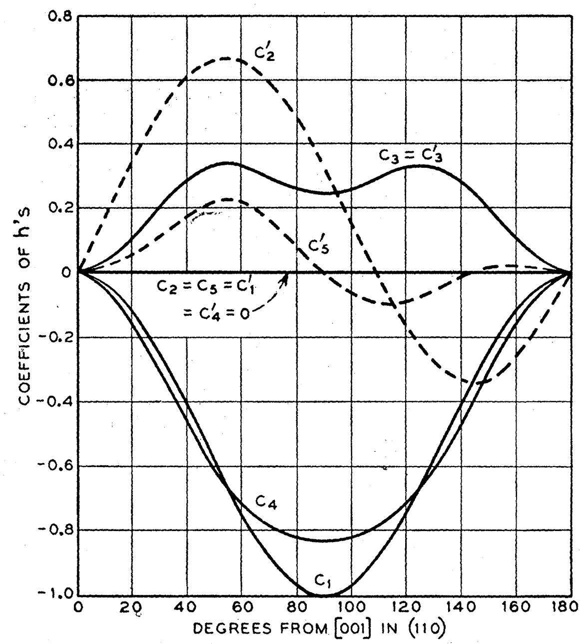

FIG. 3. Coefficients $c_1, c_2, \cdots, c_5$ of the terms in $h_1, \cdots, h_5$ of Eq. (1), for λ measured in (110) plane parallel to [001] (unprimed c's) and to $[1\bar{1}1]$ (primed c's), with magnetization at $\theta°$ from [001].

$$\lambda - \lambda_0 = h_1(-1+\cos 2\theta + \sqrt{3}\sin 2\theta)/12 + h_2(-1+\cos 2\theta + \sqrt{3}\sin 2\theta)/6 + h_4(-3+4\cos 2\theta + 4\sqrt{3}\sin 2\theta - \cos 4\theta + \sqrt{3}\sin 4\theta)/72 + h_5(\cos 2\theta + \sqrt{3}\sin 2\theta - \cos 4\theta + \sqrt{3}\sin 4\theta)/36.$$

Items (3) and (5) above were used in the computation of the matrix elements of Table I. The way in which the coefficients of the h's vary with θ is shown for these two cases in Fig. 3.