In general, however, these derivatives must be evaluated under the physical conditions obtaining when a sound wave is propagated through the body. For example, at low enough temperatures a perfect infinite lattice without impurity will have a thermal conductivity tending to infinity. Consequently, the changes which occur during the propagation of a sound wave will be isothermal, and $(\partial p/\partial V)_T$ is the appropriate derivative. At higher temperatures, however (and this will usually include room temperature), the processes will be practically adiabatic so that $(\partial p/\partial V)_S$ should be employed.

At intermediate temperatures the conditions for the first derivative will be neither isothermal nor adiabatic but lie between. Then within the limitations of the theory $\partial p/\partial V$, and hence the wave velocity and Θ, depend only on the volume. Under these circumstances, the second differentiation with respect to volume presents no ambiguity.

Comparing Eq. (22) with Eq. (12), it follows that Slater's values[6] based on experimental $p:V$ data must be diminished by 0.33. This correction then slightly improves the over-all agreement.

This work forms part of a general investigation of the effects of anharmonicity on the thermodynamic properties of simple solids and the electrical resistance of metals.

[6] See reference 4, pp. 393, 451.

PHYSICAL REVIEW VOLUME 89, NUMBER 4 FEBRUARY 15, 1953

Multiple Scattering Corrections to the Impulse Approximation in the Two-Body System*

K. A. BRUECKNER
Indiana University, Bloomington, Indiana
(Received November 10, 1952)

Exact solutions for the scattering of a fast particle by two heavy scatterers are obtained and compared with the usual treatment of the impulse approximation in which multiple scattering is neglected. It is found that the multiple scattering qualitatively changes the solution except in the extreme Born approximation limit. The methods developed are applied to the isotopic spin dependent, but spin independent, scattering of mesons in the deuteron. It is found that if high energy scattering is assumed to be in the isotopic spin $\frac{3}{2}$ state, a considerable depression in the total cross section can be expected for phase shifts larger than 45 degrees.

I. INTRODUCTION

THE impulse approximation[1–3] has been developed to deal with the scattering of a fast particle by a system of heavy scatterers where the motion of the scattering centers can be neglected during the scattering process. This approximation leads to simplified theoretical evaluations of many processes and has been applied in particular to a variety of phenomena in deuterium.[4] In these applications, it has been argued that multiple scattering effects can be neglected if the free-particle scattering amplitudes are small compared with the deuteron radius. It will be shown, however, that this criterion is incorrect and that the neglect of multiple scattering is valid only in the limit where the Born approximation is valid for the single scattering center, leading otherwise to qualitatively incorrect results. The exact solutions, in the framework of the impulse approximation, will be discussed first for the simple case of S-state scattering and secondly in the case of P-state meson scattering. In the latter case, spin independent but isotopic spin dependent scattering will be considered.

We shall not discuss the validity of the impulse approximation as such, since this has been discussed in detail particularly by Chew and Wick,[2] and by Chew and Goldberger.[3] We shall also not attempt to consider the effects of scatterings which do not conserve energy since, although such processes undoubtedly give corrections to the impulse approximation, they are distinct from the effect we wish to consider here.

The methods developed here will be applied to the evaluation of spin dependent scattering and to photomesonic phenomena in a forthcoming paper.

II. THE IMPULSE APPROXIMATION FOR S-STATE SCATTERING

We shall consider this very simple case to illustrate the consequences of an exact treatment of the impulse approximation. For the case of S-state scattering from

* This work was done in part while the author was a visiting physicist at Brookhaven National Laboratory during August and September, 1952. The hospitality of this laboratory is gratefully acknowledged. This work was also supported in part by a grant from the National Science Foundation.
[1] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[2] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952).
[3] G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).
[4] Fernbach, Green, and Watson, Phys. Rev. **82**, 980 (1951); B. Segall, Phys. Rev. **83**, 1247 (1951); W. B. Cheston, Phys. Rev. **85**, 952 (1952); Y. Fujimoto and Y. Yamaguchi, Prog. Theoret. Phys. **6**, 166 (1951); G. F. Chew and H. W. Lewis, Phys. Rev. **84**, 779 (1951); Isaacs, Sachs, and Steinberger, Phys. Rev. **85**, 803 (1952).

a two-body system with heavy point scatterers at r_A and r_B, the wave function for the system is

$$\psi(r)=e^{i\mathbf{k}_0\cdot\mathbf{r}}+\frac{Ae^{ik|\mathbf{r}-\mathbf{r}_A|}}{|\mathbf{r}-\mathbf{r}_A|}+\frac{Be^{ik|\mathbf{r}-\mathbf{r}_B|}}{|\mathbf{r}-\mathbf{r}_B|},\tag{1}$$

which is the general solution to the wave equation outside the range of the scatterers. The outgoing amplitude A is related to the total wave amplitude at r_A by the equation

$$A=\eta_A\left(e^{i\mathbf{k}_0\cdot\mathbf{r}_A}+\frac{Be^{ik|\mathbf{r}_A-\mathbf{r}_B|}}{|\mathbf{r}_A-\mathbf{r}_B|}\right),\tag{2}$$

where $k\eta_A=\exp(i\delta_A)\sin\delta_A$, and similarly for B. The resulting equations are easily solved and give for the amplitude of the outgoing wave, where for simplicity we have set $\eta_A=\eta_B=\eta$ and $R=|\mathbf{r}_A-\mathbf{r}_B|$,

$$f(\theta)=\left[\eta\left(e^{i(\mathbf{k}0-\mathbf{k})\cdot\mathbf{r}_A}+e^{i(\mathbf{k}0-\mathbf{k})\cdot\mathbf{r}_B}\right)+\eta^2\frac{e^{ikR}}{R}\left(e^{i(\mathbf{k}_0\cdot\mathbf{r}_A-\mathbf{k}\cdot\mathbf{r}_B)}+e^{i(\mathbf{k}_0\cdot\mathbf{r}_B-\mathbf{k}\cdot\mathbf{r}_A)}\right)\right]\left[1-\eta^2(e^{ikR}/R^2)\right]^{-1}.\tag{3}$$

To obtain the total cross section, we make use of the theorem[5] relating the total cross section to the forward scattering amplitude,

$$\sigma_{\text{total}}=4\pi\bar{\lambda}\,Im f(0),\tag{4}$$

and obtain

$$\sigma_{\text{total}}=2\sigma_0[1+\sin x\sin(x+2\delta)/x^2+\sin\delta\sin(2x+\delta)/x^2+\sin^2\delta\sin^2 x/x^4][(1-\sin^2\delta/x^2)^2+4\sin^2\delta\sin^2(x+\delta)/x^2]^{-1},\tag{5}$$

where we have averaged over the angles of $\mathbf{R}$ and introduced $x=kR$ and $\sigma_0=4\pi\bar{\lambda}^2\sin^2\delta$. We now note that in the limit $\delta\rightarrow 0$,

$$\sigma_{\text{total}}=2\sigma_0(1+\sin^2 x/x^2),\tag{6}$$

which is the prediction of the impulse approximation if multiple scattering is neglected. This limit is correct, however, only if $\delta\rightarrow 0$ and not if $R\rightarrow\infty$; i.e., even for widely separated scatterers, the neglect of multiple scattering is valid only if the Born approximation is applicable to the free particle case. This is particularly apparent if we take $\delta=90°$ which leads to a cross section less for all values of R than the sum of the free particle cross sections, in striking contrast to the Born approximation result. Another interesting limit is for $R\rightarrow 0$ where, for arbitrary δ,

$$\sigma_{\text{total}}=8\pi R^2.\tag{7}$$

A plot of the total cross section as a function of x, for various values of δ, is given in Fig. 1. It is apparent that the exact solution deviates rapidly from the Born approximation result even for rather small values of the phase shift. A qualitative feature of importance is that the interference is negative for all values of R for phase shifts somewhat larger than 45°.

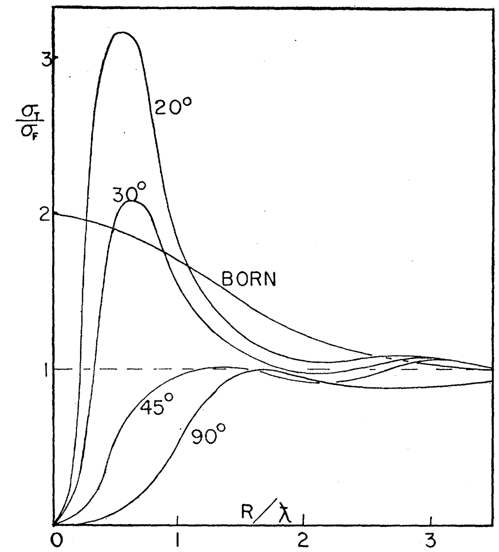

FIG. 1. S-wave scattering by two heavy point scatters, as a function of $R/\bar{\lambda}$; R is the separation of the scatterers, $\bar{\lambda}$ the wavelength. The ratio of the cross section to the sum of the free-particle cross sections is given. The phase shifts for the two particles have been assumed to be equal.

III. MESON SCATTERING

We shall restrict ourselves to the consideration of isotopic spin dependent P-state scattering. This is the prediction of pseudoscalar theory with pseudovector coupling and also is in approximate agreement with the experimental results. We shall not, however, consider spin-dependent scattering since the present experiments,[6] while they do not fix the spin dependence unambiguously, suggest that the spin dependence is weak. The spin dependence can be included in a straightforward way, but the resulting equations are considerably more difficult to solve than the simpler case we shall consider. As a consequence of this neglect, our results will not exactly relate to the actual case of meson scattering but will serve to illustrate the effects of interest.

For a single scatterer at r_A, the wave function is

$$\psi=\psi_0-ik\mathbf{A}\cdot\nabla(e^{ik|\mathbf{r}-\mathbf{r}_A|}/|\mathbf{r}-\mathbf{r}_A|),\tag{8}$$

which is a general solution of the wave equation, outside of the region of interaction, for P-wave scattering alone. The amplitude $\mathbf{A}$ is related to the incoming wave ψ_0 by

$$\mathbf{A}=-ik\bar{\lambda}^3T_A\nabla\psi_0,\tag{9}$$

where

$$T_A=a\mathbf{U}^*\cdot\mathbf{U}+ab\boldsymbol{\tau}_A\cdot\mathbf{U}^*\times\mathbf{U}\tag{10}$$

[5] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 481 (1950).

[6] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952).

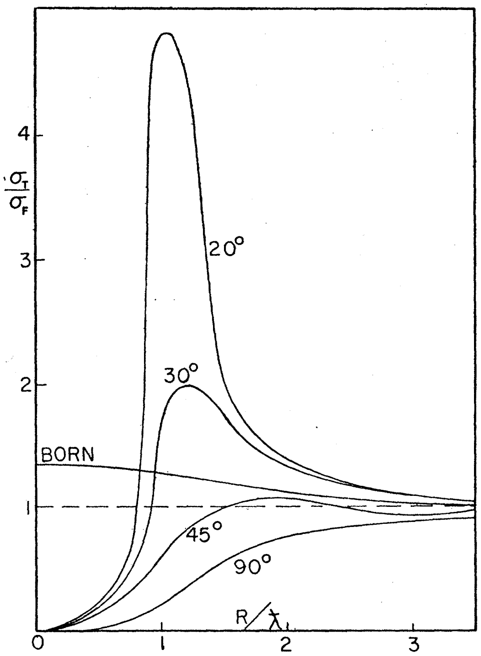

FIG. 2. P-wave isotopic spin $\frac{3}{2}$ scattering of mesons by two heavy point scatterers (neutron and proton).

is the most general isotopic spin dependent operator. The operators U_i^* and U_i create and annihilate the ith component of the meson wave, $\boldsymbol{\tau}_A$ is the usual isotopic spin operator. The coefficients a and b are related to the phase shifts for the isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$ states[7] by

$$a=2\eta_{\frac{3}{2}}+\eta_{\frac{1}{2}},\quad b=-\eta_{\frac{3}{2}}+\eta_{\frac{1}{2}},\tag{11}$$

where $\eta_t=\exp(i\delta_t)\sin\delta_t$. The asymptotic form of ψ is

$$\psi\sim\psi_0+f(\theta)e^{ikr}/r,\tag{12}$$

where

$$\begin{aligned}f(\theta)[3\bar{\lambda}\cos\theta]^{-1}&\\ =\eta_{\frac{3}{2}},&\quad (P+,P+),(N-,N-)\\ =(\eta_{\frac{3}{2}}+2\eta_{\frac{1}{2}})/3,&\quad (P-,P-),(N+,N+)\\ =(2\eta_{\frac{3}{2}}+\eta_{\frac{1}{2}})/3,&\quad (N0,N0),(P0,P0)\\ =2^{\frac{1}{2}}(\eta_{\frac{3}{2}}-\eta_{\frac{1}{2}})/3,&\quad (N0,P-),(P0,N+).\end{aligned}\tag{13}$$

The notation $(P+,P+)$ means π^+ mesons on protons, etc.

With this as an introduction, we consider next the case of two scatterers at r_A and r_B, for which the wave function is

$$\psi=\psi_0+(\hbar/i)[\mathbf{A}\cdot\nabla(e^{ik|\mathbf{r}-\mathbf{r}_A|}/|\mathbf{r}-\mathbf{r}_A|)+\mathbf{B}\cdot\nabla(e^{ik|\mathbf{r}-\mathbf{r}_B|}/|\mathbf{r}-\mathbf{r}_B|)].\tag{14}$$

ψ_0 describes the initial state of the systems and has the form

$$\psi_0=e^{i\mathbf{k}_0\cdot\mathbf{r}}t_0,\tag{15}$$

[7] For a more detailed development of the theory of the isotopic spin, see W. Heitler, Proc. Roy. Irish Acad. **51**, 33 (1946); K. M. Watson, Phys. Rev. **85**, 892 (1952); and K. A. Brueckner, Phys. Rev. **86**, 106 (1952).

where $\mathbf{k}_0$ is the momentum of the incoming wave and t_0 is an isotopic spin function specifying the initial charge state.

The amplitudes $\mathbf{A}$ and $\mathbf{B}$ are related to the total wave amplitude at r_A and r_B by the equations

$$\begin{aligned}\mathbf{A}&=\bar{\lambda}^3T_A\frac{\hbar}{i}\nabla\left[\psi_0+\frac{\hbar}{i}\mathbf{B}\cdot\nabla(e^{ik|\mathbf{r}-\mathbf{r}_B|}/|\mathbf{r}-\mathbf{r}_B|)\right]_{r=r_A},\\ \mathbf{B}&=\bar{\lambda}^3T_B\frac{\hbar}{i}\nabla\left[\psi_0+\frac{\hbar}{i}\mathbf{A}\cdot\nabla(e^{ik|\mathbf{r}-\mathbf{r}_A|}/|\mathbf{r}-\mathbf{r}_A|)\right]_{r=r_B}.\end{aligned}\tag{16}$$

The asymptotic form then is

$$\psi(r)=\psi_0+\hbar(\mathbf{A}\cdot\mathbf{k}e^{-i\mathbf{k}\cdot\mathbf{r}_A}+\mathbf{B}\cdot\mathbf{k}e^{-i\mathbf{k}\cdot\mathbf{r}_B})e^{ikr}/r.\tag{17}$$

To obtain the total cross section, we again make use of the theorem relating the cross section to the imaginary part of the forward amplitude. Since only the diagonal part contributes, we need not consider the charge exchange process in the total cross section since it is an off-diagonal process. We cannot, however, neglect the charge exchange processes in computing the values of $\mathbf{A}$ and $\mathbf{B}$, as will be apparent in the following.

TABLE I. Total cross sections for meson-deuteron scattering under the assumption of P-state scattering in the isotopic spin $\frac{3}{2}$ state. The wavelength is given in units of the meson Compton wavelength. The energy is in the laboratory system and is based on the assumption that the system in which the nucleons are assumed to be infinitely heavy is the correct center-of-mass system for the meson and one nucleon. σ_f is the sum of the free-particle cross sections, σ_{Born} is the prediction of the Born approximation calculation (neglecting multiple scattering), σ_{exact} is the result of the exact treatment of the multiple scattering. The cross sections are given in millibarns.

$\bar{\lambda}$	E	$\delta_{3/2}$	σ_f	σ_{Born}	σ_{exact}
1.000	81 Mev	20°	90	113	144
0.833	113 Mev	30°	178	218	153
0.714	150 Mev	45°	261	288	236
0.556	220 Mev	90°	315	340	246

To solve the equations for $\mathbf{A}$ and $\mathbf{B}$, we first write them in the more convenient form,

$$\begin{aligned}\mathbf{A}&=\bar{\lambda}^3\frac{\hbar}{i}T_A\left[i\mathbf{k}_0\psi_0(r_A)+\mathbf{B}f(R)+\mathbf{B}\cdot\frac{\mathbf{R}}{R^2}\mathbf{R}g(R)\right],\\ \mathbf{B}&=\bar{\lambda}^3\frac{\hbar}{i}T_B\left[i\mathbf{k}_0\psi_0(r_B)+\mathbf{A}f(R)+\mathbf{A}\cdot\frac{\mathbf{R}}{R^2}\mathbf{R}g(R)\right],\end{aligned}\tag{18}$$

where

$$\bar{\lambda}^3f(R)=-\frac{1}{x}\frac{d}{dx}\left(\frac{e^{ix}}{x}\right),\quad g(R)=x\frac{df}{dx}.\tag{19}$$

Let us now restrict our attention to the scattering of a π^+ meson by a deuteron. In this case the initial system is an isotopic singlet nucleon state and a π^+ meson: Under the action of the operators T_A and T_B, the system will undergo transitions to an isotopic triplet state.[8] We can therefore take these two states as the basis vectors for a matrix representation of the operators T_A, T_B. Calling the initial state (1) and the second state (2), we then have

$$T_A=\begin{pmatrix} a & 2^{\frac{1}{2}}b \\ 2^{\frac{1}{2}}b & a+b \end{pmatrix},\quad T_B=\begin{pmatrix} a & -2^{\frac{1}{2}}b \\ -2^{\frac{1}{2}}b & a+b \end{pmatrix}. \tag{20}$$

The two coupled equations of Eq. (18) can first be solved, after multiplying by $\mathbf{R}$, for the matrix elements of $\mathbf{A}\cdot\mathbf{R}$ and $\mathbf{B}\cdot\mathbf{R}$. These can then be substituted into the vector equations to give a solution for $\mathbf{A}$ and $\mathbf{B}$. In computing the total cross section, as remarked above, we need consider only the elements of A and B corresponding to direct scattering, i.e., A_1 and B_1. Straightforward evaluation leads to the result that

$$\sigma_{\text{total}}=8\pi\lambda\!\!\!^{-2}\, Im\frac{1}{\rho_1}\left\{a+g_1-f(a^2-2b^2+g_2)\frac{\sin x}{x}+\frac{1}{3}\frac{g}{\rho_2}[g_3-\epsilon(a^2-2b^2)-g_4\epsilon]\right\}, \tag{21}$$

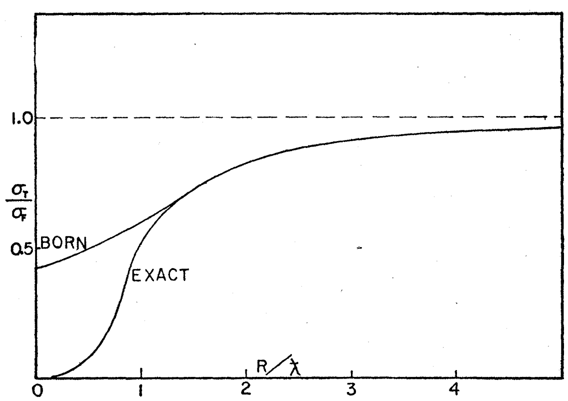

FIG. 3. P-wave scattering by two heavy point scatterers (neutron and proton) with $\delta_{\frac{3}{2}}=10°$, $\delta_{\frac{1}{2}}=-7°$.

where $h=f+g$. The functions f, g, h are now to be considered to be dimensionless functions of $x=r/\lambda\!\!\!^{-}$ alone. The quantities ρ_1, ρ_2, g_1, g_2, g_3, g_4 include the effects of the multiple scattering; they are, in general, extremely complicated function of a, b, f, and h. If the phase shifts go to zero, then the ρ's become equal to one and the g_i's go to zero. For the special case of scattering in the isotopic spin $\frac{3}{2}$ state (with $a=-2b$), they reduce to $\rho_1=1-b^2f^2$, $\rho_2=1-b^2h^2$, $g_1=g_2=0$, $g_3=2b^3(f+h)$, $g_4=(1+b^2fh)2b^2$. In this result, we have averaged over the angles of $\mathbf{R}$ and introduced the symbol

$$\tfrac{1}{3}\epsilon=\tfrac{1}{2}\int_{-1}^{1}\mu^2 e^{i\mu x}d\mu. \tag{22}$$

[8] The isotopic spin wave function for this state is

$$2^{\frac{1}{2}}(\omega_1^{\,1}t_1^{\,0}-\omega_1^{\,0}t_1^{\,1}),$$

where $t_1^{\,m}$ is the triplet spin function and $\omega_1^{\,m}$ is the meson wave function.

TABLE II. Total cross sections at 60 Mev. The phase shifts have been taken to be $\delta_{\frac{3}{2}}=10°$, $\delta_{\frac{1}{2}}=-7°$, giving total cross sections for π^+ mesons on protons and neutrons of 32.2 mb and 21.4 mb, respectively. The units are the same as for Table I.

ƛ	E	σ_f	σ_{Born}	σ_{exact}
1.284	60 Mev	53.6	36.7	30.9

Let us first consider this result in the limit $R\to\infty$. We find for the sum of the free particle total cross sections

$$\begin{aligned}\sigma_f&=8\pi\lambda\!\!\!^{-2}Im a\\&=8\pi\lambda\!\!\!^{-2}(2\sin^2\delta_{\frac{3}{2}}+\sin^2\delta_{\frac{1}{2}}),\end{aligned} \tag{23}$$

which agrees with the sum as given by Eq. (13).

Next we consider the Born approximation limit. Here we find, as a, $b\to 0$,

$$\begin{aligned}\sigma_{\text{total}}&=8\pi\lambda\!\!\!^{-2}Im[a-(a^2-2b^2)(f\sin x/x+\epsilon g/3)]\\&=\sigma_f\left[1-\frac{(2\sin^2\delta_{\frac{3}{2}}+8\sin\delta_{\frac{3}{2}}\sin\delta_{\frac{1}{2}}-\sin^2\delta_{\frac{1}{2}})}{(2\sin^2\delta_{\frac{3}{2}}+\sin^2\delta_{\frac{1}{2}})}\times Im\left(\frac{f\sin x}{x}+\frac{\epsilon g}{3}\right)\right].\end{aligned} \tag{24}$$

This result is easily shown to be identical with the usual result in which multiple scattering has been neglected. The angular integral encountered in the usual evaluation is

$$\lambda\!\!\!^{-4}\int\frac{d\Omega_k}{4\pi}\int\frac{d\Omega_R}{4\pi}(\mathbf{k}_0\cdot\mathbf{k})^2e^{i(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{R}}, \tag{25}$$

which is equal to

$$-Im(f(x)\sin x/x+\epsilon g(x)/3), \tag{26}$$

with f and g defined in Eq. (19). Again it is clear that this limit is valid only in Born approximation and is not approached as a limit for large R when the plane shifts are not small.

These results, of course, break down for small R since the meson-nucleon interaction is of finite extent, modifying the wave function of Eq. (14) for small distances. This is in addition the region where the impulse approximation is particularly bad, since it corresponds to high velocities for the nucleons. The process of meson absorption is also associated with small nucleon separation. These effects all will tend to lead to a finite contribution to the total cross section from the region of small R. Although this region is of small importance for a reasonable choice of the deuteron wave function, the uncertainties in its treatment lead to the largest uncertainties in our results.

IV. EVALUATION OF THE TOTAL CROSS SECTION FOR DEUTERIUM

We consider in detail two cases for which the phase shifts are known or can be deduced with some certainty from the experimental data. At high energies, it is known[6] that the total cross sections are approximately in the ratio $\sigma(P+, P+):\sigma(P-, P-):\sigma(N0, P-) = 9:1:2$. This fixes the scattering predominantly in the isotopic spin $\frac{3}{2}$ state. Accordingly we set $\delta_{\frac{1}{2}}=0$ in Eq. (21). The resulting total cross section is given in Fig. 2 as a function of $x=R/\lambda\!\!\!^{-}$ for various values of the phase shift, showing the rapid departure of the cross section from the Born approximation result in which multiple scattering is neglected. It is noteworthy that the net interference effect is negative for phase shifts larger than 45°, in striking contrast with the Born approximation result which gives positive interference for all values of the phase shift.

To evaluate the cross section in deuterium, it is now only necessary to average the cross sections, given in Fig. 2 as a function of R, over the deuteron wave function. For this we assume the Hulthen wave function

$$\psi(r)=\left[\frac{(e^{-\alpha r}-e^{-\beta r})}{r}\right]\left[\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}\right]^{\frac{1}{2}}. \qquad (27)$$

We need also associate a specific wavelength with the various values of the phase shift. We shall assume values which are in approximate agreement with the total cross section measurements of Fermi *et al.*[6] The values of the wavelength, laboratory system energy, phase shift, free particle cross sections, the Born approximation prediction, and the prediction with the multiple scattering properly taken into account, are given in Table I. It is apparent that as the phase shift passes 45°, one can expect a depression of the total cross section in deuterium relative to the free cross section. This is in qualitative agreement with the preliminary experimental results of Fermi.[9]

Another measurement by Isaacs, Sachs, and Steinberger[4] at 60 Mev gives a ratio of the (+, proton) total cross section (28 mb) to the (π^-, proton) total cross section (18 mb) of about 1.6, and a deuterium cross section for π^+ mesons about equal to the proton cross section alone. If we take at this energy phase shifts of 10° and −7° for the isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$ states, respectively, the resulting total deuterium cross section as a function of $x=R/\lambda\!\!\!^{-}$ is given in Fig. 3 together with the Born approximation result. The exact and approximate results agree fairly well except at small x where the multiple scattering is important. Averaging these over the deuteron wave function gives us the results tabulated in Table II. The multiple scattering does not charge the result qualitatively; however, it increases the interference from 19 millibarns to 25 millibarns to give a deuterium cross section which is less than the (+, proton) cross section alone.

[9] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 413 (1952).

V. CONCLUSIONS

The multiple scattering corrections to the impulse approximation have been evaluated in the case of two heavy scatterers for a simple example of S-wave scattering and for the isotopic-spin dependent P-wave scattering of mesons. The results show that the multiple scattering can be neglected only if the Born approximation is applicable to the free particle cross sections. Evaluation of meson scattering for the deuteron under the assumption of isotopic spin $\frac{3}{2}$ scattering leads to the conclusion that considerable depression of the total cross section relative to the free particle cross sections can be expected for phase shifts larger than 45°. Application to the case of scattering at 60 Mev where the cross sections are small, with the isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$ phase shifts 10° and −7°, respectively, show that the multiple scattering considerably enhances the destructive interference to give a deuterium cross section 58 percent of the sum of the free particle cross sections and less than the (proton, +) cross section alone. The results are approximate principally in that off-the-energy-shell scattering has been neglected, and the impulse approximation assumed.

The author is indebted to Professor L. L. Foldy, who pointed out the simple method for obtaining an exact solution to the two-body scattering problem which is developed here. The author is in addition grateful to Professor Robert Serber, Professor Geoffrey Chew, and Professor Kenneth M. Watson for valuable discussions.