Photoproduction of π^0 Mesons in Hydrogen and Deuterium*

NORMAN C. FPANCIS†
University of Rochester, Rochester, New York
(Received September 8, 1952)

Photoproduction of π^0 mesons in hydrogen and deuterium is calculated in the weak coupling theory under several different assumptions regarding the interaction Hamiltonian. All calculations are performed non-relativistically taking into account the anomalous magnetic moments of the nucleons. The contribution of the negative energy states for the nucleons and of the nucleon current are isolated. Calculations are also performed with a phenomenological Hamiltonian which yields a $(2+3\sin\theta')$ angular distribution in the center-of-mass system for the photoproduction of π^0 mesons in hydrogen. In the case of deuterium, the relative importance of elastic and inelastic photoproduction is examined as a function of photon energy, meson angle, and the theoretical results are compared with recent experimental data. Best agreement with experiment is obtained if nucleon current effects are neglected.

INTRODUCTION

THE standard weak coupling theory which was used to calculate the photoproduction of π^+ mesons in hydrogen predicts much too small a cross section for π^0 meson production.[1] However, when the anomalous magnetic moment of the proton is assumed to interact directly with the electromagnetic field and the π^0 meson is treated as a pseudoscalar particle, the cross section for π^0 production becomes comparable to the π^+ cross section.[2] If the anomalous magnetic moment of the proton is responsible for π^0 production in hydrogen, then the neutron should contribute an almost equal amount to the π^0 photoproduction in a nucleus like deuterium. Indeed, the elastic production process

$$\gamma+D\rightarrow\pi^0+D$$

at all meson angles, and the total cross section (elastic plus inelastic) for π^0 meson produced in the forward direction, are especially interesting because of their dependence, due to interference effects, on the relative sign of the neutron and proton π^0 coupling constants. In practice, it is easier to measure the total deuteron cross section at large angles with respect to the photon beam where the proton and neutron make their separate contributions.

At least two objections can be raised against the relativistic weak coupling calculation carried out by Kaplon[2] to explain the photoproduction of π^0 mesons in hydrogen. The first objection is that while the proton cross section calculated in this way agrees well with experiment[3] both as regards absolute magnitude and variation with energy, it predicts an approximately isotropic angular distribution in the laboratory system, which seems to contradict experiment. The origin of the isotropic distribution can be traced to interference between the probability amplitudes for transitions to positive and negative energy intermediate states. Since the odd part of the (PS, PV) meson-nucleon interaction Hamiltonian is needed to give a large amplitude for negative energy state transitions, if for an as yet unknown reason the odd part of the (PS, PV) meson-nucleon operator is suppressed[4] and only the $(\boldsymbol{\sigma}\cdot\mathbf{q})$ part ($\boldsymbol{\sigma}$ is the proton spin and $\mathbf{q}$ is the meson momentum) is retained, the interference between negative and positive energy transitions is eliminated and the proton crcss section favors the emission of π^0 mesons in the forward direction. For this reason the deuteron elastic π^0 production is calculated with an interaction which contains only the even part of the (PS, PV) interaction. The elastic production is also calculated with the entire (PS, PV) interaction Hamiltonian since the experimental angular distribution from hydrogen is not final.

A second possible objection to a weak coupling calculation which includes the anomalous magnetic moment of the proton is that the anomalous moment, which presumably is due to virtual charged meson currents, may be more important for the photoproduction of π^0 mesons than the Dirac moment associated with the bare proton. With this in mind, a third model is considered in which the charge of the proton is completely ignored and the proton is assumed to interact with the radiation field through its anomalous moment alone. If the proton and neutron anomalous moments are assumed equal, the photoproduction of π^0 mesons is independent of the charge state of the nucleon target. Since the explicit version of the (PS, PV) theory which is used may be incorrect, it is interesting to inquire into the most general phenomenological interaction possible within the context of weak coupling theory. It turns out that, for a pseudoscalar π-meson, one can

* This work was assisted by the U. S. Atomic Energy Commission.
† Now at Indiana University, Bloomington, Indiana.
[1] K. A. Brueckner, Phys. Rev. 79, 641 (1950).
[2] M. F. Kaplon, Phys. Rev. 83, 712 (1951), and Aidzu, Fujimoto, and Fukuda, Prog. Theoret. Phys. 6, 193 (1951).
[3] A. Silverman and M. Stearns, Phys. Rev. 83, 206 (1951).

[4] Similar *ad hoc* assumptions are needed to explain both the forward maximum in the differential cross section for the reaction $p+p\rightarrow D+n$ [see Chew, Goldberger, Steinberger, and Yang, Phys. Rev. 84, 581 (1951)] and the π^+ to π^- meson ratio of about one in the reactions

$$\gamma+D\rightarrow\pi^++n+n$$

and

$$\gamma+D\rightarrow\pi^-+p+p$$

[see R. E. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952)].

use a linear combination of the following five terms: $(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}')$, $(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}')(\mathbf{k}'\cdot\mathbf{q}')$, $(\boldsymbol{\sigma}\cdot\mathbf{k}')(\mathbf{q}'\cdot\boldsymbol{\lambda}')$, $(\boldsymbol{\sigma}\cdot\mathbf{q}')(\mathbf{q}'\cdot\boldsymbol{\lambda}')$, $[\mathbf{q}'\cdot(\mathbf{k}'\times\boldsymbol{\lambda}')]$, where $\boldsymbol{\sigma}$ is the spin of the nucleon, $\mathbf{q}'$ is the meson momentum, $\mathbf{k}'$ and $\boldsymbol{\lambda}'$ are the photon momentum and polarization, respectively—all quantities measured in the center-of-mass system. Coefficients are, in general, arbitrary functions of q' and k'.

If, in particular, the phenomenological interaction is chosen as

$$2\mathbf{q}'\cdot(\mathbf{k}'\times\boldsymbol{\lambda}')+i[(\boldsymbol{\sigma}\cdot\mathbf{k}')(\mathbf{q}'\cdot\boldsymbol{\lambda}')-(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}')(\mathbf{q}'\cdot\mathbf{k}')],$$

the proton cross section is proportional to $(2+3\sin^2\theta')$. This angular dependence is especially interesting because it agrees with the results of a quantum mechanical, strong coupling calculation[5] as well as corresponding to the angular dependence which is associated with the $l=1$, $j=\frac{3}{2}$ state (l being the orbital and j the total angular momentum) of the meson-nucleon system.[6]

THE METHOD

The production of π^0 mesons in photon-nucleon collisions is calculated with four nonrelativistic interaction Hamiltonians. The first of these may be obtained directly from the relativistic (PS, PV) Hamiltonian. To lowest order in $1/M$ (M is the nucleon mass), the Hamiltonian for a proton may be written

$$H_1(p)=H_{MN}(p)+H_{RN}(p)+H_{MRN}(p), \qquad (1)$$

where

$$H_{MN}(p)=g_P(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})U/\mu,$$

$$H_{RN}(p)=-e\mu_P[\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}\times\mathbf{A})]/2M-e(\mathbf{p}\cdot\mathbf{A})/M,$$

$$H_{MRN}(p)=-eg_P(\boldsymbol{\sigma}\cdot\mathbf{A})\frac{\partial U}{\partial t}\Big/M\mu.$$

Here $\boldsymbol{\sigma}$ and μ_P are the spin and magnetic moment of the proton, respectively; $\mathbf{A}$ is the vector potential evaluated at the position of the proton; μ is the meson mass; U is the π^0 meson wave function; e is the charge of the proton, and g_P is the coupling constant of the proton to the π^0 meson field.

$H_{MN}(p)$ and $H_{RN}(p)$ are the usual nonrelativistic approximations to the pure meson-nucleon and radiation-nucleon interactions, respectively. The catastrophic term, $H_{MRN}(p)$, can be shown to result from negative energy state transitions. If the catastrophic term is neglected, the interaction becomes

$$H_2(p)=H_{MN}(p)+H_{RN}(p). \qquad (2)$$

Similarly, if the anomalous moment of the proton is assumed to be entirely responsible for the π^0 production, the interaction Hamiltonian takes the form

$$H_3(p)=\frac{g_P'}{\mu}(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})U-\frac{e\mu_P'}{2M}[\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}\times\mathbf{A})], \qquad (3)$$

where μ_P' is the anomalous moment of the proton and g_P' is a constant.

Of all the interaction terms, H_{MRN} is the only one which has a diagonal matrix element; all the others require second-order calculations. It is therefore convenient to define an effective interaction $(H)_e$ which enables the process to be calculated in first order. Clearly, to determine $(H)_e$, we need only sum the second order matrix element over all of the intermediate states. After performing this calculation we obtain the following effective Hamiltonians:

$$[H_1(p)]_e=\frac{ie}{M\mu}\frac{2\pi}{(k\epsilon)^{\frac{1}{2}}}e^{i\mathbf{K}\cdot\mathbf{r}}\left\{\boldsymbol{\sigma}\cdot\left[-\epsilon\boldsymbol{\lambda}+\frac{(\mathbf{q}\cdot\boldsymbol{\lambda})\mathbf{q}}{\epsilon}-\frac{\mu_P\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\lambda})}{k}\right]\right\}g_P, \qquad (4)$$

$$[H_2(p)]_e=\frac{ie}{M\mu}\frac{2\pi}{(k\epsilon)^{\frac{1}{2}}}e^{i\mathbf{K}\cdot\mathbf{r}}\left\{\boldsymbol{\sigma}\cdot\left[\frac{(\mathbf{q}\cdot\boldsymbol{\lambda})\mathbf{q}}{\epsilon}-\frac{\mu_P\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\lambda})}{k}\right]\right\}g_P, \qquad (5)$$

$$[H_3(p)]_e=-\frac{ie}{M\mu}\frac{2\pi}{(k\epsilon)^{\frac{1}{2}}}e^{i\mathbf{K}\cdot\mathbf{r}}\{\boldsymbol{\sigma}\cdot[\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\lambda})]\}\mu_P'g_P', \qquad (6)$$

where $\mathbf{q}$ and ϵ are the momentum and energy of the meson, respectively; $\boldsymbol{\lambda}$ is the photon polarization whose momentum is $\mathbf{k}$; r is the position vector of the proton and $\mathbf{K}=\mathbf{k}-\mathbf{q}$.

As a fourth effective Hamiltonian we take the following:

$$[H_4(p)]_e=\frac{eG_P}{M\mu^2}\frac{2\pi}{\{k'\epsilon'\}^{\frac{1}{2}}}e^{i\mathbf{K}'\cdot\mathbf{r}'}[2\mathbf{q}'\cdot(\mathbf{k}'\times\boldsymbol{\lambda}')+i\boldsymbol{\sigma}\cdot\mathbf{q}'\times(\mathbf{k}'\times\boldsymbol{\lambda}')], \qquad (7)$$

where the primes indicate that the variables are measured in the center-of-mass system and G_P is a constant. In the center-of-mass system, this phenomenological interaction gives rise to a $(2+3\sin^2\theta')$ angular distribution, and this is at present the sole reason for choosing it.

After squaring the matrix element of the effective Hamiltonians, performing the usual summations and averages, and introducing the statistical factor, we obtain the following proton cross sections in the laboratory system:

$$\left(\frac{d\sigma}{d\Omega}\right)_1^P=\frac{\Lambda_1^2g_P^2}{2}\left\{\frac{2\epsilon^2}{q^2}+\frac{q^2\sin^2\theta}{\epsilon^2}+\mu_P^2(1+\cos^2\theta)-2\sin^2\theta-4\frac{\epsilon}{q}\mu_P\cos\theta\right\}, \qquad (8)$$

[5] Y. Fujimoto and H. Miyazawa, Prog. Theoret. Phys. **5**, 1052 (1950).

[6] K. A. Brueckner and K. Watson, Phys. Rev. **86**, 923 (1952).

where

$$\Lambda_1^2=\left(\frac{e^2q^3}{\mu^2M^2k}\right)\bigg/\left[1+\frac{\epsilon}{M}\frac{(q-k\cos\theta)}{q}\right],$$

$$\Lambda_2^2=\left(\frac{e^2q^3}{\mu^2M^2k}\right)\bigg/\left[1+\frac{\epsilon}{2M}\frac{(q-k\cos\theta)}{q}\right];$$

$$\left(\frac{d\sigma}{d\Omega}\right)_2^P=\frac{\Lambda_1^2F_P}{2}, \quad (9)$$

$$F_P=g_P^2[\mu_P^2(1+\cos^2\theta)+(\sin^2\theta)q^2/\epsilon^2];$$

$$\left(\frac{d\sigma}{d\Omega}\right)_3^P=\frac{\Lambda_1^2\mu_P'^2g_P'^2(1+\cos^2\theta)}{2}, \quad (10)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_4^P=\frac{q^2q'^2k'^2[2+3(\sin^2\theta)q^2/q'^2]e^2G_P^2}{2\mu^4kM(k+M)[q-k\epsilon\cos\theta/(k+M)]}, \quad (11)$$

with θ the angle between $\mathbf{k}$ and $\mathbf{q}$.

The cross section $(d\sigma/d\Omega)_4{}^P$ is accurate to the first power in $1/M$, while Eqs. (8), (9), and (10) are accurate to lowest order in $1/M$ since they are obtained from effective Hamiltonians which are also accurate to lowest order in $1/M$. However, the statistical factor in these and in several of the following equations contain higher order corrections to insure a meaningful comparison between these results and the plane wave approximation to the total cross section.

The effective Hamiltonian for a neutron target may be written

$$[H_1(n)]_e=[H_2(n)]_e=[H_3(n)]_e=\frac{\mu_Ng_N'}{\mu_P'g_P'}[H_3(p)]_e, \quad (12)$$

where μ_N and g_N' are the neutron's magnetic moment and coupling constant, respectively.

We may assume that

$$[H_4(n)]_e=[H_4(p)]_e. \quad (13)$$

The proton and the neutron effective Hamiltonians are indeed equal if the π^0 meson is produced in $\frac{3}{2}$ isotopic spin state.[7]

The cross sections for the process

$$\gamma+n\rightarrow\pi^0+n$$

are, therefore,

$$\left(\frac{d\sigma}{d\Omega}\right)_1^N=\left(\frac{d\sigma}{d\Omega}\right)_2^N=\left(\frac{d\sigma}{d\Omega}\right)_3^N=\frac{(g_N'\mu_N)^2}{g_P'^2\mu_P'^2}\left(\frac{d\sigma}{d\Omega}\right)_3^P, \quad (14)$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_4^P=\left(\frac{d\sigma}{d\Omega}\right)_4^N. \quad (15)$$

The first three neutron cross sections are equal because the catastrophic term vanishes for a neutron target and the neutron moment is entirely anomalous.

[7] K. Watson, Phys. Rev. 85, 852 (1952).

ELASTIC PRODUCTION

The cross section for the process[8] $\gamma+D\rightarrow\pi^0+D$ is calculated with the effective Hamiltonians defined by Eqs. (4) through (7) and Eqs. (12) and (13). However, since these interactions were derived for a free nucleon at rest, their application in the deuteron calculation must be justified.

In deriving Eqs. (4) to (6), it was necessary to sum over free nucleon intermediate states. For the deuteron problem, on the other hand, the intermediate states are no longer free but are the states of a two-nucleon system with a potential. However, the energy denominator to lowest order in $1/M$ is independent of the energy of the nucleons in the intermediate state and therefore the application of closure over all intermediate states results in the free nucleon effective Hamiltonians.

A further objection to the employment of Eqs. (4) to (7) and Eqs. (12) and (13) is that the momentum operator p appearing in the corresponding Hamiltonians is evaluated for a target at rest. The validity of this approximation has been checked, and the error in this instance is also negligible.

Furthermore, with the use of the effective Hamiltonian, one neglects processes in which one particle in the deuteron absorbs the photon and the other emits the meson. This two-body effect clearly depends strongly on the behavior of the nucleon potential at small distances. If the Hulthen potential is assumed and if the energy of the incident photon is 300 Mev, the two-body terms may be calculated and these terms may also be neglected.

Finally, in this work the D state admixture to the deuteron wave function is neglected. If the Hulthen potential

$$-(\beta^2-\alpha^2)/(e^{(\beta-\alpha)r}-1)$$

is assumed, the deuteron wave function becomes

$$\psi_0(\mathbf{r})=N(e^{-\alpha r}-e^{-\beta r})/r,$$

where $-\alpha^2/M$ is the binding energy of the deuteron and where β and N are related to the triplet effective range r_0.

The effective Hamiltonians which were used to calculate the proton and neutron cross sections may be used to obtain the deuteron elastic result if we label particles 1 and 2 the proton and neutron, respectively. Then the $\mathbf{r}$ and $\boldsymbol{\sigma}$ appearing in $[H(p)]_e$ and $[H(n)]_e$ will have subscripts 1 and 2, respectively. The deuteron matrix element may now be written

$$\mathfrak{M}_D{}^i={}^3\chi_{m'}{}^*\int\psi_0(\mathbf{r})e^{-i\mathbf{P}\cdot\mathbf{R}}\{[H_i(p)]_e+[H_i(n)]_e\}\times\psi_0(\mathbf{r})d\mathbf{r}_1d\mathbf{r}_2\,{}^3\chi_m, \quad (16)$$

where ${}^3\chi_m$ is the triplet spin function, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, and

[8] N. Francis and R. E. Marshak, Phys. Rev. 85, 496 (1952); Heckrotte, Henrich, and Lepore, Phys. Rev. 85, 490 (1952).

$\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$. The superscript i indicates which of the four effective Hamiltonians is being considered. When $i=1$, 2 or 3, the evaluation of the matrix element is straightforward and requires no further discussion. To find $\mathfrak{M}_D{}^4$, however, $[H_4(p)]_e$ and $[H_4(n)]_e$ must be expressed in the laboratory system. In the laboratory system

$$[H_4(p)]_e=\frac{eG_Pe^{i\mathbf{K}\cdot\mathbf{r}}}{M\mu^2(k\epsilon)^{\frac{1}{2}}}[2\mathbf{q}'\cdot(\mathbf{k}'\times\boldsymbol{\lambda}')+i\boldsymbol{\sigma}\cdot\mathbf{q}'\times(\mathbf{k}'\times\boldsymbol{\lambda}')], \quad (17)$$

where $\mathbf{q}'=\mathbf{q}-\boldsymbol{\beta}\epsilon$, $\mathbf{k}'=\mathbf{k}-\boldsymbol{\beta}k$, $\beta=(\mathbf{p}+\mathbf{k})/(M+k)$. The momentum of the target nucleon is p. If the gauge is chosen so that the fourth component of the vector potential vanishes either in the laboratory or in the center-of-mass system, $\boldsymbol{\lambda}=\boldsymbol{\lambda}'$. It should also be remarked that in calculating the deuteron matrix element, the $\mathbf{p}$ appearing in $\boldsymbol{\beta}$ must be treated as the momentum operator which acts on the proton coordinate if the $\mathbf{p}$ appears in $[H_4(p)]_e$ and on the neutron coordinate if the $\mathbf{p}$ appears in $[H_4(n)]_e$.

After the conventional manipulations the elastic cross section for the four different effective Hamiltonians may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_1^D=\tfrac{1}{3}\Lambda_2{}^2I^2(\tfrac{1}{2}\mathbf{K})\left\{\frac{g_P{}^2q^2\sin^2\theta}{\epsilon^2}+2g_P{}^2\frac{\epsilon^2}{q^2}+(\mu_Pg_P+\mu_Ng_N)^2(1+\cos^2\theta)-2g_P{}^2\sin^2\theta-4(\mu_Pg_P+\mu_Ng_N)g_P\frac{\epsilon}{q}\cos\theta\right\}; \quad (18)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_2^D=\tfrac{1}{3}\Lambda_2{}^2I^2(\tfrac{1}{2}\mathbf{K})[F_P+F_N+2F_{PN}]; \quad (19)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_3^D=\tfrac{1}{3}\Lambda_2{}^2I^2(\tfrac{1}{2}\mathbf{K})\mu_N{}^2(g_P'-g_N')^2(1+\cos^2\theta); \quad (20)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_4^D=\frac{4}{3}\Lambda_2{}^2I^2(\tfrac{1}{2}\mathbf{K})G_P{}^2\frac{k^2}{\mu^2}\left\{(5\sin^2\theta+2)\left[1-\frac{3k+\epsilon}{2(M+k)}\right]-\frac{q^2+3k\epsilon}{q(M+k)}\cos\theta\right\}; \quad (21)$$

where

$$I(\mathbf{K})=\int e^{\pm i\mathbf{K}\cdot\mathbf{r}}\psi_0{}^2(\mathbf{r})d\mathbf{r},$$

F_P is defined above, $F_N=g_N{}^2\mu_N{}^2(1+\cos^2\theta)$, and $F_{PN}=\mu_Pg_P\mu_Ng_N(1+\cos^2\theta)$. In the elastic cross sections, we have neglected $1/M^2$ terms in $(d\sigma/d\Omega)_4{}^D$ and have neglected $1/M$ terms in the other three cross sections.

TOTAL CROSS SECTION

An upper and lower limit to the total cross section, elastic plus inelastic, for the photoproduction of π^0 mesons in deuterium will be obtained assuming a central nuclear potential that vanishes for odd parity states ("Serber mixture"). The effect of the distorted S state will be considered but all other partial waves will be assumed undistorted. Since the plane wave cross section may be considered to be a sum of terms, each with a different orbital angular momentum, and since the distortion is considered in only the final S states, the total cross section will be the plane wave cross section less its S part plus the cross section calculated with the distorted final S state. That is,

$$\left(\frac{d\sigma}{d\Omega}\right)_T=\int_\mu^{\epsilon_{\max}}\left[\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_{P.W.}-\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_S\right]d\epsilon+\left(\frac{d\sigma}{d\Omega}\right)_S', \quad (22)$$

where $(d\sigma/d\Omega)_T$ is the total cross section, $(d^2\sigma/d\Omega d\epsilon)_{P.W.}$ is the spectrum derived assuming plane waves for the final nuclear state, $(d^2\sigma/d\Omega d\epsilon)_S$ is the S part of the plane wave result and $(d\sigma/d\Omega)_S'$ is the cross section calculated with the distorted S state. The plane wave spectrum and its S part can be derived with relative ease but the calculation of the cross section for the distorted S state is tedious. We will therefore determine $(d^2\sigma/d\Omega d\epsilon)_{P.W.}$ and $(d\sigma/d\Omega d\epsilon)_S$ exactly and find an upper limit to $(d\sigma/d\Omega)_S'$.

Although the following procedure may be applied to a calculation involving any of the four effective Hamiltonians defined above, for definiteness we shall calculate the upper and lower limits to the total cross section using $H_e=[H_2(p)]_e+[H_2(n)]_e$. The effective Hamiltonian may be written

$$H_e=(\boldsymbol{\sigma}_1\cdot\mathbf{L}_1)e^{i\mathbf{K}\cdot\mathbf{r}_1}+(\boldsymbol{\sigma}_2\cdot\mathbf{L}_2)e^{i\mathbf{K}\cdot\mathbf{r}_2}, \quad (23)$$

where

$$\mathbf{L}_1=\frac{ie}{M\mu}\frac{2\pi}{(k\epsilon)^{\frac{1}{2}}}g_P\left[\frac{(\mathbf{q}\cdot\boldsymbol{\lambda})}{\epsilon}\mathbf{q}-\mu_P\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\lambda})\right],$$

and

$$\mathbf{L}_2=-\frac{ie}{M\mu}\frac{2\pi}{(k\epsilon)^{\frac{1}{2}}}g_N\mu_N\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\lambda}).$$

Using the effective Hamiltonian defined by Eq. (23), the cross section for transitions to plane wave final states is easily found to be

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_{P.W.}=\frac{\Lambda^2MN^2}{K}\{A(F_P+F_N)+\tfrac{2}{3}BF_{PN}\}, \quad (24)$$

where

$$\Lambda^2=\frac{e^2q^3}{M^2\mu^2k};\quad A=\frac{1}{a-pK}-\frac{1}{a+pK}+\frac{1}{b-pK}-\frac{1}{b+pK}-\frac{2}{b-a}\log Q;$$

$$B=\frac{b-a}{b+a}\left[\frac{1}{a}\log\left(\frac{a+pK}{a-pK}\right)-\frac{1}{b}\log\left(\frac{pK+b}{-pK+b}\right)\right];$$

$$a=\alpha^2+p^2+K^2/4;\quad b=\beta^2-\alpha^2+a;\quad \mathbf{p}=(\mathbf{p}_1-\mathbf{p}_2)/2,$$

and

$$Q=\frac{a+pK}{a-pK}\frac{b-pK}{b+pK}.$$

The part of the plane wave spectrum corresponding to final S states is

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_S=\Lambda^2N^2M[F_P+F_N+\tfrac{2}{3}F_{PN}]\times(\log^2Q)/2K^2p.\quad (25)$$

The matrix element for transitions to distorted S states may be written

$$\mathfrak{M}_S'=\chi_f^*\int\psi_f^*(\mathbf{r})e^{-i\mathbf{P}\cdot\mathbf{R}}[(\boldsymbol{\sigma}_1\cdot\mathbf{L}_1)e^{i\mathbf{K}\cdot\mathbf{r}_1}+(\boldsymbol{\sigma}_2\cdot\mathbf{L}_2)e^{i\mathbf{K}\cdot\mathbf{r}_2}]\times\psi_0(\mathbf{r})d\mathbf{r}_1d\mathbf{r}_2{}^3\chi_m,\quad (26)$$

where ψ_f is the distorted S state wave function. The cross section for distorted final S states is as follows:

$$\left(\frac{d\sigma}{d\Omega}\right)_S'=\tfrac{1}{3}\sum_f{}'\left\{\frac{\epsilon q\sum_m\langle|\mathfrak{M}_S'|^2\rangle}{(2\pi)^2[1+(\epsilon/2M)(q-k\cos\theta)/q]}\right\},\quad (27)$$

where $\sum'$ implies a summation over all final spin states and only those nuclear final states that conserve energy. An upper limit to $(d\sigma/d\Omega)_S'$ is obtained by replacing q by its maximum value and summing over the complete set of final nuclear states. We finally obtain

$$\left(\frac{d\sigma}{d\Omega}\right)_S'\leq\left[\left\{\frac{8\pi\Lambda_2{}^2N^2J}{K^2}\right\}\left\{F_P+F_N+\tfrac{2}{3}F_{PN}\right\}\right]_{q=q_{\max}}\equiv\left(\frac{d\sigma}{d\Omega}\right)_S',\quad (28)$$

where

$$J=\left\{\frac{K}{2}\left[\tan^{-1}\frac{K}{2\alpha}+\tan^{-1}\frac{K}{2\beta}-2\tan^{-1}\frac{K}{\alpha+\beta}\right]-\frac{\alpha}{2}\log\frac{4\alpha^2+K^2}{4\alpha^2}+\frac{\beta}{2}\log\frac{4\beta^2+K^2}{4\beta^2}+\frac{(\alpha+\beta)}{2}\log\frac{(\alpha+\beta)^2+K^2}{(\alpha+\beta)^2}\right\}.$$

Therefore,

$$\left(\frac{d\sigma}{d\Omega}\right)_T\leq\left(\frac{d\sigma}{d\Omega}\right)_S''+\int_\mu^{\epsilon_{\max}}\left[\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_{P.W.}-\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_S\right]d\epsilon.\quad (29)$$

If the π^0 meson spectra calculated with and without distortion are compared, it is apparent that the effect of the distortion is to increase the spectrum for low nucleon energies and decrease it for high nucleon energies. Due to the closure principle, the integral over all relative nuclear energies of the undistorted and distorted spectra are equal. Therefore, if the integration over the relative momentum extends from zero to a finite upper limit, the distorted integral exceeds the undistorted and the plane wave result is a lower limit to the total cross section.[9]

Since both an upper and a lower limit to the total cross section have been derived, it is instructive to determine when one limit may be more accurate than the other. For mesons produced in the forward direction, the distorted S state spectrum is sharply peaked at a very high meson energy. Consequently, above photon energies of about 1.5μ, when the half-width of the spectrum is small compared to the entire energy range, the error introduced by replacing q by $q_{\max}$ and summing over a complete set of final states is quite small and the upper limit to the total cross section is very accurate. However, as the meson angle increases, the distorted S state spectrum becomes diffuse and remains appreciable from $\epsilon_{\max}$ to ϵ_0 where $\epsilon_{\max}=(q_{\max}{}^2+\mu^2)^{\frac{1}{2}}$ and ϵ_0 is the energy of a π^0 meson photoproduced by a nucleon at rest. ($\epsilon_{\max}-\epsilon_0=K_0{}^2/4M$ where K_0 is the momentum transfer to a free nucleon.) Therefore for large meson angles, when K_0 is also large, $(d\sigma/d\Omega)_S''$ is a good deal larger than $(d\sigma/d\Omega)_S'$. This difference, though significant compared to $(d\sigma/d\Omega)_S'$, will be small compared to $(d\sigma/d\Omega)_T$ if many partial waves contribute to the photoproduction as they generally do.

If a single curve representing the total cross section as a function of angle is desired, a reasonably accurate result could be obtained if it is assumed that

$$\left(\frac{d\sigma}{d\Omega}\right)_T=\frac{1}{2\pi}\left[\theta\left(\frac{d\sigma}{d\Omega}\right)_{P.W.}+(2\pi-\theta)\left\{\left(\frac{d\sigma}{d\Omega}\right)_S''+\int_\mu^{\epsilon_{\max}}\left[\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_{P.W.}-\left(\frac{d^2\sigma}{d\Omega d\epsilon}\right)_S\right]d\epsilon\right\}\right].\quad (30)$$

A simple and fast method,[10] which yields an upper limit to the total cross section approximately equal to the upper limit obtained with the procedure mentioned above, will now be applied to the problem when

$$H_e=[H_4(p)]_e+[H_4(n)]_e.$$

This method, which is applicable when the energy of the incident photon is high, involves the replacement of the rigorous conservation of energy equation which appears in the cross section as the argument of a δ-function with the conservation of energy condition for a free nucleon at rest. Then, if the summation over

[9] This argument is due to G. Chew (private communication).
[10] M. Lax and H. Feshbach, Phys. Rev. **81**, 189 (1951).

final states is extended to include the complete set of final states, the result is an upper limit to the total cross section and may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_T \leq \frac{\Lambda_1^2 k^2}{\mu^2}\left\{\eta^2\left[(2+3\cos^2\theta)-\frac{4k\epsilon\cos\theta}{q(k+M)}\right] + \frac{I(\mathbf{K})}{3}\left[\left(1-\frac{(k+\epsilon)}{k+M}\right)(11\sin^2\theta+2) - \frac{2(q^2+k\epsilon)\cos\theta}{q(k+M)}\right]\right\}_{t=t_0},$$

where $\eta = M/(M+k)$. This result is quite near the correct total cross section for small θ but is about 20 percent high in the backward hemisphere.

RESULTS AND CONCLUSIONS

The most recent measurement[11] of the angular dependence of the photoproduction of π^0 mesons in hydrogen indicates that for mesons produced between 45° and 135° in the laboratory system, the yield is large at 45° and decreases sharply as the meson angle increases to 135°. This experimental result should be

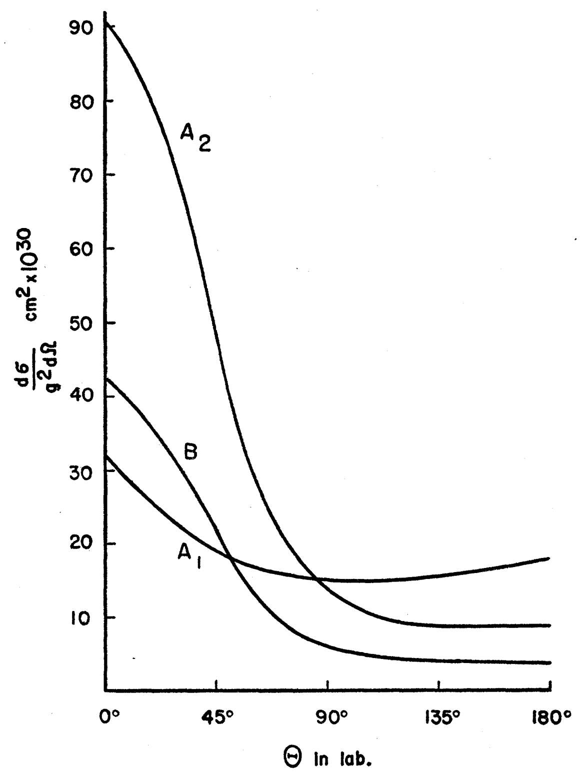

FIG. 1. Curves A_1 and A_2 give the differential cross section at $k=300$ Mev/c for the process $\gamma+p\rightarrow\pi^0+p$, considering and neglecting negative energy state transitions, respectively. The curve B gives the differential cross section at $k=300$ Mev/c for the process $\gamma+n\rightarrow\pi^0+n$.

[11] G. Cocconi and A. Silverman, private communication.

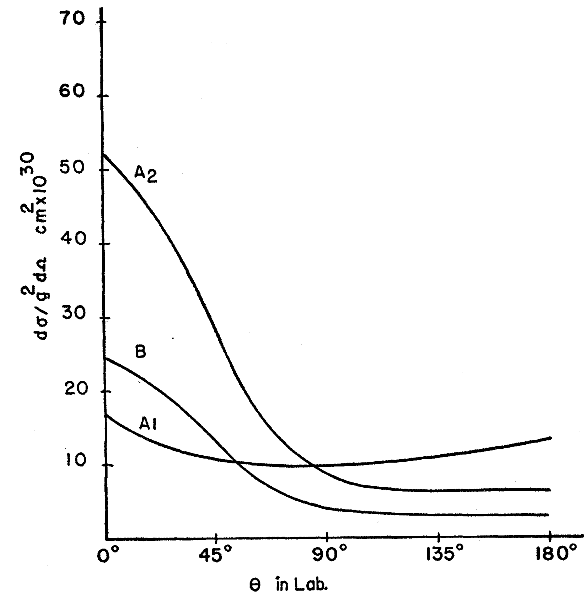

FIG. 2. Same as Fig. 1 at $k=250$ Mev/c.

compared with the theoretical predictions which were calculated for a photon energy of 300 Mev since the experiment was performed with a photon beam whose average energy was 290 Mev. The cross sections calculated for $k=300$ Mev/c using both $[H_1(p)]_e$ and $[H_2(p)]_e$ are shown in Fig. 1 as curves A_1 and A_2, respectively. The curve A_1 is far too isotropic in the angular region in question to agree with the experimental results. The curve A_2 increases steeply from $\theta=90°$ to 45° in qualitative agreement with experiment, but its behavior from $\theta=90°$ to 135° is too slowly varying. The closest agreement is obtained if the cross section is assumed to have the form $(2+3\sin^2\theta')$ in the center-of-mass system. If this theoretical cross section is transformed to the laboratory system, the resulting differential cross section in the laboratory decreases monotonically as θ increases from 45° to 135° and is shown in Fig. 9 as curve P.

The curve A_1 of Fig. 1 is a plot of the proton cross section calculated nonrelativistically with the catastrophic term included and, therefore, may be compared with the results of the relativistic calculation.[2] At $\theta=0$, where $k=\epsilon$, the relativistic and nonrelativistic results are equal. However, as θ increases, the relativistic curve remains constant while curve A_1 of Fig. 1 decreases. The difference in the shapes of the two curves is a result of neglecting terms of order $1/M$ in the nonrelativistic calculation. The error is especially large because of the destructive interference present when the catastrophic term is included.

If $\mu_P'=-\mu_N$, and if the effect of the proton charge is neglected, the proton and neutron cross sections are equal and their angular dependence is the same as curve B of Figs. 1 and 2. These curves, like A_2 of Fig. 1, are too flat from $\theta=90°$ to 135°.

TABLE I. Ratio R of deuteron elastic cross section when $g_P=-g_N$ to that when $g_P=g_N$.[a]

			R_1	R_2	R_3
k	300	0°	28.5	221	∞
		90°	16.7	11.2	∞
k	250	0°	28.5	123	∞
		90°	19.4	10.3	∞

[a] The subscript on R defines which effective Hamiltonian is used. The R_1 and R_2 are calculated neglecting and considering negative energy state transitions, respectively, while R_3 is calculated assuming that the anomalous moment alone contributes.

The deuteron elastic cross section depends strongly on the relative sign of the neutron and proton coupling constants. If we define $(d\sigma/d\Omega)_D^-$ and $(d\sigma/d\Omega)_D^+$ to be the elastic cross sections when $g_P=-g_N$ and when $g_P=g_N$, respectively, and if we define their ratio to be R, we obtain the values given in Table I.

The significant feature of Table I is the large values of R for all three theories. Therefore, if $g_P=-g_N$, the elastic deuteron cross section is appreciable, but if $g_P=g_N$, the calculated elastic result is so small that it is probably not measurable with the present techniques.

Figures 3 and 4, and Fig. 9, curve D, show the angular dependence of the cross section for the reaction

$$\gamma+D\rightarrow\pi^0+D.$$

For meson angles near zero, the relatively well-known low momentum components of the deuteron wave function contribute while at large meson angles, where the momentum transfer K is not small, the high momentum components of the deuteron wave function are needed. Therefore, at the small meson angles the elastic cross section is quite accurate as well as large. At the large meson angles, where the cross section is small, the result is uncertain because the high momentum components of the deuteron S state wave function are not well known and because the D state admixture of the deuteron wave function, which we

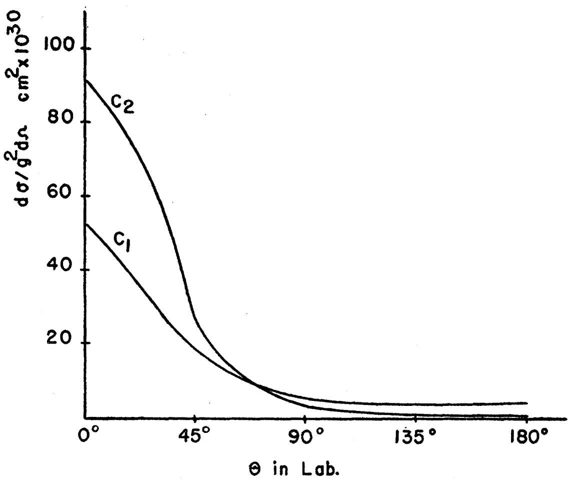

FIG. 4. Same as Fig. 3 at $k=250$ Mev/c.

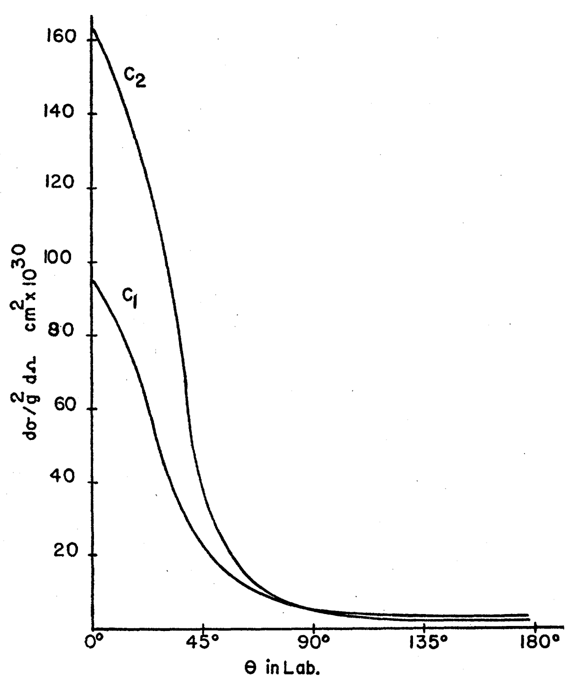

FIG. 3. Curves C_1 and C_2 give the differential cross section at $k=300$ Mev/c for the process $\gamma+D\rightarrow\pi^0+D$, assuming $g_P=-g_N$ and considering and neglecting negative energy state transitions, respectively.

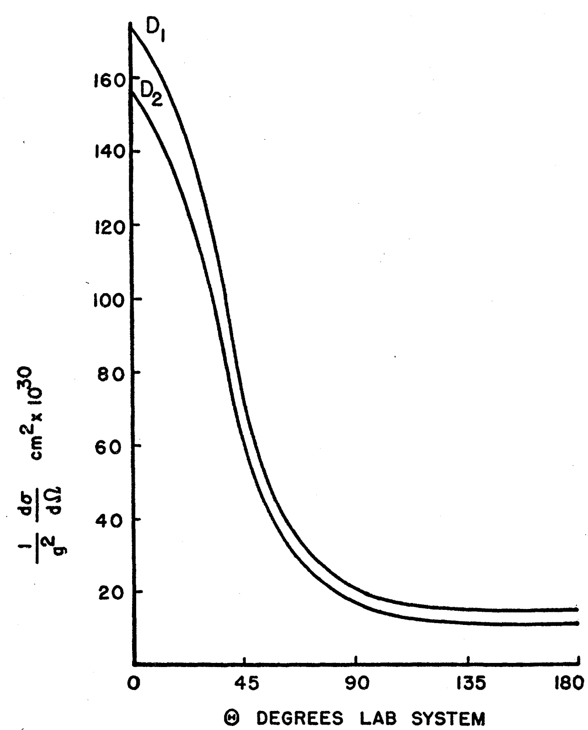

FIG. 5. Curves D_1 and D_2 give the upper and lower limits to the total cross section for the photoproduction of π^0 mesons in deuterium at 300 Mev/c with $g_P=-g_N$ and neglecting negative energy state transitions.

have neglected, might introduce an appreciable correction to the cross section.[12]

The point at $\theta=0°$ of Fig. 9, curve D, is uncertain due to the cancellation of the first and second order terms in $1/M$.

Figures 5–8 show the variation with angle of the upper and lower limits of the total cross section for the photoproduction of π^0 mesons in deuterium calculated with $[H_2(p)]_e+[H_2(n)]_e$, that is, neglecting negative energy state transitions. At the meson angles near zero, where transitions to S states are important, the upper limit is quite accurate. For mesons emitted in the backward direction, many partial waves contribute to the cross section and therefore the lower limit, which is actually the plane wave result, is likely to be as close to the total cross section as the upper limit.

We have observed that the measurement of the elastic production at any angle determines the sign of g_P/g_N. That is, the coherent production determines the relative phase of the proton and the neutron ampli-

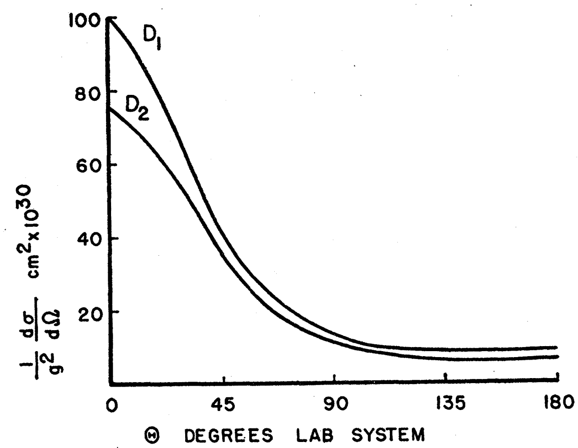

FIG. 6. Same as Fig. 5 at $k=250$ Mev/c.

tudes. For $\theta>90°$, the total cross section is chiefly incoherent and is, therefore, essentially independent of the relative sign of g_P and g_N. However, the cross section at these angles is quite sensitive to the neutron contribution and, thus, the comparison of the theoretical results with experiment at large meson angles offers a good method for determining the neutron contribution to the π^0 production.

Cocconi and Silverman[11] have measured the ratio of the deuterium total cross section to the hydrogen cross section when the maximum photon energy is 310 Mev and for $\theta=45°$, 90°, and 135° in the laboratory system. We will first consider the 90° ratio. The measured result is

$$\left[\left(\frac{d\sigma}{d\Omega}\right)_T \Big/ \left(\frac{d\sigma}{d\Omega}\right)_P\right]_{\theta=90°}=1.91\pm0.09.$$

[12] Chew, Goldberger, Steinberger, and Yang, Phys. Rev. 84, 581 (1951).

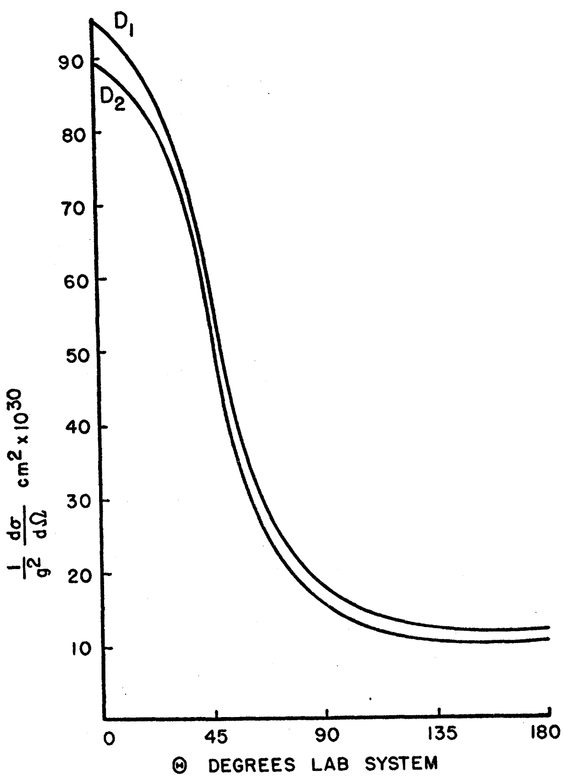

FIG. 7. Same as Fig. 5 with $g_P=g_N$.

The upper limits to the calculated ratios assuming no negative energy state transitions are 1.51 and 1.29 when $g_P=-g_N$ and when $g_P=g_N$, respectively. The larger calculated ratio is smaller than the experimental result by about 20 percent which is outside the range of the experimental error. The reason why the calculated cross section is small is because the neutron con-

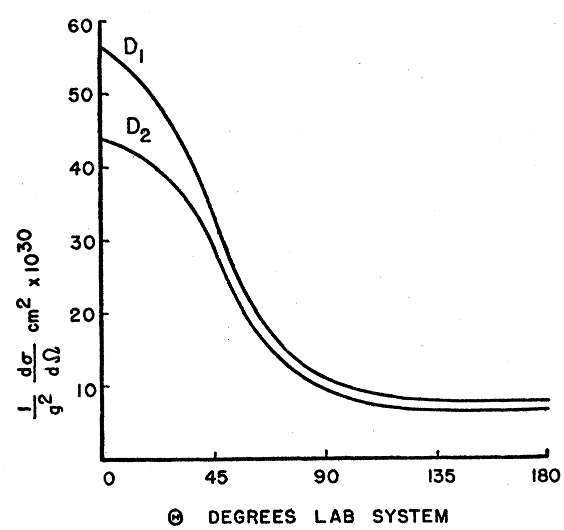

FIG. 8. Same as Fig. 6 with $g_P=g_N$.

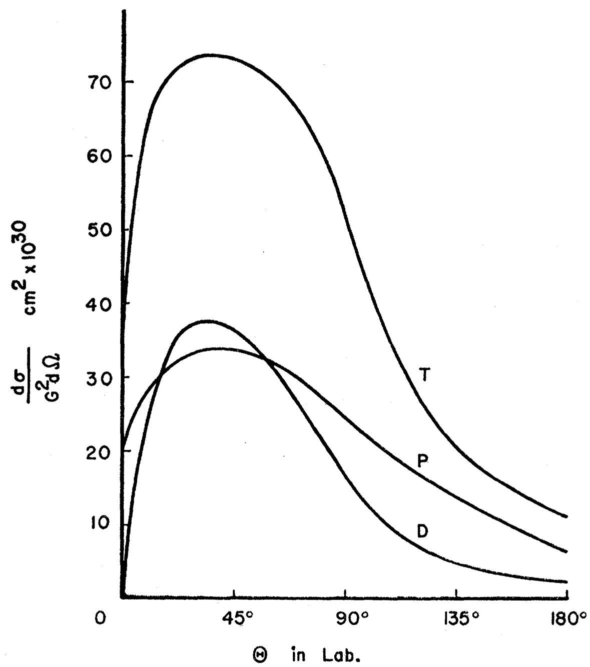

FIG. 9. Curves P, D, and T are the proton, elastic deuterium, and upper limit to the total deuterium cross section, respectively, for $k=300$ Mev/c, assuming an interaction which gives a $(2+3\sin^2\theta')$ proton cross section in the center-of-mass system.

tribution is only about one-half that of the proton when the measured magnetic moment of the latter is used.

If we assume that $\mu_P'=-\mu_N$ and that the proton current does not contribute, however, and furthermore that $g_P'=-g_N$, the upper limit to the ratio of the total deuteron to the proton cross sections at $\theta=90°$ may be estimated to be about 2 in rough agreement with the experimental value (see Fig. 9). When $\theta=135°$ the lower limit to the ratio of the total deuteron cross section to the proton cross section is about 1.5, which may be compared with an experimentally measured result of 1.53. Therefore, the measured ratio of the total cross sections is not inconsistent with the hypothesis that the anomalous moment alone contributes to the photo π^0 meson production. The ratio of the upper limit to total cross section in deuterium to the hydrogen result is 2.1 at 90° and 2.0 at 135° when the proton's cross section is $(2+3\sin^2\theta')$ in the center-of-mass system. The ratio at 135° is approximately 33 percent above the experimental value, which is not inconsistent considering that the experiment at 135° is uncertain to about 20 percent and the upper limit is probably large by about 10 to 20 percent.

Finally, we may predict that if the anomalous moments of the proton and neutron alone contribute to the photoproduction of π^0 mesons, then for $k=300$ Mev and for $\theta=90°$

$$\left(\frac{d\sigma}{d\Omega}\right)_D \Big/ \left(\frac{d\sigma}{d\Omega}\right)_P = 0.5.$$

A ratio of 0.65 is obtained if the calculation is performed with the interaction which yields a $(2+3\sin^2\theta')$ distribution for the proton.

In conclusion, the only interactions which yield a ratio of about 2 in agreement with experiment for $(d\sigma/d\Omega)_T/(d\sigma/d\Omega)_P$ at $\theta=90°$ are the interactions which give approximately equal proton and neutron cross sections. The production of π^0 mesons in photon-nucleon collisions thus appears to be charge independent, although further experiments are clearly necessary.

The author wishes to thank Dr. R. E. Marshak for suggesting this problem and for his continued stimulating guidance. He is also indebted to Dr. C. Longmire and Dr. R. Scaletar for their aid in the early stages of this research.