Polarization Effects in Nucleon-Nucleon Scattering*

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If a beam of unpolarized nucleons is scattered from a target of unpolarized nucleons, the scattered particles are polarized (in a direction normal to the scattering plane) provided that the interaction contains tensor or spin-orbit forces. The polarization can be detected by means of a second similar scattering since the cross section then contains an azimuthal dependence:

$$I(\theta, \phi)=I_0(\theta)(1+\epsilon \cos\phi),$$

where $\epsilon(\theta)$ is essentially the square of the polarization. Calculations are carried out by the author for a double $p-p$ scattering using the tensor interaction described in the preceding paper, and for a double $n-p$ scattering using the central and tensor potential of Christian and Hart (containing the "half-exchange" dependence proposed by Serber). The polarization produced by the first scattering at the optimum angle of $\theta\approx 50°$ was found to vary from 6 percent at 40 Mev to 33 percent at 285 Mev for $n-p$ scattering and from 10 percent 129 Mev to 15 percent at 350 Mev for $p-p$ scattering. The $n-p$ results (previously published) are consistent with the azimuthal asymmetry detected in a double scattering experiment reported by L. Wouters.

SCATTERING OF A POLARIZED BEAM

FOR a single nucleon-nucleon collision in a definite initial spin state χ_i, the intensity of the scattered state is given by $(\bar{S}\chi_i, \bar{S}\chi_i)$, the expectation value of $\bar{S}^\dagger\bar{S}$. S is the 3×3 triplet spin scattering matrix defined in the Appendix of the preceding paper[1]; $\bar{S}$ (4×4 dimensions) is the same with singlet states included. The result of a measurement to which many scattering events contribute is necessarily the average expectation value of the measured quantity taken over an ensemble of all possible initial states of the system. The totality of information concerning a system can be expressed in terms of the q-dimensional density matrix, $\rho_{ji}^{(q)}=\sum_\alpha g_\alpha(a_i^\alpha)^* a_j^\alpha$, where $\sum_i a_i^\alpha u_i$ is the wave function of the system in the state α, g_α is probability of occurrence, and u_i a complete set of expansion functions. Following the method of Wolfenstein and Ashkin,[2,3] let $\rho^{(4)}$ refer to the initial spin states of the two-nucleon system; then the differential scattering cross section is given by $\mathrm{Tr}(\rho^{(4)}\bar{S}^\dagger\bar{S})$. Consider for the moment an ensemble of one particle (spin $\frac{1}{2}$) systems; a measurement of spin will yield the result $\langle\boldsymbol{\sigma}_1\rangle=\mathrm{Tr}(\rho^{(2)}\boldsymbol{\sigma}_1)$, from which it follows that the (two-dimensional) density matrix can be written $\rho^{(2)}=\frac{1}{2}[1+\langle\boldsymbol{\sigma}_1\rangle\cdot\boldsymbol{\sigma}_1]$. The four-dimensional density matrix describing a spin state ensemble of two-particle systems is given by the "direct product"[4] of the density matrices for the one-particle ensembles, provided that the states of one particle are not correlated with those of the other:

$$\rho^{(4)}(1, 2)=\rho^{(2)}(1)\times\rho^{(2)}(2),$$

or

$$(\rho^{(4)})_{ij;i'j'}=[\rho^{(2)}(1)]_{ii'}[\rho^{(2)}(2)]_{jj'}.$$

Hence,

$$\rho^{(4)}=\tfrac{1}{4}(1+\langle\boldsymbol{\sigma}_1\rangle\cdot\boldsymbol{\sigma}_1)\times(1+\langle\boldsymbol{\sigma}_2\rangle\cdot\boldsymbol{\sigma}_2). \tag{1}$$

The differential cross section for a beam of particles of polarization $\mathbf{P}_1=\langle\boldsymbol{\sigma}_1\rangle/I_1$ scattered from an unpolarized target $\langle\boldsymbol{\sigma}_2\rangle=0$ is therefore given by

$$\begin{aligned}\mathrm{Tr}(\rho^{(4)}\bar{S}^\dagger\bar{S})&=\tfrac{1}{4}I_0\,\mathrm{Tr}(\bar{S}^\dagger\bar{S})+\tfrac{1}{4}\langle\boldsymbol{\sigma}_1\rangle\cdot\mathrm{Tr}(\boldsymbol{\sigma}_1\times 1\bar{S}^\dagger\bar{S})\\&=\tfrac{1}{4}I_0\,\mathrm{Tr}(\bar{S}^\dagger\bar{S})+\tfrac{1}{8}\langle\boldsymbol{\sigma}_1\rangle\cdot\mathrm{Tr}(\boldsymbol{\sigma}S^\dagger S),\end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}$ is the triplet spin operator and I_0 the intensity of the incident beam. The second equality follows from the absence of matrix elements in S between triplet and singlet states; hence the latter do not contribute to the "polarization term" $\frac{1}{8}\langle\boldsymbol{\sigma}_1\rangle\cdot\mathrm{Tr}(\boldsymbol{\sigma}S^\dagger S)$.

For an interaction of the form

$$[A(r)+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 B(r)][a+bP_x],$$

S is proportional to the (triplet) unit matrix and so the polarization term vanishes. In the case of a tensor or spin-orbit force, it follows from Eq. (A2)[1] (or can be proved by symmetry arguments[5]) that the polarization term in Eq. (2) is nonvanishing and proportional to the component of polarization of the incident beam normal to the scattering plane. Detection of an azimuthal dependence of this type in the nucleon-nucleon scattering cross section would therefore constitute direct evidence for the presence of noncentral forces. The problem now to be considered is that of producing the incident polarized beam of high energy (S states alone do not contribute to polarization) nucleons.

If an unpolarized beam strikes an unpolarized target, the polarization of the scattered beam is given by

$$(\bar{S}\chi_i, \boldsymbol{\sigma}_i\bar{S}\chi_i)/(\bar{S}\chi_i, \bar{S}\chi_i)=\mathrm{Tr}(\boldsymbol{\sigma}SS^\dagger)/\mathrm{Tr}(\bar{S}\bar{S}^\dagger),$$

where $\rho^{(4)}=\frac{1}{4}1$ is the density matrix describing the initial system. A proof, based on the transformation

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[1] Don R. Swanson, Phys. Rev. **89**, 740 (1953). The notation of this reference will be used throughout.

[2] L. Wolfenstein, Phys. Rev. **76**, 541 (1949).

[3] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

[4] H. Weyl, *Theory of Groups and Quantum Mechanics* (Methuen and Company, Ltd., London, 1931), Chap. II.

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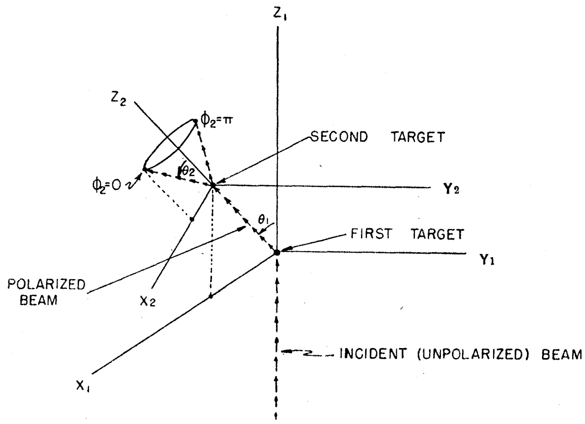

FIG. 1. Coordinate axes for double scattering problem.

properties of S, that

$$\mathrm{Tr}(\boldsymbol{\sigma}SS^\dagger)=\mathrm{Tr}(\boldsymbol{\sigma}S^\dagger S)$$

has been given by Wolfenstein and Ashkin.[3] An algebraic *tour de force*, however, using the form (A2) (reference 1) for S, yields the equality

$$\mathrm{Tr}(\sigma_y S^\dagger S)-\mathrm{Tr}(\sigma_y SS^\dagger)=\frac{\tan\theta\cos\phi}{4k^2}\times Im\sum_J[P_{J+1}{}^2+J(J+1)P_{J+1}][(A_{J+1}{}^J-A_{J-1}{}^J)-(B_{J+1}{}^J-B_{J-1}{}^J)][A-E-C]=0, \quad (3)$$

which vanishes immediately for purely central or $\mathbf{S}\cdot\mathbf{L}$ forces (uncoupled, therefore $\delta_l{}^{Jm_s}=\delta_l{}^J$) and does so for tensor forces as a consequence of the Wronskian conditions (A9), (A17b), reference 1.

If the z_1 direction is taken as that of the incident beam, then $\mathrm{Tr}(\sigma_{z_1}S^\dagger S)\equiv 0$ may be readily confirmed. Placing the x_1 axis in the scattering plane, $(\phi_1=0)$, the polarization is given by

$$P_{1y_1}(\theta_1,\phi_1=0)=\frac{1}{8I_1}\mathrm{Tr}(\sigma_{y_1}S_1{}^\dagger S_1)=\frac{Q_1(\theta_1)}{I_1(\theta_1)};$$
$$P_{x_1}=P_{z_1}=0;\quad I_1=\tfrac{1}{4}\mathrm{Tr}(S_1{}^\dagger S_1), \quad (4)$$

where the subscripts 1 will be used throughout to denote the first scattering.

In the first scattering, introduce the subscript (b) to represent the particles originally in the incident beam, and (t) to denote those from the target. The polarization of the two scattered beams is the same:

$$\langle\boldsymbol{\sigma}_b\rangle=\mathrm{Tr}(\boldsymbol{\sigma}_b\times 1\bar{S}^\dagger\bar{S})=\mathrm{Tr}(1\times\sigma_t\bar{S}^\dagger\bar{S})=\langle\boldsymbol{\sigma}_t\rangle=\tfrac{1}{2}\mathrm{Tr}(\boldsymbol{\sigma}S^\dagger S). \quad (5)$$

The nucleons emerging at some laboratory angle (Θ,Φ) will be used to form the incident beam for a second scattering. If particles (b) are to be used, the center-of-mass angles are $\theta=2\Theta$ and $\phi=\Phi$; for particles (t), however, $\theta=\pi-2\Theta$ and $\phi=\Phi+\pi$. Consider, for example, the experiment of Wouters[6] in which incident protons produce a neutron beam by means of a (p, n) reaction. The (p, n) collision is described by $S(\theta,\phi)$, and the polarization of neutrons observed at Θ, Φ is $\langle\sigma_t\rangle(\theta,\phi)/I(\theta)$ where $\theta=\pi-2\Theta$ and $\phi=\Phi+\pi$. The scattering matrix itself carries all information on the exchange nature of the interaction. In the case of two protons the S matrix is antisymmetric, so it is of course immaterial whether $\theta=2\Theta$, $\phi=\Phi$ or $\theta=\pi-2\Theta$, $\phi=\Phi+\pi$ is used.

The subscript 1 will be used hereafter in place of (b) or (t) to indicate that the operator in question refers to once-scattered particles which form an incident beam for the second scattering.

THE DOUBLE SCATTERING PROBLEM

The coordinate system for the second scattering $(x_2y_2z_2)$ is obtained by rotating $(x_1y_1z_1)$ about the y_1 axis until the z axis lies along the new incident beam (Fig. 1). Hence $P_{y_1}=P_{y_2}$ is unchanged and represents (in the form of $\langle\boldsymbol{\sigma}_1\rangle$) just the quantity that must appear in the density matrix for the new initial state

$$\rho^{(4)}=\rho_1{}^{(2)}\times\rho_2{}^{(2)}=\tfrac{1}{4}(1+\langle\boldsymbol{\sigma}_1\rangle\cdot\boldsymbol{\sigma}_1)\times(1+\langle\boldsymbol{\sigma}_2\rangle\cdot\boldsymbol{\sigma}_2). \quad (6)$$

The subscript 2 refers to particles of the second target. The latter is supposed to be unpolarized, so that $\langle\boldsymbol{\sigma}_2\rangle=0$. The differential cross section for the second scattering

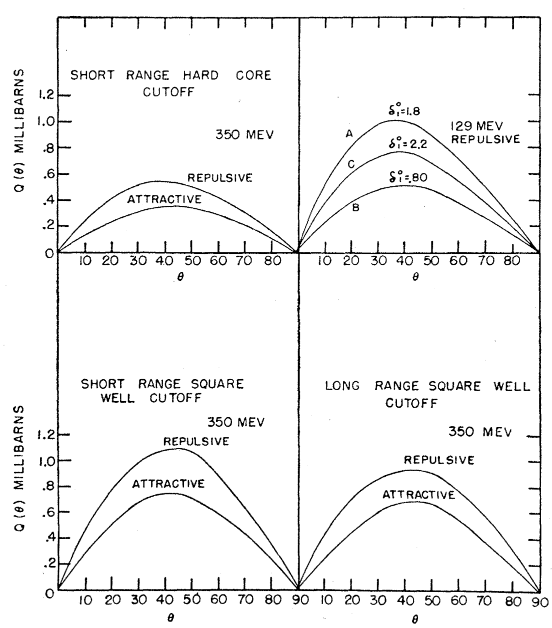

FIG. 2. Values of $Q(\theta)=\frac{1}{8}\mathrm{Tr}(\sigma_y S^\dagger S)$ for $p-p$ scattering at (lab. system) energies of 350 Mev, 129 Mev. θ=scattering angle in center-of-mass system. The interactions indicated (cut-off singular tensor) are those for which cross sections were computed in reference 1 (Figs. 2, 3).

[6] L. Wouters, Phys. Rev. 84, 1069 (1951).

is obtained from (6), (2), (3), and (4):

$$(d\sigma/d\Omega)_2 = J(\theta_1, \theta_2, \phi_2) = I_1(\theta_1)I_2(\theta_2) + Q_1(\theta_1)Q_2(\theta_2)\cos\phi_2. \quad (7)$$

$I_1(\theta_1)$ and $I_2(\theta_2)$ are the differential cross sections with polarization terms omitted.

In the case of $p-p$ scattering, or $n-p$ scattering with exchange dependence $1\pm P_x$, so that interaction occurs only in orbital angular momentum states of the same parity, then the condition

$$(S^\dagger S)(\theta, \phi) = (S^\dagger S)(\pi-\theta, \phi+\pi)$$

implies $Q(\theta) = -Q(\pi-\theta)$ so that $Q(\pi/2)=0$. The contribution to the polarization at $\theta=\pi/2$ must therefore come exclusively from odd-even interference terms; the possibility of such a measurement suggests a test of the $1+P_x$ dependence proposed by Serber.

Ignoring for the moment the fact that the second scattering occurs at a somewhat lower energy than the first, and assuming the two involve the same types of particles (i.e., both $n-p$ or both $p-p$), then the measured ratio at the optimum angles $\theta_1=\theta_2=\theta_{\max}$ is

$$R \equiv \frac{J(\phi_2=0)}{J(\phi_2=\pi)} = \frac{1+(Q/I)^2}{1-(Q/I)^2} \geq 1. \quad (8)$$

Barring a somewhat remarkable dependence of $Q(\theta)$ on energy, a ratio greater than 1 should in general be expected as the experimental result whenever $\theta_1 \approx \theta_2$. A relationship which led to Eq. (3),

$$\sqrt{2}(B-D)\cot\theta-(A-C-E) = \sum_J [P_{J+1}{}^2 + J(J+1)P_{J+1}][(A_{J+1}{}^J - A_{J-1}{}^J) - (B_{J+1}{}^J - B_{J-1}{}^J)] = 0, \quad (9)$$

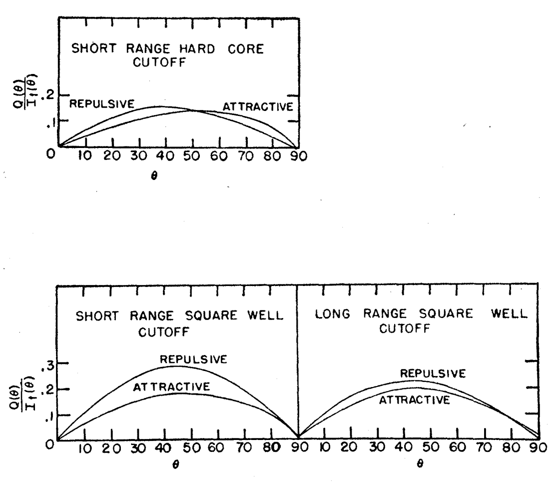

FIG. 3. Values of $Q(\theta)/I_t(\theta)$ for $p-p$ scattering at 350 Mev. $I_t(\theta)$ = triplet cross section. Polarization is given by

$$Q(\theta)/[I_t(\theta)+I_s(\theta)];$$

$I_s(\theta)$ = singlet cross section. The function plotted hence represents the polarization at those angles ($\theta > 50°$ for Christian and Noyes model) for which singlet scattering is negligible. The interactions indicated (cut-off singular tensor) are those for which cross sections were computed in reference 1.

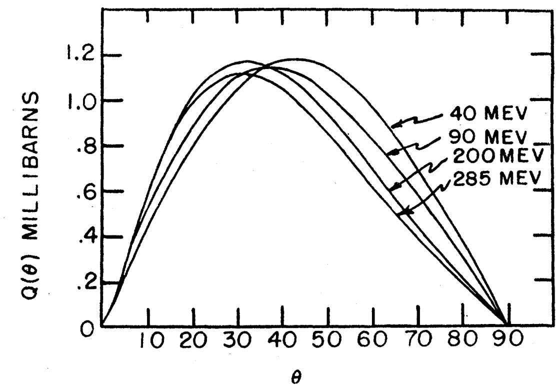

FIG. 4. Values of $Q(\theta)=\frac{1}{8}\operatorname{Tr}(\sigma_y S^\dagger S)$ for $n-p$ scattering at the energies indicated. The interaction used is that of Christian and Hart. A similar plot of polarization (Q/I) is given in reference 7.

can be used to simplify appreciably the form of $Q(\theta)$ by eliminating $A-C$.

$$Q(\theta) = \frac{\epsilon_4}{8k^2} Im[E'(B'+D')^* + D^{*\prime}B'\cot\theta] + \frac{\epsilon_2}{4k}(C_+ + F)\Re(B+D) \equiv Q' + Q_I. \quad (10)$$

ϵ_2, ϵ_4 are defined in Eq. (A17) of reference 1.

RESULTS AND CONCLUSIONS

For $p-p$ scattering, $Q(\theta)$ is plotted in Fig. 2 for all cases considered in reference 1 except the long-range hard core model which has been omitted because the coupled phase shifts were found only roughly. The dominant term of Eq. (10), which alone yields a value of $Q(\theta)$ correct to within 50 percent or so is quite simple; for singular potentials $(C_K - C_L)$ is very small, so that $Q' \gg Q_I$; only P states have been kept:

$$Q(\theta) \approx Im\{B_1'^{2*}[B_1'^0 + \tfrac{3}{2}(A_1'^1 - A_1'^2)]\} P_2^1(\cos\theta). \quad (11)$$

The importance of obtaining accurate values for the coupled ${}^3P_2$ phase shifts is clear; even rigorously there is no contribution to the polarization from the ${}^3P_0$ and ${}^3P_1$ states alone. The polarization,

$$P(\theta, \phi=0) = Q(\theta)/I(\theta),$$

is plotted in Fig. 3; the value of $I(\theta)$ was taken in all cases to be the predicted triplet cross section for the potential model used; that is, the singlet scattering is assumed negligible for $\theta \geq 50°$. If, instead, it is assumed that singlet scattering can be introduced in such a way as to bring the cross section in each case up to the experimental value of 4 millibarns, then Fig. 2, rather than Fig. 3, shows more clearly the dependence of polarization on choice of cutoff. With the potential given by Eq. (6) of reference 1, the polarization (at $\theta \approx 50°$) is 10 percent ($R \approx 1.02$) at 129 Mev and 15 percent ($R \approx 1.05$) at 350 Mev.

For $n-p$ scattering, the tensor and central interaction of Christian and Hart (containing the "half-exchange" dependence proposed by Serber) is used. In Fig. 4, $Q(\theta)$ is plotted for energies of 40, 90, 200, 285 Mev. A similar plot of the polarization $Q(\theta)/I(\theta)$ was given in an earlier report.[7] A comparison of $Q(\theta)$ with Q/I illustrates the point that $I(\theta)$ alone carries almost the entire energy dependence of the polarization.

If odd-state forces were introduced into the triplet $n-p$ interaction (by changing the $1+P_x$ dependence), the polarization could be considerably larger because of the contribution from $S-P$ interference:

$$Q_{SP}=(1/8k^2)Im\{B_0^1[B_1^2-B_1^0+\tfrac{3}{2}(A_1^2-A_1^1)]^*\}\sin\theta. \quad (12)$$

[7] Don R. Swanson, Phys. Rev. 84, 1068 (1951).

To obtain some idea of the magnitude of this term, suppose the same amount of triplet odd-state interaction were introduced into the $n-p$ Hamiltonian as was used for the $p-p$ interaction in the preceding paper.[1] Interpolating the $p-p$ phase shifts to obtain rough values at 200 Mev, the result is $Q_{SP}\approx 0.5\sin\theta$ millibarns leading to $R(\pi/2)\approx 1.03$. Hence, although the asymmetry is appreciably influenced by the presence of odd states, the quoted uncertainty in the experimental results of Wouters[6] is too great to permit any sharp conclusions to be drawn on the question of the exchange dependence of the $n-p$ interaction. The desirability of further experiments on $n-p$ double scattering is, however, indicated.

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Isomeric Transitions in Tc^{93} and Tc^{96}†

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The 390-kev γ-line reported to be associated with Tc^{92} is found to be Tc^{93} and is shown to be a $M4$ isomeric transition. Mass assignment to Tc^{96} is made for the isomeric transition previously known to have 34.4-kev energy and 51.5-minute half-life, and a weak positron branching was found.

MASS assignment of the many technetium activities which are usually produced by irradiation of molybdenum is quite difficult because the latter element has seven stable isotopes of roughly equal natural abundance. From the standpoint of nuclear shell structure, knowledge of these activities is of particular interest since according to this model, technetium with 43 protons lies in a region of isomerism. Until now characteristics of isomeric transitions were known for the odd-even isotopes 95, 97, and 99. This report concerns isomeric states in the nuclei 93 and 96.

The three strongest activities produced when enriched Mo^{92} is bombarded with protons of energies between 5 and 10 Mev decay with half-lives of 4.5 minutes, 43.5 minutes, and 2.7 hours.[1–3] At the proton energies used (p, n) and (p, γ) reactions are the most probable, so the activities are expected to be associated with Tc^{92} or Tc^{93}. In order to attempt to fix the mass of the 390-kev γ-transition of 43.5-minute half-life as being Tc^{93}, the activity was produced by two additional separate reactions. Thresholds calculated from the empirical mass formula[4] for (d, n) and $(d, 2n)$ reactions on Mo^{92} are -2 Mev and $+10$ Mev. The 390-kev line was observed in 1-mil molybdenum foils throughout the full range of 20-Mev deuterons; however, it could not be produced with neutron bombardment on the same type of foil. Thresholds for $(\alpha, 4n)$ and $(\alpha, 5n)$ on Nb^{93} are calculated to be 34 and 45 Mev, respectively. The activity was found to be produced in slight amount with α's between 39 and 40 Mev. Although this evidence cannot be considered conclusive, it appears to indicate that the 390-kev line is Tc^{93} instead of Tc^{92} as previously reported.

The multipole order of the 390-kev line was determined by measurement of the K/L ratio and the conversion coefficient. The activity for the multipole order measurements was produced by bombardment of enriched Mo^{92} with 9.5-Mev protons from the 60-inch cyclotron and the activity observed in a β-spectrometer. Chemical separation of the Tc activities was made by heating the bombarded molybdenum oxide in a glass tube open at one end. By controlling the temperature, the technetium oxide can be made to condense in the cooler part of the tube whereas the less volatile molybdenum oxide is not affected. The activity is removed with a drop of dilute ammonium hydroxide and mounted on a thin Tygon foil. Figure 1 shows the K- and L-

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[4] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950), p. 7.