

The integration in x can be manipulated to give

$$\int_0^{\infty} \frac{x^2 e^x dx}{(x^2 + 4\alpha^2)^{3/2} (e^x - 1)^2}$$

If one takes advantage of the smallness of α , this integral can be shown to have the value $\log(\epsilon/\alpha)$, where $\epsilon = 2.718 \dots$. We have then

$$(\mu\kappa\theta/p_0)^2 \int_0^{\pi/2} (9 \cos^3\theta + 2 \cos\theta \sin^2\theta) \log(2\epsilon\kappa\theta \tan\theta/\beta^3\mathcal{C}) d\theta.$$

This integration can be carried out and gives finally

$$(20/3)(\mu\kappa\theta/p_0)^2 \log(\kappa\theta\epsilon^{0.65}/\beta^3\mathcal{C}).$$

We will now compute the contribution of the second term in $F(\lambda, \lambda', \theta)$. We make the same substitution

and analysis as before and obtain similarly

$$\left(\frac{\mu\beta^3\mathcal{C}}{p_0}\right)^2 \int_0^{\pi/2} \int_0^{\infty} \frac{(9 \cos^3\theta \cot^2\theta - \cos^3\theta)x^2 e^x dx d\theta}{(x^2 + 4\alpha^2)^{3/2} (e^x - 1)^2}.$$

If we integrate in x , observing as before the smallness of α for most all θ , this expression is

$$(\mu\kappa\theta/p_0)^2 \int_0^{\pi/2} (9 \cos^3\theta - \cos\theta \sin^2\theta) d\theta.$$

We obtain on further integration in θ

$$(17/3)(\mu\kappa\theta/p_0)^2 = (20/3)(\mu\kappa\theta/p_0)^2 \log\epsilon^{0.85}.$$

On inserting the sum of the two terms we have evaluated into (55), we obtain (56).

Induced Conductivity in Luminescent Powders. II. AC Impedance Measurements*

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Alternating current measurements of the impedance changes induced by ultraviolet, infrared, x-ray, and gamma-ray irradiation in (Zn:Cd)S luminescent powders are reported. These results are in agreement with previously reported dc measurements, and indicate that the observed impedance changes are primarily due to the change of the electron density in the conductivity band. Whereas the capacity changes show a monotonic increase, the Q values (X/R) show a pronounced minimum with increasing intensity. A model consisting of radiation sensitive and nonsensitive powder portions is proposed, and theoretical calculations based on such a model are shown to be in agreement with most of the experimental results. This ac measurement technique can be used to measure small exciting intensities (down to almost 1/1000 erg/sec cm²).

1. INTRODUCTION

IN a previous investigation¹ of the conductivity induced in luminescent type materials (in powder form) by ultraviolet radiation it was found that a non-ohmic relationship between the induced current and the applied field existed at all voltages except when the highest field strengths were used. Similar non-ohmic relationships were found in later experiments in which x-rays were used as the exciting radiation. With light excitation the powders are not uniformly excited and this nonuniform excitation is certainly responsible for at least part of the non-ohmic relationship observed. The x-rays used for these experiments penetrated these thin layers relatively uniformly; nevertheless, similar strong non-ohmic relationships were observed and this indicates that this non-ohmic characteristic is inherent to the powder structure. Since conductivity experiments with single crystals^{2,3} irradiated with x-rays showed an

ohmic relationship, this supports the idea that the non-ohmic characteristics are inherent to the grain structure.

With these results in mind it is assumed here, as a working hypothesis, that the powder has to be considered as an inhomogeneous system. Some parts of the powder sample (grain boundaries and air spaces) remain highly resistive even under strong excitation, while other parts, especially the interior of the grains, become highly conductive under excitation, much more than observed from dc measurements of the total resistance. To test this idea it was decided to duplicate many of the conductivity measurements with powders under ultraviolet, infrared, and high energy particle radiation using an alternating current field of low field strength instead of a dc field and measuring the effective resistive and reactive components of the impedance, hoping, in this way, to detect the true conductivity of the single grains.

Somewhat similar ac measurements have been reported by Garlick and Gibson,⁴ who have thoroughly

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¹ H. Kallmann and B. Kramer, *Phys. Rev.* **87**, 91 (1952).

² H. Kallmann and R. Warminsky, *Ann. Physik* **4**, 57 (1948) and **4**, 69 (1948); see also J. Fassbender, *Ann. Physik* **6**, 33 (1950) and R. Warminsky, Ph.D. thesis, Berlin Tech. Univ. (1948) (unpublished).

³ H. Kallmann, Signal Corps Engineering Report E-1036 (Fort Monmouth, New Jersey) (unpublished).

⁴ G. F. J. Garlick and A. F. Gibson, *Proc. Roy. Soc. (London)* **A188**, 485 (1947).

investigated the apparent change in dielectric constant and conductance of such luminescent powders in the frequency range of 0.1 to 10 megacycles. Their interpretation of such measurements is based upon a model in which the trapped electrons, assumed to be capable of remaining trapped while moving distances over 1000 times the inter-atomic spacing, are responsible for the impedance changes observed. de Groot⁵ reports some ac measurements on luminescent powders and attributes the impedance changes to the electrons in the conductivity band; his measurements, however, are not extensive enough to confirm his interpretation.

The results of the ac measurements on luminescent powders reported in this paper are completely in agreement with the previously reported dc conductivity measurements¹ if, as assumed above, the inhomogeneity, that is, the relative insensitivity of parts of the powder to external radiation, is taken into account. This can be strikingly seen from the results obtained by the addition of infrared light during or after ultraviolet excitation. Corresponding to the observed initial dc current stimulation followed by current quenching, the ac measurements show an increase and then a decrease in the capacitance change values.

In our inhomogeneous system the impedance changes observed are ascribed to the fact that certain portions of the powder remain relatively nonconducting while other portions, especially the interior of the grains may become highly conductive (many electrons in the conduction band) under the influence of exciting radiation. Such a system could be treated theoretically by assuming the sample to be composed of some dielectric medium in which are embedded particles whose conductivity depends on the exciting radiation. The important feature of such a matrix is the increase of capacity to a saturation value with increasing excitation since the particles, when conductive, effectively decrease the spacing between the electrodes. The Q of such a system would be large (for a perfect dielectric) at low excitation, decreasing when the radiation sensitive particles become slightly conductive, and rising to a high value again when the particles become very conductive under strong excitation. Such an inhomogeneous system would exhibit an infinite dc resistivity; this is not in contradiction with the dc measurements since a resistivity $>10^{12}$ ohm-cm was obtained if the field strength was of the same value as that used in the ac measurements (~ 100 volt/cm).⁶

Since the computation of the impedance of such an inhomogeneous system would entail the use of various unknown parameters involving the shape and size of the particles as well as their spacing in the dielectric and would still be only an approximation, a very simplified model composed of layers of radiation sensitive portions alternating with high resistance radiation nonsensitive

layers will be used. The high resistive layers essentially represent the high resistance grain boundaries and the air spaces between the grains while the radiation sensitive ones represent the interior of the grains. Such a layer model shows all the characteristic features of the above-described inhomogeneous system, and can be easily shown (see appendix) to be equivalent to two condensers in series, one of which is shunted by a radiation sensitive resistance. As this resistance decreases with increasing excitation, the effect of the capacitance in parallel with it becomes increasingly negligible, and finally only the capacitance of the nonconductive layer remains, which means the effective capacitance of the entire sample has increased. Increasing the intensity of excitation further can cause no further changes in the capacitance, and a saturation value is obtained. The effective resistive component of the impedance, however, shows no such monotonic change with intensity. As the radiation sensitive layer becomes slightly conducting, the shunt resistance decreases, and the effective series resistance of the sample increases. With much larger intensities this layer simply becomes a very low resistance in series with the nonconducting layer; the resistive component of the total impedance decreases. Thus, on the basis of this simple layer model, the resistive component may vary greatly and show a decided maximum. Since the capacitance changes measured are less than 100 percent, the ratio of the reactive to the resistive component of the impedance should show a definite minimum. The measurements described below show both the capacitance increase (also observed by Garlick and Gibson) and the predicted minimum of the " Q " value.

It should be noted that the inhomogeneous system described above is subject to at least two modifications which can also be illustrated by the layer model. First, the so-called nonconductive portion may also be somewhat responsive to the exciting radiation; this would lead to a nonsaturated change in capacitance as well as a Q value at higher radiation intensities that remained lower than the unexcited Q value. Secondly, the radiation may not be absorbed uniformly throughout the powder thickness. This is certainly true for ultraviolet radiation at around 3660A because of strong absorption in some of these samples as well as scattering in the powder. This effect tends to spread out the variation of capacitance and Q with intensity without changing the basic characteristics of such dependence curves.

On the basis of this simple layer model the following formulas (see appendix) for the capacitance change and Q values as functions of the exciting intensity are obtained for uniform excitation:

$$\Delta C = C - C_0 = \epsilon A d_2 / \{ (d_1 + d_2) \times [d_1 + (d_1 + d_2) \omega^2 \epsilon^2 / \gamma^2 I^2] \}; \quad (A5)$$

$$Q = (\omega \epsilon / \gamma I) (1 + d_1 / d_2) + \gamma I (d_1 / d_2) / \epsilon \omega. \quad (A7)$$

⁵ W. de Groot, *Physica* 12, 402 (1946).

⁶ The high dc photoconductivity occurs only at high field strengths where the nonconductive layers are bridged.

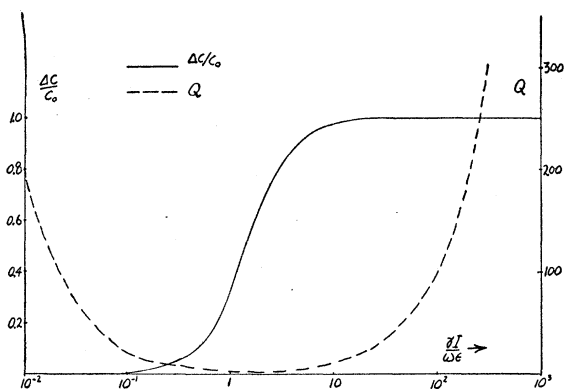


FIG. 1. Theoretical curves showing the capacitance change and Q value versus exciting intensity ($d_1=d_2$).

Curves showing relations (A5) and (A7) are given in Fig. 1 for $d_1=d_2$.

For nonuniform excitation the total impedance may be written as (see appendix)

$$Z = \frac{(d_2/d_0 A \alpha \omega \epsilon) (\tan^{-1}\{(\omega \epsilon / \gamma I)(e^{\alpha d_0} - 1)\} / [1 + e^{\alpha d_0} (\omega \epsilon / \gamma I)^2])}{-j(d_0 / A \omega \epsilon) \{1 - (d_2 / 2 \alpha d_0^2) \times \ln[(\omega^2 \epsilon^2 + \gamma^2 I^2) / (\omega^2 \epsilon^2 + \gamma^2 I^2 e^{-2 \alpha d_0})]\}} \quad (\text{A11})$$

The ΔC and Q curves obtained from (A11) are shown in Figs. 2 and 3, again for $d_1=d_2$. Since (A5), (A7), and (A11) give ΔC and Q as functions of I/ω , the curves should be shifted to lower intensities with decreasing frequencies.

This proposed representation does not delve into an atomistic explanation of the nonconductive behavior of the grain boundaries, but it is believed that both the grain boundaries and the air space may be considered as capacities with very high interior resistances only slightly influenced by radiation, while the grains themselves provide the capacities with an excitation sensitive interior resistance. The nonconductive portion may also be partly due to electron depleted barrier layers near the electrodes, but the capacitance saturation obtained

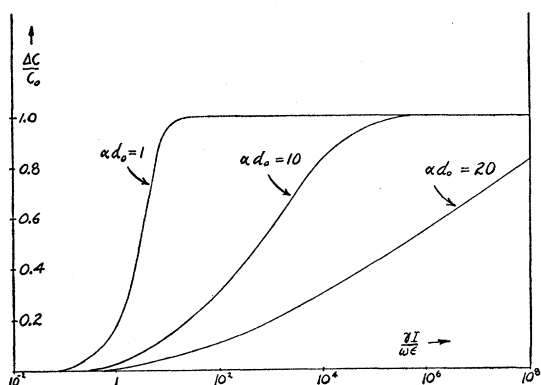


FIG. 2. Theoretical curves showing the capacitance change versus exciting intensity for various values of αd_0 ($d_1=d_2$).

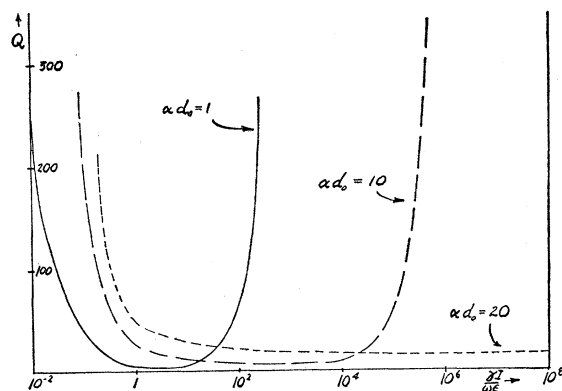


FIG. 3. Theoretical curves showing the Q values versus exciting intensity for various values of αd_0 ($d_1=d_2$).

at high exciting intensities indicates that such layers are not important in the powders investigated.

In Sec. 2 the experimental results of such impedance measurements are described. These investigations deal with the impedance changes induced in the powders by ultraviolet, x-ray, and gamma-radiation, as well as the influence of infrared radiation.

2. EXPERIMENTAL RESULTS

Experiments were performed at 1 kc using a General Radio Bridge 650-A and at 4.6 kc, 80 kc, and 800 kc with a Boonton 160-A Q -meter.

The powders investigated were deposited on a piece of conductive glass, and a thin aluminum foil (about 4 cm^2 in area) was used as the other electrode. This method of preparing powders for conductivity measurements has previously been described.¹

A. Results with Ultraviolet Irradiation

The ultraviolet source used was of the mercury discharge type (maximum energy around 3650Å); about 10 microwatts per square centimeter was incident upon the powder. Calibrated neutral filters were used to decrease this intensity by a factor of up to 10^5 .

In Fig. 4 the capacitance change and Q values for powder L (19 mg/cm^2), a long-persistence [Zn: Cd]-

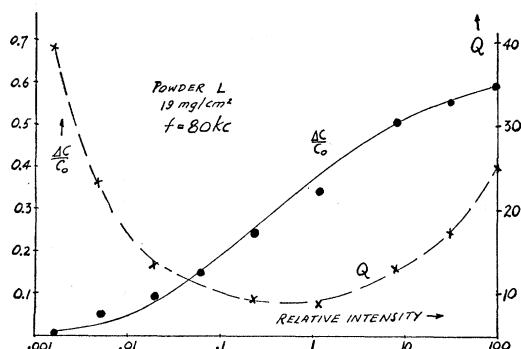


FIG. 4. Capacitance change and Q values for powder L versus ultraviolet exciting intensity.

S(Pb, Ag) material, are shown over a range of exciting intensities at a frequency of 80 kc. The characteristics of these curves are similar to those of the theoretical curves. The capacitance change curve shows the typical slow rise, relatively steep middle portion, and tendency toward saturation with increasing intensity, while the Q curve shows the pronounced minimum to be expected from the theoretical calculations.

Figures 5 and 6 show the capacitance change and Q values of powder L (19 mg/cm^2) at frequencies of 4600 cycles, 80 kc, and 800 kc for various values of exciting intensities. The capacitance change curve clearly shows how an increase in frequency shifts the curves to increased intensities; thus, a value of $\Delta C/C_0$ of 20 percent is reached at about $(5 \times 10^{-5})I_{\text{max}}$ for $f=4600$ cps, at about $(10^{-3})I_{\text{max}}$ for $f=80$ kc, and at $(10^{-1})I_{\text{max}}$ for $f=800$ kc. For this powder at these intensities, the saturation part of the curve can be seen only at low frequencies, while the slow increase from zero capacitance change can be seen best at the highest frequency. At 80 kc, the intermediate frequency, the $\Delta C/C_0$ curve

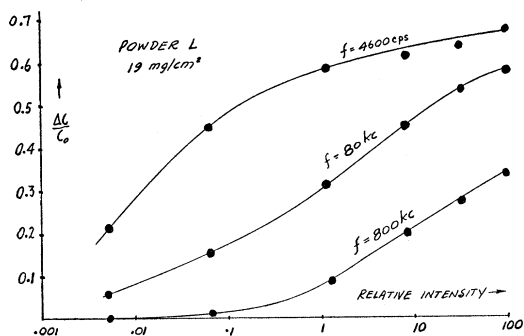


Fig. 5. Capacitance change values for powder L versus ultraviolet exciting intensity at various applied frequencies.

is approximately linear with $\log I$ over most of the measured range, similar to the curves obtained by Garlick and Gibson.

Although the theoretical calculations lead to the same value for $\Delta C/C_0$ at given values of I/ω , the experimental results indicate that the percentage change in I is considerably larger than the percentage change of ω to obtain the same capacitance change. The theoretical results were obtained by assuming ρ to vary inversely as I ; if this dependence is changed to $\rho \propto 1/I^n$, where n is less than one, the theoretical curves would be in more accord with the measurements. In the results on dc measurements,¹ all powders showed a deviation from a linear relationship between the induced current and the exciting intensity; if the current density is expressed as $J = aI^n$, $n > 1$ for the previously reported measurements. This is at variance with the relationship obtained from Fig. 5 where $n < 1$ is a better fit for the experimental results. It may be that the shape of the dc current-intensity curves were considerably influenced by other factors, such as potential barriers and non-

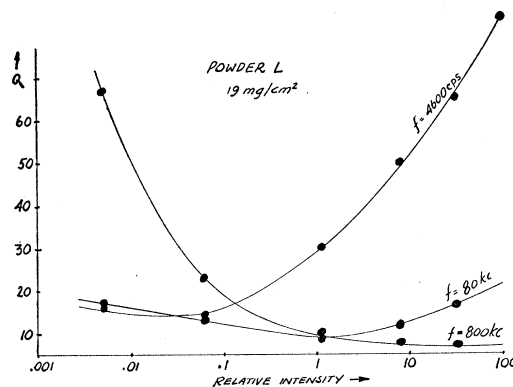


Fig. 6. Q values for powder L versus ultraviolet exciting intensity at various applied frequencies.

uniform field strength, which do not show up appreciably in the ac measurements. The theoretical calculations carried out previously¹ indicate that there are regions where $n > 1$, $n = 1$, and $n < 1$.

Measurements of fluorescent emission (E) versus exciting intensity showed, in some of these powders, a dropping off from a linear relation at low intensities, indicating a relationship $E \propto I^n$, where $n > 1$. Further investigations are being carried out to find explanations for these different behaviors.

Figure 6 shows the Q curves for powder L , and again the shift toward higher intensities with higher frequencies is easily noted. For the highest frequency used, 800 kc, the minimum of the Q curve is just reached, while at the lowest frequency (4600 cps), the minimum can just be seen at lowest intensities. The shifts are more difficult to determine because of the broadness of the curves; in addition, the Q values are probably not as accurate as the capacitance measurements.

Figure 7 shows the capacitance change and Q curves for powder N , a ZnS(Ag) long persistence luminescent powder of 26 mg/cm^2 thickness. The curve of $\Delta C/C_0$ versus the log of the exciting intensity is linear over a range of more than 5 orders of intensity magnitudes, and there is no sign of saturation. The departure from a straight line would probably be evident at both lower

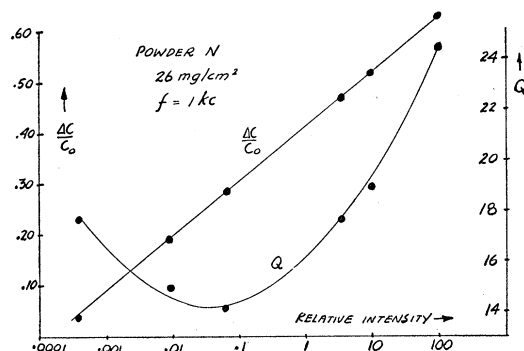


Fig. 7. Capacitance change and Q values for powder N versus ultraviolet exciting intensity.

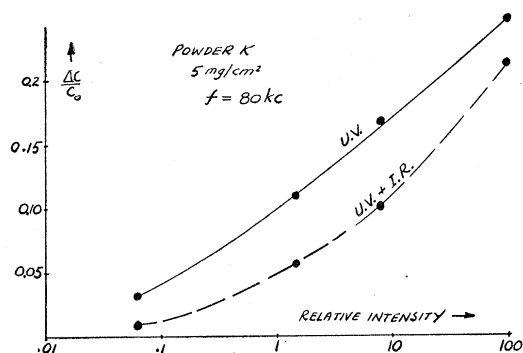


FIG. 8. Capacitance change values for powder *K* versus ultraviolet exciting intensity with and without infrared irradiation.

and higher frequencies. The Q curve shows the typical pronounced minimum.

Other $\Delta C/C_0$ and Q curves of other powders under ultraviolet excitation are shown and discussed together with ultraviolet plus infrared curves in the following section.

B. Results for Simultaneous Ultraviolet and Infrared Irradiation

The capacitance and Q values of these powders were measured when infrared irradiation was added to the ultraviolet excitation. The infrared source was a 250-watt heating lamp with a 7-56 Corning filter, and 1 milliwatt/cm² was incident upon the powder. The wavelength of the transmitted radiation was too long to excite the powder. In all powders measured, the equilibrium curves obtained with simultaneous infrared and ultraviolet radiation shifted as if the actual ultraviolet intensity were decreased; that is, a higher intensity was required to obtain a corresponding value of the ordinate. Thus, a definite "quenching" was observed, analogous to the current quenching observed with dc measurements. This shift is more pronounced at lower ultraviolet intensities; again in agreement with previous dc measurements where quenching always increases with decreasing excitation intensity. Figures 8 and 9 show the shift due to addition of infrared irradiation in the capacitance change and Q curves for a 5

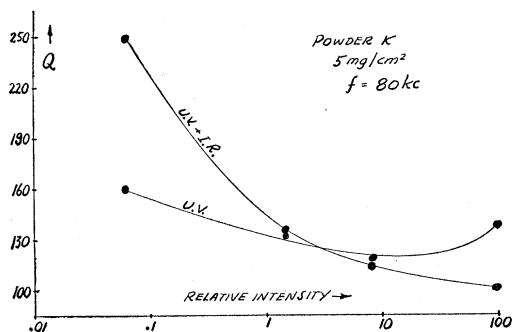


FIG. 9. Q values for powder *K* versus ultraviolet exciting intensity with and without infrared irradiation.

mg/cm² sample of powder *K*; a [Zn: Cd]S(Pb, Cu, Ni) short-persistence luminescent powder. From Fig. 8 it can be seen that at high intensities the quenching is about 50 percent while at low intensities it is almost 90 percent, which checks with the dc measurements (where quenching is defined as $[(J_{UV} - J_{UV+IR})/J_{UV}] \cdot 100$ percent).

The Q curve (Fig. 9) also shows the same shift, but in addition there is now a definite change in the value of the minimum Q ; this value changing from about 120 to 100, when infrared radiation is added to the existing ultraviolet excitation. Since an effective change of intensity should only shift the curve, this change in the minimum value indicates that the infrared radiation has some additional effect on the "nonconducting" portion of the powder; an effect not taken into account in the theoretical models proposed above.

Previous dc measurements have shown that a current stimulation occurs before quenching sets in when infra-

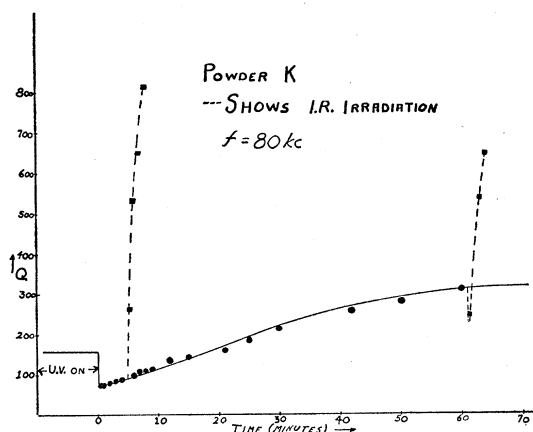


FIG. 10. Q values for powder *K* versus time after cessation of ultraviolet excitation showing effect of infrared irradiation at two points on decay curve.

red irradiation is added to ultraviolet excitation, especially for powder *K*. Such a stimulation is difficult to note when using a Q meter or impedance bridge because the stimulation peak is rather fast and some time is required to obtain an impedance measurement. It was definitely noted, however, that at lower intensities (to the left of the cross-over point between the two Q curves of Fig. 9), the application of additional infrared irradiation caused a momentary decrease in the Q value before the equilibrium value was reached, while at intensities to the right of this point, first an increase and then a decrease in the Q value was noted. Thus, the infrared induced stimulation peak observed in dc measurements is also found in these impedance investigations. Since the infrared irradiation acts to empty the filled traps, the change in Q should be monotonic if only the filled traps influence the impedance measurements; if, however, the effects are primarily due to electrons in the conduction band, a nonmonotonic change may occur,

since first there is an increase and then a decrease of the electron density in the conduction band due to the stimulation and overriding quenching effects of the infrared irradiation. This stimulation effect is more easily noted after ultraviolet excitation (during the decay period) and will be discussed below.

C. Measurements after Ultraviolet Excitation

Figure 10 shows the Q values obtained with powder K during and after ultraviolet excitation. As soon as the ultraviolet excitation is removed there is a sharp drop in the Q value followed by a slow build up to a Q value much higher than the Q with UV excitation. Thus the Q values again pass through a minimum, this time due to the decrease in conductivity with time rather than with intensity; effectively the Q values change from the extreme right-hand point of the UV curve (Fig. 9) through the minimum to the extreme left. The effects of infrared irradiation after ultraviolet excitation

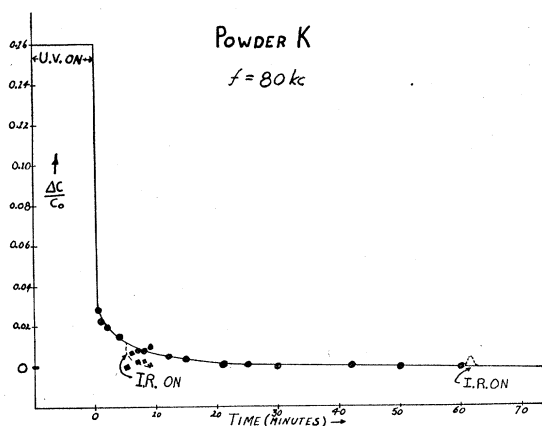


FIG. 11. Capacitance change values for powder K versus time after cessation of ultraviolet excitation showing effect of infrared irradiation at two points on decay curve.

are indicated by the dotted lines and are completely similar to the dc effects. Immediately after the ultraviolet radiation has been stopped, the effect of the infrared is to further decrease the conductivity. However, further along the decay curve (about 1 hour after ultraviolet excitation), a temporary decrease, then an increase of Q can be noted. This corresponds to a stimulation and subsequent quenching of the conductivity, respectively as in dc measurements. The stimulation is difficult to observe if the infrared is switched on at early portions of the decay curve; after longer time intervals the stimulation becomes more extended and can be noted on the Q meter.

The capacitance change (for this same sample) as a function of time after ultraviolet excitation is shown in Fig. 11. Corresponding to the fast decay of the conductivity a sharp decrease in the capacitance is seen. Infrared irradiation causes a further decrease in the capacitance if applied soon after ultraviolet excitation

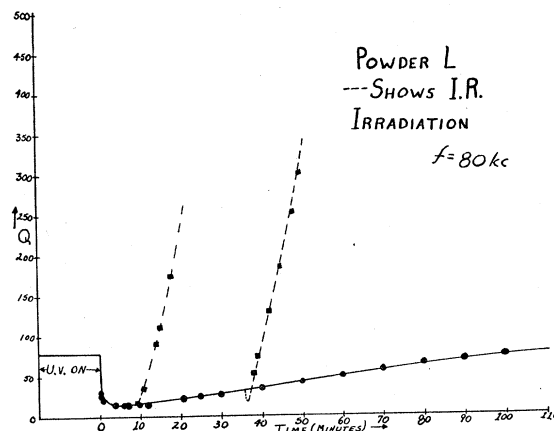


FIG. 12. Q values for powder L versus time after cessation of ultraviolet excitation showing effect of infrared irradiation at two points on decay curve.

has ceased, but if 1 hour elapses before infrared irradiation is applied, an increase in capacitance is noted. The latter is due to an increase of conductivity, corresponding to the decrease in Q seen in Fig. 10. Figures 12 and 13 show similar effects for powder L . Again, stimulation is observed only after a considerable time lapse after ultraviolet radiation has ceased. The decay of the capacitance change curve is considerably longer than that of powder K .

D. X-Ray and Gamma-Irradiation

Figures 14 and 15 show the effect of x-ray excitation on powder L at 80 kc.

X-ray experiments were undertaken to prove that the previously described results obtained with ultraviolet excitation are not primarily due to the non-uniform excitation of the powder. X-ray irradiation, although it excites these powders nearly uniformly (small α , Eq. (A9)), shows almost the same shape

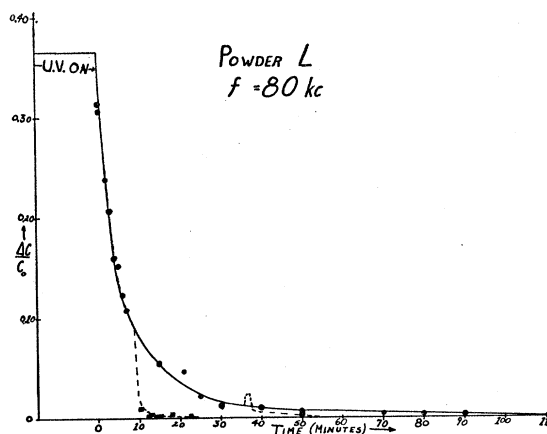


FIG. 13. Capacitance change values for powder L versus time after cessation of ultraviolet excitation showing effect of infrared irradiation at two points on decay curve.

of the $\Delta C/C_0$ and Q curves so that one can be sure that these effects are inherent to this type of material. X-rays of 50 kv were used, and intensities up to 300 r/min were obtained, which is roughly equivalent to the same amount of absorbed energy as in the light experiments. The $\Delta C/C_0$ curve shows a very sharp increase to its saturation value; this occurs over a range of about 2 magnitudes of intensity in comparison with about 3 orders of magnitude for the fastest rising curve for ultraviolet excitation. This is to be expected from the theoretical curves (Fig. 2), in which the smallest absorption (α) gives the steepest rise. Similarly, the Q curve is sharper than any of the other Q curves given above, again in agreement with the theoretical curves of Fig. 3.

Under gamma-irradiation, powder L evidenced a slow change in Q from over 1000 to 30 after a 150-minute exposure (irradiation of 10 r/hr). This gamma-irradiation corresponds to a very small ultraviolet excitation since the absorbed energy in the powder is of the order of 1/100 erg per second. The equilibrium Q value under gamma-irradiation would therefore correspond to the lowest intensity of Fig. 6. A similar slow build up in conductivity was noted in the dc measurements, where weak ultraviolet excitation was required for hours before an equilibrium current was reached, due to the fact that a large number of empty traps must be filled before an appreciable density of conduction band electrons becomes evident.

3. CONCLUSIONS

The above described experiments indicate that the changes of impedance induced by external radiation in (Zn: Cd)S powders can be essentially ascribed to a change in the density of electrons in the conduction band if the sample is considered to be an inhomogeneous material, a portion of which remains highly resistive even under strong irradiation. The observed impedance changes should, therefore, not be described as changes in the dielectric properties of the material, and the assumption of Garlick and Gibson of polarizable traps

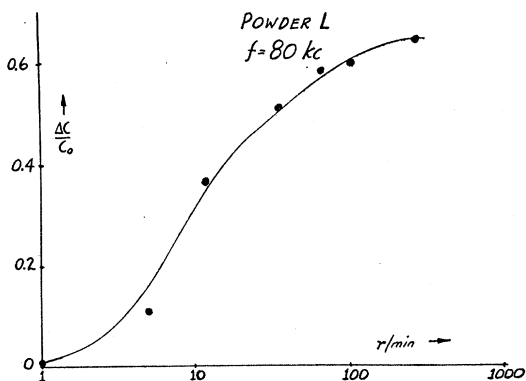


FIG. 14. Capacitance change for powder L values versus x-ray exciting intensity.

is not necessary, at least for these powders. It may be noted that the particular shape of the Q curve reported here has not yet been explained by the polarizable trap model. Such a shape could, to some extent, also be explained on the basis of nonuniform excitation without assuming an intrinsic inhomogeneity of the powder. From the x-ray experiments (Sec. 2, Part D) where the powders are uniformly excited and the Q curve has the same shape, it is concluded that it is the intrinsic inhomogeneity of the powder that is responsible for the shape of the Q curve.

The close parallelism which was found between the radiation induced conductivity in dc measurements and the induced impedance changes in the ac measurements also strongly supports the above-mentioned conclusion. This parallelism is especially noticeable in the experiments where infrared light is directed at the powder after previous ultraviolet excitation. Corresponding to the observed dc current increase followed by a quenching there is first an increase, then a decrease in the capacitance (and a corresponding change in the Q value). If the impedance changes were due to electrons in traps, infrared irradiation would constantly decrease this amount, and there would be only a decrease in the capacitance and no such change in Q as found in Figs. 10 and 12.

Using the simplified layer model, an order of magnitude evaluation can also be made of the conductivity of the radiation sensitive portion. The result thus obtained is larger than that observed in the dc measurements (as is to be expected because of grain contact resistance), but is about the same as the experimental value found for single CdS crystals.^{2,3}

APPENDIX

A. Uniform Excitation

If the powder can be considered to consist of a radiation sensitive portion and a permanently nonconducting portion, the impedance may be written as

$$Z = -j/\omega C_1 + R_2/(1 + R_2^2 \omega^2 C_2^2) - jR_2^2 \omega C_2/(1 + R_2^2 \omega^2 C_2^2), \quad (A1)$$

since the arrangement described above is equivalent to two capacities in series.⁷ C_1 is the capacitance of the nonconducting layer, C_2 is the capacitance of the radiation sensitive layer and R_2 is the shunt resistance of the radiation sensitive layer. If one considers the resistance of the conducting layer to vary inversely as the exciting intensity the impedance may be written as

$$Z = -jd_1/\omega\epsilon A + (d_1/\gamma IA)/(1 + \omega^2\epsilon^2/\gamma^2 I^2) - (j\omega\epsilon d_2/\gamma^2 I^2 A^2)/(1 + \omega^2\epsilon^2/\gamma^2 I^2), \quad (A2)$$

where we have used $C_1 = \epsilon A/d_1$; $C_2 = \epsilon A/d_2$; $R_2 = \rho d_2/A = d_2/\gamma IA$, and where A is the area, d_1 and d_2 are the thicknesses of the nonconducting and conducting layer

⁷ See any textbook on electrical circuits.

respectively, and ϵ is assumed constant. Separating into real and imaginary components, the impedance may be written

$$Z = (d_2/A\gamma I)/(1 + \omega^2\epsilon^2/\gamma^2 I^2) - (j/\omega\epsilon A)\{[d_2/(1 + \gamma^2 I^2/\omega^2\epsilon^2)] + d_1\}. \quad (\text{A3})$$

The apparent capacitance of the powder is obtained from the reactive part:

$$C = 1/\omega X = \epsilon A/\{[d_2/(1 + \gamma^2 I^2/\omega^2\epsilon^2)] + d_1\}. \quad (\text{A4})$$

The apparent change in capacitance of the powder is

$$\Delta C = C - C_0 = (\epsilon A d_2)/\{(d_1 + d_2) \times [d_1 + (d_1 + d_2)\omega^2\epsilon^2/\gamma^2 I^2]\}, \quad (\text{A5})$$

where C_0 is the capacitance at zero intensity. The largest capacitance change obtainable (very large I) is

$$\Delta C_{\text{sat.}} = \epsilon A d_2/d_1(d_1 + d_2). \quad (\text{A6})$$

The Q of the circuit (X/R) is

$$Q = (\omega\epsilon/\gamma I)(1 + d_1/d_2) + \gamma I(d_1/d_2)/\omega\epsilon. \quad (\text{A7})$$

Thus, Q becomes infinite for both $I=0$ and $I=\infty$, with a minimum given by

$$(\gamma I/\omega\epsilon)^2 = (d_1 + d_2)/d_1, \quad (\text{A8})$$

at which point it has a value of $2[d_1(d_1 + d_2)]^{1/2}/d_2$.

B. Nonuniform Excitation

If the incident radiation is strongly absorbed by the luminescent powder, there will be a nonuniform excitation throughout the powder. With x-rays, high energy beta-particles, and gamma-radiation such absorption is negligible and the powder can be considered uniformly excited. With ultraviolet or alpha-particle excitation, strong absorption sets in, and the layer of the powder facing the source is more strongly excited than the other side.

To consider the case of nonuniform excitation, the model again consists of N_1 nonconductive and N_2 conductive layers.

The impedance element of one layer of the radiation sensitive portion of the powder may be expressed as

$$dZ = dx/A(j\omega\epsilon + \gamma I e^{-\alpha x}), \quad (\text{A9})$$

where $A\epsilon/dx$ and $dx/\gamma I e^{-\alpha x}A$ are the capacitance and resistance of the element considered; where the intensity of the radiation is assumed to fall off exponentially in the powder; and where x is the position of the layer. Each radiation insensitive layer has the same value as for uniform excitation and together have the im-

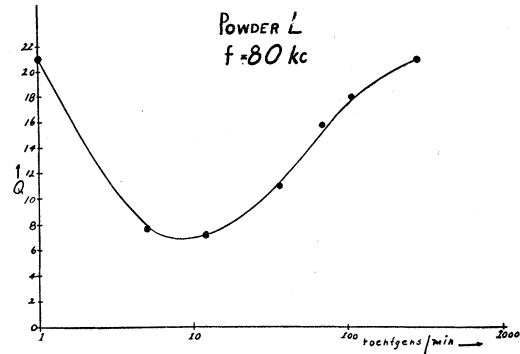


FIG. 15. Q values for powder L versus x-ray exciting intensity.

pedance $1/j\omega C_1$. Therefore, the total impedance is

$$Z = (d_2/d_0 A) \int_0^{d_0} dx/(j\omega\epsilon + \gamma I e^{-\alpha x}) + 1/j\omega C_1, \quad (\text{A10})$$

where $1/C_1 = \sum 1/C_n$, for C_n the capacitance of each radiation insensitive layer. The integral is to be evaluated over the entire thickness of the powder (d_0); since the radiation sensitive portion is only the fraction d_2/d_0 of the entire sample, this factor must be placed in front of the integral. The integration is straightforward and yields

$$Z = (d_2/d_0 A \alpha \omega\epsilon) (\tan^{-1}\{(\omega\epsilon/\gamma I)(e^{\alpha d_0} - 1)/[1 + e^{\alpha d_0}(\omega\epsilon/\gamma I)^2]\}) - (j d_0/A \omega\epsilon) \{1 - (d_2/2\alpha d_0^2) \times \ln[(\omega^2\epsilon^2 + \gamma^2 I^2)/(\omega^2\epsilon^2 + \gamma^2 I^2 e^{-2\alpha d_0})]\}. \quad (\text{A11})$$

For α approaching zero (negligible absorption) this equation reduces to (A3) above. However, the intensity dependence of Q and $\Delta C/C_0$ for intermediate values of intensity is considerably different from that obtained from (A3) for considerable absorption ($\alpha \geq 1/d_0$).

If, instead of using the simple model consisting of a conducting and nonconducting portion of the powder, these portions are differentiated by having different conductivities (γ_1 and γ_2) differing by at least several orders of magnitude, Eq. (A10) becomes

$$Z = (d_1/d_0 A) \int_0^{d_0} dx/(j\omega\epsilon + \gamma_2 I e^{-\alpha x}) + (d_1/d_0 A) \int_0^{d_0} dx/(j\omega\epsilon + \gamma_1 I e^{-\alpha x}). \quad (\text{A12})$$

The change in the $\Delta C/C_0$ and Q curves using (A12) instead of (A10) would be that (1) the $\Delta C/C_0$ curve would not reach a saturation value but would continue to increase with intensity; (2) the Q curve, after going through a minimum, would tend to reach a maximum and then to decrease again at higher intensities.