The Continuous Gamma-Ray Spectrum Accompanying Beta-Decay*

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The spectral distribution and absolute intensity of the internal bremsstrahlung emitted in beta-decay was investigated in P^{32}, Y^{91}, and RaE. The gamma-radiation was detected by means of a NaI(Tl) crystal, phototube, linear amplifier and one-channel discriminator. Care was taken to avoid spurious effects, and corrections were made for several factors. Both the spectral distributions and the absolute intensities of the continua are in excellent agreement with theoretical predictions. The differences in the spectra due to allowed or forbidden types of the beta-transition and to different forms of interaction are too small to be detected in these experiments. Observations on the angular correlation of the internal bremsstrahlung of beta-emitters and on the internal bremsstrahlung emitted in K capture are also reported.

I. INTRODUCTION

THE spectral distribution of the continuous gamma-ray spectrum accompanying beta-decay has been reported previously.[1–4] The work reported here represents more complete results and extends the range of investigations to 20 kev. Some results on the angular correlation between the beta-ray and its associated bremsstrahlung are also given.

The theory of the spectral distribution of the internal bremsstrahlung was first given by Knipp and Uhlenbeck,[5] and Bloch.[6] Further calculations on the bremsstrahlung from first and second forbidden beta-transitions were made by Chang and Falkoff[7] for scalar interactions. A calculation of the first forbidden case for tensor interaction with the selection rule ($\Delta J=2$; yes) was made by Madansky *et al.*[8] The expressions for the spectral distribution of the gamma-rays for the allowed and forbidden cases are given below.

The following expressions utilize the notation of Chang and Falkoff,[7] where $x=W_0-k$.

Allowed beta-transition:

$$S(k)=\frac{\alpha G^2|M|^2}{4\pi^4}\frac{1}{k}\left\{\left[W_0^2\left(\frac{2}{3}x^3+x\right)-W_0\left(x^4+x^2-\frac{1}{8}\right)+\frac{7}{15}x^5-\frac{3}{8}x\right]\ln[x+(x^2-1)^{\frac{1}{2}}]-\left[W_0^2\left(\frac{11}{9}x^2+\frac{4}{9}\right)-W_0\left(\frac{7}{4}x^3+\frac{1}{8}x\right)+\frac{689}{900}x^4-\frac{1021}{1800}x^2-\frac{8}{75}\right](x^2-1)^{\frac{1}{2}}\right\}.$$

First forbidden beta-transition; scalar interaction:

$$S(k)=\frac{\alpha G^2|M|^2}{12\pi^4}\frac{1}{k}\left\{\left[W_0^4\left(\frac{2}{3}x^3+x\right)+W_0^3\left(-\frac{7}{3}x^4-3x^2+\frac{1}{8}\right)+W_0^2\left(\frac{18}{5}x^5+5x^3\right)+W_0\left(-\frac{13}{5}x^6-\frac{14}{3}x^4-\frac{15}{8}x^2-\frac{5}{24}\right)+\left(\frac{76}{105}x^7+\frac{5}{3}x^5+\frac{11}{6}x^3+\frac{3}{8}x\right)\right]\ln[x+(x^2-1)^{\frac{1}{2}}]-\left[W_0^4\left(\frac{11}{9}x^2+\frac{4}{9}\right)+W_0^3\left(-\frac{151}{36}x^3-\frac{73}{72}x\right)+W_0^2\left(\frac{163}{25}x^4+\frac{152}{75}x^2+\frac{4}{75}\right)+W_0\left(-\frac{4303}{900}x^5-\frac{5783}{1800}x^3-\frac{2441}{1800}x\right)+\left(\frac{14{,}741}{11{,}025}x^6+\frac{8133}{4900}x^4+\frac{135{,}853}{88{,}200}x^2+\frac{136}{2205}\right)\right](x^2-1)^{\frac{1}{2}}\right\}.$$

First forbidden beta-transition; tensor interaction; special case of selection rule ($\Delta J=2$; yes):

$$S(k)=\frac{\alpha G^2|M|^2}{12\pi^4}\frac{1}{k}\left\{\left[W_0^4\left(\frac{2}{3}x^3+x\right)+W_0^3\left(-2x^4-2x^2+\frac{1}{4}\right)+W_0^2\left(\frac{43}{15}x^5+\frac{8}{3}x^3-\frac{3}{8}x\right)+W_0\left(-\frac{31}{15}x^6-\frac{7}{3}x^4-\frac{3}{8}x^2-\frac{1}{12}\right)+\left(\frac{64}{105}x^7+\frac{8}{15}x^5+\frac{1}{12}x\right)\right]\ln(x+(x^2-1)^{\frac{1}{2}})\right.$$

(formula continued on next page)

* Work supported by the U. S. Atomic Energy Commission.
[1] L. Madansky and F. Rasetti, Phys. Rev. **83**, 187 (1951), also includes references to earlier work.
[2] T. B. Novey, Phys. Rev. **83**, 145 (1951).
[3] T. B. Novey, Phys. Rev. **86**, 619 (1952).
[4] D. Maeder and P. Preiswerk, Phys. Rev. **84**, 595 (1951).
[5] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).
[6] F. Bloch, Phys. Rev. **50**, 272 (1936).
[7] C. S. Wang Chang and D. L. Falkoff, Phys. Rev. **76**, 365 (1949).
[8] Madansky, Lipps, Bolgiano, and Berlin, Phys. Rev. **84**, 596 (1951).

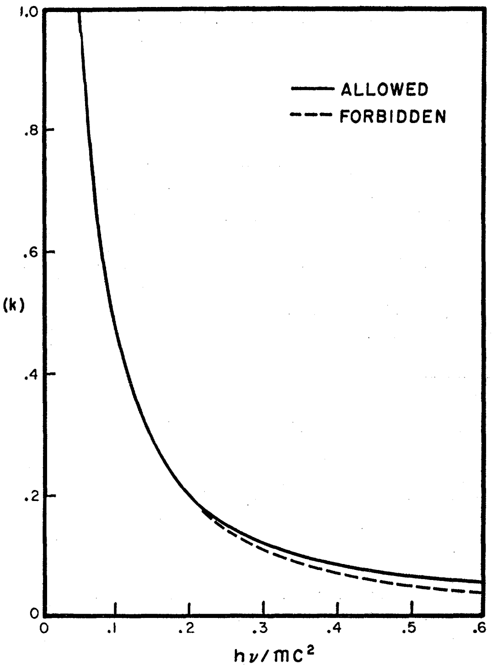

FIG. 1. Energy distribution of gamma-rays following allowed and first forbidden beta-transitions. $W_0=4$ in relativistic units. Ordinate $S(k)$ normalized to unity for $k=0.05$. Abscissa k measured in relativistic units. The dotted curve represents both the scalar theory and the tensor theory. The solid curve represents the allowed theory.

(*formula continued from preceding page*)

$$-\left[W_0^4\left(\frac{11}{9}x^2+\frac{4}{9}\right)+W_0^3\left(-\frac{7}{2}x^3-\frac{1}{4}x\right)+W_0^2\left(\frac{4481}{900}x^4+\frac{97}{600}x^2+\frac{4}{225}\right)+W_0\left(-\frac{363}{100}x^5-\frac{1337}{1800}x^3-\frac{437}{900}x\right)+\left(\frac{1306}{1225}x^6-\frac{769}{22,050}x^4+\frac{7733}{44,100}x^2+\frac{24}{1225}\right)\right](x^2-1)^{\frac{1}{2}}\Bigg\}.$$

A plot of the gamma-ray spectrum is shown in Fig. 1. In the energy range of 20–200 kev, one finds that the shapes of the allowed and forbidden cases are almost identical. The ratio of the total intensity of gamma-rays (in units of mc^2) to the total number of beta-rays for an allowed beta-transition can also be computed theoretically and is tabulated in Sec. 4.

Phosphorus 32, yttrium 91, and radium E are all pure beta-emitters and, hence, were chosen for the experiment. P^{32} exhibits an allowed shape for its beta-spectrum.[9] Yttrium 91 has the unique α-2 type spectrum[9] where the form of the beta-interaction is tensor or axial vector with the selection rule ($\Delta J=2$; yes).

The radium E beta-spectrum also differs from the allowed shape. A mixture of interactions[10] gives a reasonable fit to the shape of its beta-spectrum. However, no calculation using this particular mixture of interactions (tensor and pseudoscalar) has been made for the internal bremsstrahlung. No calculations have been made employing the correct Z dependence for these three cases. Since the allowed and forbidden cases yield internal bremsstrahlung spectra which differ by only a small fraction of our experimental error in the energy range investigated, the experimental data have been compared with the allowed theory in each case. The results are described in Sec. 3.

The angular correlation of the beta-ray and its bremsstrahlung also has been derived for the case of an allowed beta-transition,[5,6] and for the unique α-2 first forbidden transition.[8] The correlation function depends on the range of beta-ray and gamma-ray energies used in the experiment. For a given range of energies the calculations show that for the special first forbidden beta-transition[8] discussed, the angular correlation functions differ from the allowed ones. Recent work by Horowitz,[11] however, indicates that if one considers an alternative mechanism for the bremsstrahlung, namely,

$$N_0 \nearrow N_0^*+\gamma \searrow N_1+\nu+e+\gamma, \qquad N_0 \searrow N_1^*+e+\nu \nearrow N_1+\nu+e+\gamma,$$

in contrast to the usual one,

$$N_0\rightarrow N_1+\nu+e'\rightarrow N_1+\nu+e+\gamma,$$

then for a forbidden transition the angular correlation function becomes practically identical with the allowed one. This nuclear effect could certainly be present.

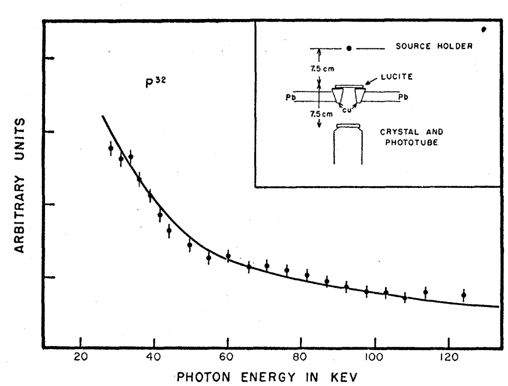

FIG. 2. Corrected theoretical spectrum for P^{32} and experimental data. Experimental arrangement is shown in inset.

[9] C. S. Wu, Revs. Modern Phys. **22**, 386 (1950).

[10] A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952). C. S. Wu, Proc. Intern. Conf. Nuclear Phys. and Phys. Fundamental Particles, p. 160 (Sept. 1951).

[11] J. Horowitz, J. phys. et radium **13**, 429 (1952); and private communication.

However, this result is also based on calculations which do not include the Z dependence.

The experiments described below were designed to verify the theoretical predictions of the shapes of the internal bremsstrahlung continua. Some experiments were also conducted to obtain the qualitative behavior of the internal bremsstrahlung angular correlation.

II. EXPERIMENTAL PROCEDURE FOR MEASURING ENERGY SPECTRA

The experimental arrangement used with the beta-sources is shown in Fig. 2. The source was mounted about 15 cm above the NaI crystal. In order to stop the beta-particles and to prevent their being scattered into the crystal, a lead table top (1×20×20 inches) covered with ½-inch plywood was placed with its top surface half-way between the source and crystal. The center of the table contained a copper plug with a hole about ½ inch in diameter to pass the radiation to be examined and covered with a $\frac{3}{16}$-inch Lucite plate to stop the beta-particles in such a manner as to avoid admixture of the ordinary bremsstrahlung produced in stopping the beta-rays.[1]

Sources of the order of 0.1 millicurie were used. The gamma-radiation was measured by means of NaI(Tl) crystals (about 2×2×1 cm), a 5819 phototube, a linear amplifier and a one-channel discriminator. The P^{32} and Y^{91} sources were evaporated on small circular disks of 3-mg/cm^2 tissue paper about 10 mm in diameter. The RaE source was deposited electrochemically on one side of an 0.2-mil nickel foil strip of similar area.

By backing the source with mica and Lucite, it was determined that the external bremsstrahlung originating in the source holder was at most 1 percent of the total count observed. The relative contribution of external bremsstrahlung originating in the Lucite plate "beta-stopper" was ascertained to be of the order of 3 percent by varying the solid angle which it subtended at the scintillation crystal. Small amounts of lead in the vicinity of the crystal were sufficient to enhance the observed spectrum by ten percent or more by reradiation of the lead K x-ray. Accordingly, the only Pb shielding used was the thick lead table provided with a copper-lined aperture. Source and crystal were enclosed together in the same light-tight covering, since as little as two strips of black scotch electrical tape between source and crystal absorbed sufficient radiation to alter the spectrum by about ten percent. The crystal was calibrated in the region from 20 to 60 kev with monochromatic x-rays from a Bragg-focusing x-ray spectrometer, and at higher energies with the line spectra of Cd^{109} (22.8-kev x-ray and 89-kev gamma-ray) and Os^{185} (62.8-kev x-ray). The pulse-height spectra obtained were Gaussian within 2 or 3 percent, and at the higher energies showed a smaller peak of about 30 kev less energy, owing to escape of the iodine K x-ray, in quantitative agreement with the data of West, Meyerhof, and Hofstadter,[12] and with a theoretical computation for the particular geometry used. The line-width dependence on energy is shown in Fig. 3.

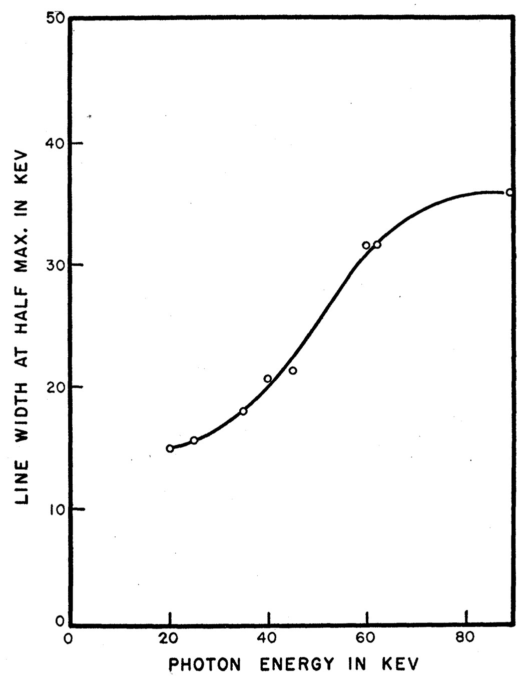

FIG. 3. Plot of the width at half-maximum of the pulse-height distribution for monochromatic gamma-rays.

III. CORRECTIONS

For comparison with experiment the theoretical spectra were corrected for absorption in the Lucite beta-stopper and aluminum foil, crystal efficiency, K x-ray escape, and instrumental resolving power. Measured x-ray absorption coefficients given by Compton and Allison[13] were used to correct for Lucite absorption and crystal efficiency. Absorption in the Lucite and the strong photoelectric absorption of iodine combined to make these particular corrections small over most of the energy ranges examined. It has been pointed out by West, Meyerhof, and Hofstadter[12] that near the K-absorption limit of iodine the K x-rays of iodine emitted subsequent to a photoelectric gamma-ray absorption will frequently escape from the crystal instead of being absorbed. The result is that a fraction of the gamma-rays will give rise to pulses of lower height. A quantitative prediction of this effect which considers the fluorescence yield of iodine as measured by Martin[14] was used to correct the spectrum for this effect. Figure 4 shows the theoretical spectrum of RaE both with and without the above corrections. The third curve shown has been corrected for instrumental resolving power by

[12] West, Meyerhof, and Hofstadter, Phys. Rev. **81**, 141 (1951).
[13] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935).
[14] L. H. Martin, Proc. Roy. Soc. (London) **A115**, 420 (1927).

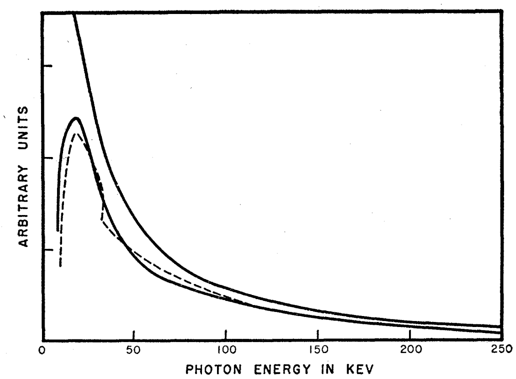

FIG. 4. Effect of corrections on the shape of the spectrum. The upper solid curve is the pure theoretical spectrum. The dotted curve includes all corrections except the folding due to the spread in pulse height for a monochromatic gamma-ray. The lower solid curve shows the final fully corrected spectrum.

folding it with a Gaussian curve whose empirical half-width varies with energy as shown in Fig. 3.

IV. RESULTS

The spectral shape is found to agree with the theory within the experimental error from 30 to 150 kev. Figures 2, 5, and 6 show corrected theoretical curves compared with the experimental data above 30 kev. No experimental points are shown below this limit, since the rapid rise of the theoretical curve combined with the fast increase of the absorption coefficient in Lucite, make it impossible to state with any assurance that the theory is verified within better than 20 percent in this region. Measurement of the total energy in the bremsstrahlung spectrum (above 90 kev) per electron emitted has been repeated in the manner previously reported.[1] The ratios I_γ/N_β of the gamma-ray energy (in units of mc^2) in the entire spectrum per beta-particle (assuming that the ratio of gamma-energy above 90 kev to that below 90 kev is the one theoretically predicted for an allowed transition) are given in Table I for the three sources investigated.

The theoretical ratio was computed using the formula for I_γ given by Chang and Falkoff (reference 7, p. 367), which was obtained by exact integration of their exact formula (5). Novey[3] finds a somewhat higher gamma-intensity from RaE, but it is believed this discrepancy might be explained by our uncertainties involved in evaluating the effects of scattering of the low energy beta-particles from RaE. Levinger[15] has computed theoretically that a Po x-ray of strength $0.64/Z^2$ per beta-particle should be present in addition to the RaE bremsstrahlung spectrum. The expected effect of such an x-ray is shown in Fig. 6 and is seen to be within our experimental error. Novey has reported the existence of such an x-ray.[4]

The bremsstrahlung accompanying K capture has also been observed for Fe^{55}. We find an end-point energy of 200±10 kev, in good agreement with the end-point energy of 205 kev reported by Maeder and Preiswerk.[4] The correction procedure described above was applied to the theoretical spectrum to obtain this figure.

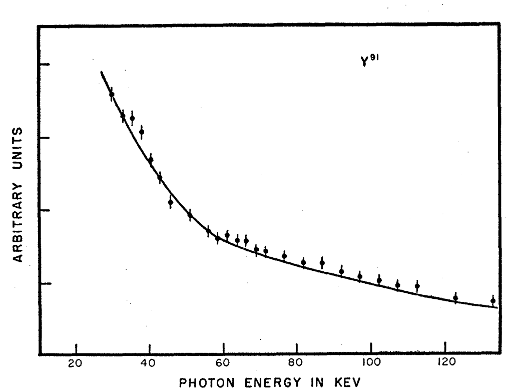

FIG. 5. Corrected theoretical spectrum for Y^{91} and experimental data.

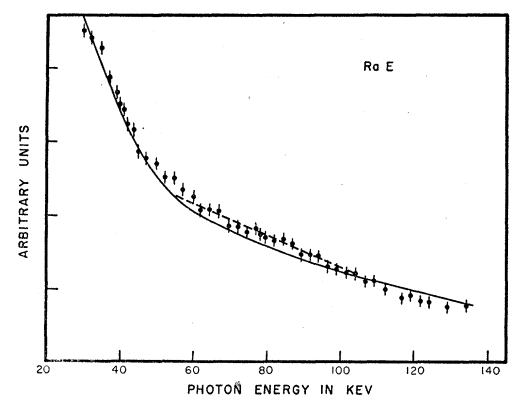

FIG. 6. Corrected theoretical spectrum for radium E and experimental data. The dotted portion shows the additional effect of Po x-rays from the photoeffect due to the beta-decay as calculated by Levinger.

V. ANGULAR CORRELATION EXPERIMENTS

An experimental determination of the angular correlation between the beta-particles and the bremsstrahlung from P^{32} and Y^{91} was also attempted. The source was mounted on the axis of a cylindrical evacuated Lucite chamber. The β-particles were counted with a 5819 phototube and a cylindrical stilbene crystal 1 inch in diameter and $1\frac{13}{16}$ inches long with a $\frac{1}{2}$-inch diameter hole running $\frac{3}{4}$ inch into the crystal to admit the particles and avoid their being scattered out of the crystal. This crystal was mounted within the vacuum system and separated from the phototube by a vacuum tight Lucite covering about $\frac{1}{16}$ in. thick. The photons were counted with a solid stilbene crystal $1\frac{5}{8}$ in. in diameter and $1\frac{1}{4}$ in. in length which together with a 5819 phototube could be rotated about the evacuated chamber. This crystal was placed with its front face

[15] J. S. Levinger, thesis, Cornell University, unpublished (1949) and private communication.

an inch and a half further from the source than the beta-counter and shielded with $\frac{1}{2}$-inch lead to avoid scattering between the crystals.

Coincidences were detected electronically with a coincidence circuit of 2×10^{-8} second resolving time. The gamma-counter pulses were inverted and delayed electronically by approximately 100 mμ-seconds with respect to those from the beta-counter, and both displayed on a synchroscope with a 50 mμ-seconds per cm sweep triggered by the coincident circuit. The resulting traces were photographed and analyzed with respect to pulse heights and separations.

A rough calibration of the beta-crystal was made by finding the pulse heights corresponding to the upper end points of known beta-spectra. Known gamma-sources were used to calibrate the gamma-counter. An experiment was performed in which all beta-pulses corresponding to electrons of energies greater than 250 kev, and all gamma-pulses corresponding to photons of energies greater than 100 kev were recorded. The results of this experiment were compared with the distribution derived from quantum mechanical allowed theory, suitably integrated and corrected for this experimental situation, and are as shown in Fig. 7. Also shown is the distribution derived from the classical theory[7] as integrated for a similar situation by Novey.[2] The experiment appears to favor the correct quantum mechanical theory. The experimental point at 40° might be explained by scattering of photons greater than 300 kev from the beta-crystal into the gamma-crystal and from external bremsstrahlung originating in the beta-crystal from the impinging beta-particle. Since it was not possible to obtain a pulse-height resolution better than 30 percent for given beta-energies, no experimental curves are shown for the correlation associated with beta-rays of selected pulse heights. It was true, however, that Y^{91} and P^{32}, which have similar beta end-point energies, gave correlations which did not differ from each other within an experimental error of 10 percent at each position when the correlation was plotted for beta-counter pulses of only a 20 percent range of pulse amplitudes corresponding on the average to kinetic energies in the region of the order of 500 kev. Novey[3] has recently reported good agreement with the theoretical result for allowed transitions.

TABLE I. Gamma-ray energy per beta-particle, I_γ/N_β.

Source	Maximum beta-energy Mev	I_γ/N_β in units of mc^2 Theory (allowed transition)	Experimental
P^{32}	1.72	2.38×10^{-3}	2.32×10^{-3}
Y^{91}	1.5	1.97×10^{-3}	1.90×10^{-3}
RaE	1.17	1.11×10^{-3}	0.84×10^{-3}

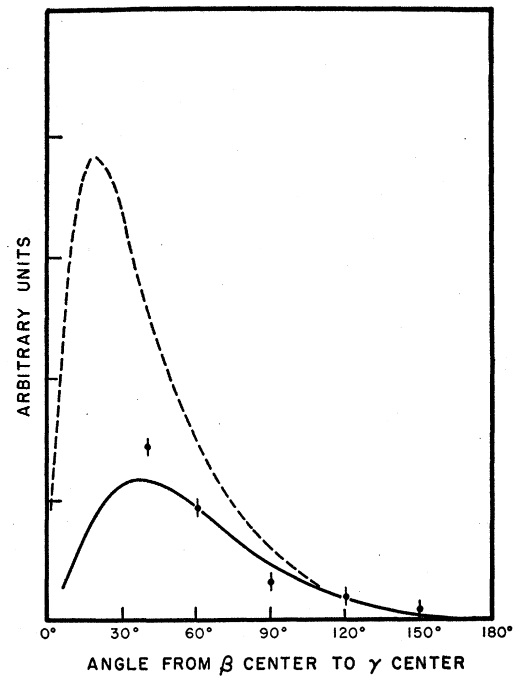

FIG. 7. Theoretical angular correlation curves for an allowed transition. The solid curve is calculated for the condition that one accepts all β-rays above 250 kev and all γ-rays above 100 kev. The dotted curve represents the classical correlation function derived for the same conditions. Both curves are corrected for solid angle of the counter.

VI. CONCLUSIONS

The experiments show excellent agreement with the predicted internal bremsstrahlung spectra. The calculations of Levinger on the relative amount of K-electron ejection appear to fall within our experimental error. The interesting question of the existence of the "Horowitz effect," however, cannot be answered by our experiments. It would, therefore, be of extreme interest to perform the correlation experiment with a beta-ray spectrograph providing good energy resolution.

We wish to thank Professor J. A. Bearden and Mr. C. Roos for their generous help with the x-ray spectrometer.