

The left side refers to the three orthogonal susceptibilities of the solid polycrystalline sample, the right to the single crystal susceptibility.

An apparatus of the Faraday type which satisfies the condition of uniformity for  $H$  and  $\text{grad}|H|$  over a suitable volume has been designed<sup>1</sup> and applied to the measurement of polycrystalline graphite. A Honda-Owen correction has been employed to determine the effect of ferromagnetic impurities, using field strengths from 8000 to 16 000 oersteds. The total susceptibility  $\Sigma\chi_m$  has been measured for various grades running from various petroleum base Atcheson graphites to natural and lampblack graphites. Although the accuracy of the measurements is of the order of 1 percent, the precision and relative values are correct to better than 0.5 percent. Small cubical samples  $\frac{1}{8}$  inch on edge were employed. For a given type of graphite the invariance of the total susceptibility was verified to 0.5 percent, and all the Atcheson graphites examined fell in the range  $-20.5 \times 10^{-6}$  to  $-21.2 \times 10^{-6}$  cgs. For comparison, Guha and Roy<sup>2</sup> find for the spur of a single crystal the value  $-23.5 \times 10^{-6}$  cgs. The "anisotropy ratio" for the manufactured graphites ranged from 1:1 to 9:1, whereas that reported for the single crystal was of the order of 60/1. One may interpret these results as suggesting that not more than 10 percent of the material in the artificial graphites is of a nongraphitic nature.

One should expect a single determination of the susceptibility of a powdered, and presumably randomly oriented, artificial graphite to be one-third of  $-21 \times 10^{-6}$ , or about  $-7 \times 10^{-6}$ . Values reported in the literature range from  $-3 \times 10^{-6}$  to  $-5 \times 10^{-6}$ . These low values are probably due to orientation of the powder either in packing or in the magnetic field, or possibly to a ferromagnetic impurity. The present results demonstrate that manufactured graphite yields the same susceptibility as natural graphite, up to a minor correction for intercrystallite carbon, or crystallites too minute to develop a crystalline field.

\* This report is based on studies conducted for the U. S. Atomic Energy Commission.

<sup>1</sup> J. J. Donoghue (to be published).

<sup>2</sup> B. C. Guha and B. P. Roy, Indian J. Phys. 17, 348 (1934).

## The Statistics of a Two-Dimensional, Hexagonal Net

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IN a recent paper,<sup>1</sup> Gurney discusses the specific heat of graphite and shows that for low temperatures, the specific heat goes as  $T^2$  and is consistent with the model he proposes to describe the distribution of normal modes. In this model, the only contributing modes at these low temperatures are those that propagated in the plane of the hexagonal network of the carbon atoms and whose displacements are perpendicular to these planes. It is the two-dimensional nature of the problem that leads to the  $T^2$  law, and the analysis of the problem is completed using a Debye frequency distribution and a Debye temperature.

However, for this problem, a closed analytic expression can be found for the frequency distribution by methods previously described<sup>2</sup> for the case of nearest neighbor interactions. The coupling constant between nearest neighbors can be evaluated from the comparison of the value of the distribution function at zero frequency and the low temperature behavior of the specific heat.

The distribution function is written as  $g(\bar{\omega}^2)d(\bar{\omega}^2)$ , where  $\bar{\omega}^2$  is the dimensionless frequency whose range is  $0 \leq \bar{\omega}^2 \leq 1$  and is related to the maximum frequency  $\omega = \omega_{\max}\bar{\omega}$ . In this expression  $\omega_{\max} = (6\alpha)^{\frac{1}{2}}$ , where  $\alpha$  is the ratio of the force constant and the atomic mass and is described by writing the contribution to the acceleration of the  $n$ th atoms as  $-\alpha(u_n - u_N)$ , where  $u_n$  is the atomic coordinate and  $u_N$  is the coordinate of a nearest neighbor.

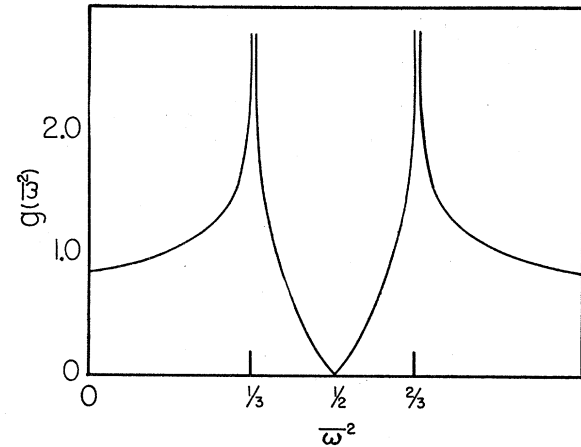


FIG. 1. The distribution function  $g(\bar{\omega}^2)$  vs  $\bar{\omega}^2$  for the two-dimensional hexagonal net in nearest neighbor interaction in normal vibration to the plane of the net.

With this notation, the expression for  $g(\bar{\omega}^2)$  is

$$g(\bar{\omega}^2) = \frac{9}{\pi^2} \left[ \frac{1-2\bar{\omega}^2}{3} \right]^{\frac{1}{2}} K \left[ \frac{\bar{\omega}^2(2-3\bar{\omega}^2)^{\frac{1}{2}}}{(1-2\bar{\omega}^2)} \right]^{\frac{1}{2}}, \quad 0 \leq \bar{\omega}^2 \leq \frac{1}{3};$$

$$g(\bar{\omega}^2) = \frac{9}{\pi^2} \frac{1-2\bar{\omega}^2}{[3\bar{\omega}^2(2-3\bar{\omega}^2)^{\frac{1}{2}}]^{\frac{1}{2}}} K \left[ \frac{1-2\bar{\omega}^2}{\bar{\omega}^2(2-3\bar{\omega}^2)^{\frac{1}{2}}} \right]^{\frac{1}{2}}, \quad \frac{1}{3} \leq \bar{\omega}^2 \leq \frac{2}{3}; \quad (1)$$

where

$$K(p) = \int_0^1 \frac{dx}{[(1-x^2)(1-p^2x^2)]^{\frac{1}{2}}}$$

and the function is symmetric about  $\bar{\omega}^2 = \frac{1}{2}$ .

Figure 1 is a plot of  $g(\bar{\omega}^2)$ . There are two symmetric infinite peaks whose asymptotic behavior is described by

$$g(\bar{\omega}^2) \simeq \frac{9}{2\pi^2} \log \frac{2}{1-3\bar{\omega}^2}, \quad (2)$$

$$\bar{\omega}^2 \rightarrow \frac{1}{3}.$$

The specific heat at low temperatures is determined by  $g(0)$ , whose value is

$$g(0) = 9/2\pi\sqrt{3}. \quad (3)$$

The specific heat per mole at low temperatures can be found from the following expression for a two-dimensional lattice:

$$C_V = 3.606 N k g(0) T^2 / \theta^2, \quad (4)$$

where  $g(0) \neq 0$  and is finite and  $\theta = \hbar\omega_{\max}/2k$ . Using the data<sup>3</sup> from reference 1, we find  $\alpha = 13.34 \times 10^{26} \text{ sec}^{-2}$ , and  $\omega_{\max} = 8.95 \times 10^{13} \text{ sec}^{-1}$ .

The details of the calculation will be published elsewhere.

<sup>1</sup> Ronald W. Gurney, Phys. Rev. 88, 465 (1952).

<sup>2</sup> W. A. Nierenberg, J. Chem. Phys. 19, 659 (1951).

<sup>3</sup> It can be remarked that the first entry in reference 1, Table I, for  $\theta^2$  should be 373 000, improving the  $T^2$  fit and changing the mean  $\theta^2$  to 375 200.

## Normal Fluid Concentration in Liquid Helium II below 1°K\*

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MEASUREMENTS of second sound velocity  $u_2$  recently obtained by the authors<sup>1</sup> permit one to calculate the normal fluid concentration  $\rho_n/\rho$  at temperatures below 1°K. This has been done, employing the well-known relationship

$$\rho_n/\rho_s = \frac{S^2 T}{C} \frac{1}{u_2^2} \quad (1)$$