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Pion-Nucleon Scattering When the Coupling Is Weak and Extended*

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The symmetrical pseudoscalar meson theory, with an extended pseudovecto coupling to the nucleon and neglect of nucleon recoil effects, is shown to be capable of explaining the existing data on the P-wave pion-nucleon scattering. The required sizes of the source radius and coupling constant correspond to a relatively weak pion-nucleon coupling, but reactive effects in the scattering are nevertheless so important that the lowest order approximation in a perturbation treatment is completely inadequate. The Dancoff approximation, on the other hand, is accurate to within 10 or 20 percent.

THE point-coupling *perturbation* theory prediction for the symmetrical pseudoscalar pion-nucleon P-wave scattering phase shifts can be stated as follows:

$$\alpha_{33}=2y,\quad \alpha_{13}=\alpha_{31}=-y,\quad \alpha_{11}=-4y, \tag{1}$$

where

$$y=\tfrac{2}{3}(f^2/\mu^2)(k_0^3/\omega_0), \tag{2}$$

if k_0, ω_0, and μ are the momentum, total energy, and mass, respectively, of the pion, and f is the pion-nucleon coupling constant. The first phase shift subscript refers to $2T$ and the second to $2J$, if T and J are the total isotopic and ordinary angular momenta, respectively. Nucleon recoil has been neglected so that S and D scattering is ignored in this discussion. The purpose of this note is to point out that the formulas (1), (2), based on a second-order perturbation calculation, do not necessarily represent a correct evaluation of the theory even when the coupling is weak, so that their failure to agree with experiment is no indication that a weak coupling approach is invalid. Indeed, a straightforward correction of the most obvious defects of (1), (2) leads to theoretical predictions in good agreement with experiment.

In the first place, the above formulas correspond to a point interaction between pion and nucleon. With neglect of nucleon recoil, such a theory does not exist in any sense. It is absolutely necessary to spread out the region of interaction, or in other words to introduce a cutoff in momentum space, in order that the theory be meaningful. The extended source or cutoff in familiar in strong coupling theory,[1] but it is just as necessary in weak coupling if the nucleon is to be held fixed. An immediate result of introducing a cutoff is to replace the function $y(k)$ by

$$x(k)=v(k)y(k), \tag{2'}$$

where $v(k)$ is the cut-off factor, i.e., the Fourier transform of the source function. ($v(k)$ is usually taken to be a monotonically decreasing function of k, normalized to unity at $k=0$.) A much more important consequence of the cutoff, however, is that it allows an evaluation of higher order effects in the scattering. Such an evaluation[2] shows that even under conditions of coupling normally characterized as "weak," that is, $f^2/\mu a\ll 1$, where a is the source "radius," it is possible for reactive effects in the scattering to be extremely important. This situation comes about because of higher order intermediate states containing only a single pion. When the single intermediate pion has an energy close to that of the incident one, the associated energy denominator becomes very small and the size of the higher order term anomalously large. The contribution of terms in which the intermediate energy is *exactly* equal to the incident energy is sometimes called the Heitler damping.[3] These terms can be taken into account by replacing each of the phase shifts in formulas (1) by its tangent. As is well known, however, the Heitler damping terms are out of phase with the first-order terms, so that the "in phase" contribution of terms near but not on the energy shell can be equally if not more important.

To calculate the reaction due to higher order one-pion states, the Dancoff approximation[4] is appropriate. In its application to pion-nucleon scattering, one neglects configurations containing three or more pions but treats the zero, one, and two pion problem "exactly." Such a procedure leads to an integral equation, which is to be discussed in a forthcoming paper by Lee and Christian.[5] In the case of interest here, i.e., when $f^2/\mu a\ll 1$, this equation is closely equivalent to an extremely simple problem: the scattering of a pion by a velocity and spin-dependent potential, the matrix elements of which are just the usual second order scattering matrix elements calculated without damping. It will be recognized that this result is analogous to the widely used procedure of using as the nucleon-nucleon potential an operator whose matrix elements are given by a weak coupling meson theory calculation without reaction.

The results of the calculation of the "potential" scattering of pions are as follows[6]:

* This research was supported by the Office of Naval Research and Atomic Energy Commission.

[1] See, for example, W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946), p. 12.

[2] J. S. Blair and G. F. Chew (to be published).

[3] W. Heitler, Proc. Cambridge Phil. Soc. **37**, 291 (1941); also see M. L. Goldberger, Phys. Rev. **84**, 929 (1951) for a recent discussion of the Heitler damping.

[4] S. M. Dancoff, Phys. Rev. **78**, 382 (1950).

[5] T. D. Lee and R. Christian (to be published).

[6] The formulas (1') are based on a variational principle which will be discussed in a forthcoming paper by Chew and Low. In the

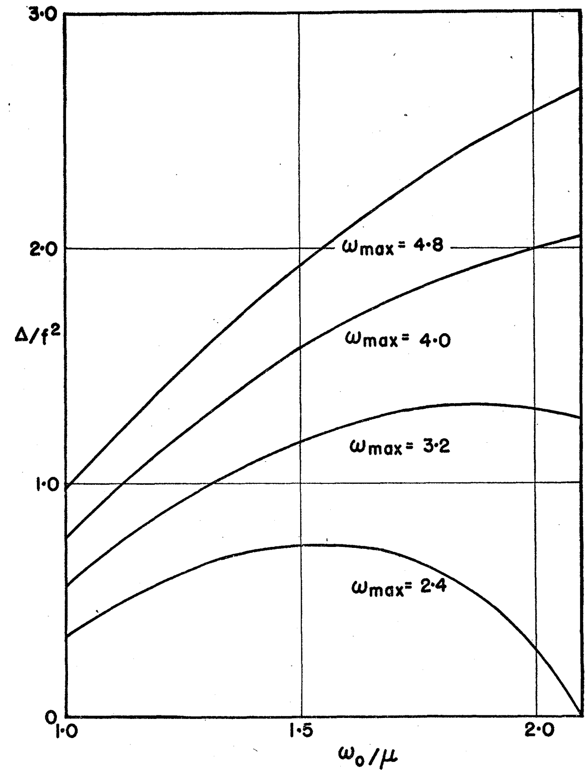

FIG. 1. The reaction function, Δ/f^2, as a function of ω_0 for a "square" cut-off factor. The cutoff occurs at a pion energy denoted by $\omega_{\max}$.

$$\tan\alpha_{33}=2x/(1-2\Delta),$$

$$\left.\begin{matrix}\tan\alpha_{31}\\ \tan\alpha_{13}\end{matrix}\right\}=-x/(1+\Delta), \qquad (1')$$

$$\tan\alpha_{11}=-4x/(1+4\Delta),$$

where

$$\Delta=\frac{2}{3\pi}\frac{f^2}{\mu^2}\int_\mu^\infty d\omega\frac{k^3}{\omega^2}\frac{\omega_0}{\omega-\omega_0}v^2(k); \qquad (3)$$

the principal value of the integral is to be taken. The coupling constant, f, is to be assigned its renormalized value, in the conventional sense of perturbation theory.

For fairly low values of ω_0, the quantity Δ is positive, so that if Δ can be as large as 0.2 the violent disagreement with experiment of the nondamped results can be removed, that is, α_{33} can become substantially larger than α_{11} in absolute value. If Δ were as large as 0.5, there would be a "resonance" in the $\frac{3}{2}$, $\frac{3}{2}$ state as was proposed by Brueckner[7] on the basis of strong coupling results. The possibility of a resonance in this one state can be traced to the sign of the nondamped phase shifts (1). The only positive phase shift is α_{33}, showing that only in the $\frac{3}{2}$, $\frac{3}{2}$ state is the "potential" effectively attractive.

In Fig. 1, the quantity Δ/f^2 is plotted as a function of ω_0 for a "square" cut-off factor, i.e., $v(k)=1$ for $k<k_{\max}$, $v(k)=0$ for $k>k_{\max}$. Four different values of $\omega_{\max}=(k_{\max}^2+\mu^2)^{\frac{1}{2}}$ are shown. Definite values of f^2 and $\omega_{\max}$ were fixed by the experimentally measured total cross sections for the scattering of positive and negative pions by protons.[8–10] Choosing $f^2=0.2$ and $\omega_{\max}=3.2\mu$ gives the fit shown in Fig. 2, which seems satisfactory.[11] Since S and D scattering is ignored in the preliminary calculations, a detailed fit to the angular distribution (which depends much more on the S and D phase shifts than does the total cross section) has not yet been attempted. However, it has been verified that the main features, except for the asymmetry about 90°, are correctly given. An attempt is now being made to include the S and D scattering in a simple way.

It remains to be checked that the required values of f^2 and $k_{\max}$ satisfy the requirement that 3-pion and higher configurations are actually unimportant. The direct perturbation calculation[2] shows that fourth-order contributions of this kind tend to increase α_{33} and decrease α_{11} still further, but the effect is only 10 percent in the former case and 20 percent in the latter (at 135 Mev). α_{13} and α_{31} are both increased by 6 percent.

The fact that $\omega_{\max}$ is considerably less than the nucleon rest energy means that our nonrelativistic treatment of the nucleon has been justified *a posteriori.* A lack of important nucleon recoil effects has also been outstanding in the observed anomalous electromagnetic properties of single nucleons, i.e., the anomalous magnetic moments and the photopion production cross

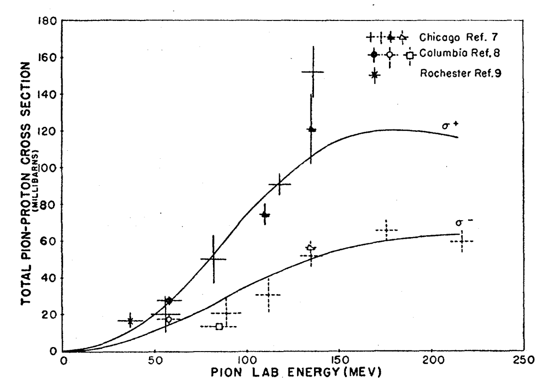

FIG. 2. Comparison of theory with the experimental pion-proton total cross sections. The solid crosses are the positive pion and the dashed crosses the negative pion experimental values.

present application the accuracy of these formulas is believed to be better than 10 percent.

[7] K. Brueckner, Phys. Rev. 86, 106 (1952).

[8] Anderson, Fermi, Long, Martin, and Nagle, Phys. Rev. 85, 934 and 936 (1952); Anderson, Fermi, Nagle, and Yodh, Phys. Rev. 86, 793 (1952).

[9] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. 82, 958 (1951); Isaacs, Sachs, and Steinberger, Phys. Rev. 85, 803 (1952).

[10] Barnes, Clark, Perry, and Angell, Phys. Rev. 87, 669 (1952).

[11] In our computations, we have taken ω_0 equal to the pion energy in the center-of-mass system.

sections.[12] It is perhaps worth mentioning at this point, therefore, that the absolute value of the *mesonic* contribution to the nucleon magnetic moments, given by the above values for f^2 and $k_{\max}$ is 1.8 nuclear magnetons.[13] An analysis of the photopion production problem, as well as the neutron-electron interaction, is proceeding on the same basis.

In conclusion, it should be emphasized that even if the success of the calculations reported here is not accidental, the particular form of the Yukawa theory (e.g., pseudoscalar or pseudovector coupling) has by no means been established. *Any theory with a coupling linear in the pion field, which has a nonrelativistic limit for the nucleon*, will lead to the results described here.

The author is indebted to R. Serber and T. D. Lee for the idea of using the Dancoff approximation. Discussion with J. S. Blair, K. Brueckner, and N. Kroll also contributed significantly to the progress of the problem. Complete details of the calculations reported here, together with applications of the same general approach to other problems, will be published at a later time.

[12] J. S. Blair and G. F. Chew, Ann. Rev. Nuc. Sci. 2 (1952).
[13] M. H. Friedman (to be published).

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Heat Conduction of the Boundary Layer in Liquid Helium II*

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In experiments on the thermal conductivity of liquid helium II, an anomalous heat conduction has been found in the vicinity of the heat source. The magnitude of this effect is given as a function of temperature and is shown to be independent of the heat flux. The effect has been ascribed to a thin layer of liquid helium in the vicinity of the energy source having a poor heat conduction. The existence of this layer is probably a consequence of the finite rate of conversion from superfluid to normal particles.

AN investigation carried out in this laboratory[1] on the thermal conductivity of liquid helium II and He^3-He^4 mixtures yielded, in the former case, anomalously low values of heat conduction without the characteristic maximum between 1.2°K and the λ-point. The apparatus used in this connection (solid lines in Fig. 1) consisted of a thin-walled Monel capillary, 0.01-cm inner diameter and 17.5 cm long, with the lower end embedded 0.635 cm in a copper block on which were wound a heater and phosphor bronze thermometer. After exit through the vacuum jacket, the capillary passed through another copper block in contact with a liquid helium bath. The observed temperature differences between these two blocks were used to calculate the values of thermal conductivity. In order to determine unambiguously that the low results arose from an end effect originating in the heat input wall, the measurements were repeated with the following changes:

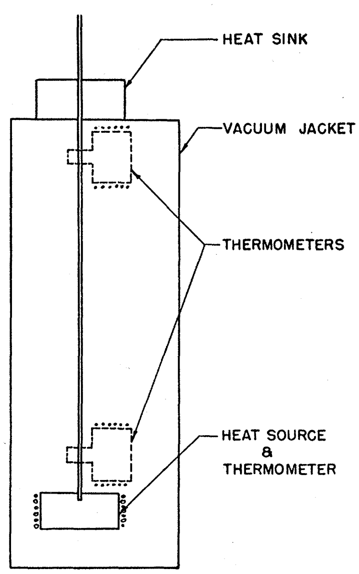

FIG. 1. Diagram of apparatus.

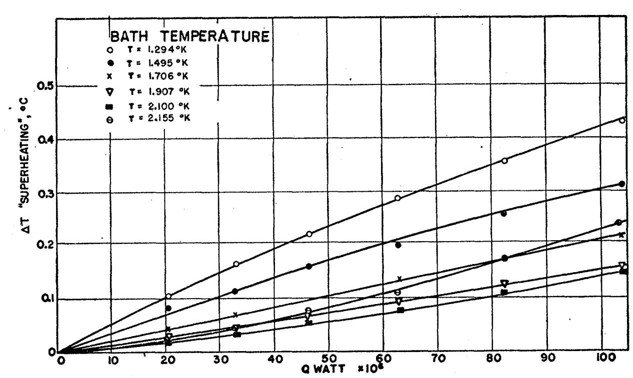

FIG. 2. "Superheating", ΔT, as a function of energy input Q and temperature T.

* This work was supported in part by the Office of Naval Research under contract with the Ohio State University Research Foundation.

[1] R. E. Probst, Massachusetts Institute of Technology Low Temperature Conference (1949), unpublished.