then these data would lead one to expect that the discontinuities in the specific heat and thermal expansion would be finite at the normal λ-point. This is the present indication from direct measurements.[16]

[16] See reference 2, pp. 206 and 245.

This work was made possible in part by a grant from the Harvard Foundation for Advanced Study and Research. I wish to thank Mr. G. Bjorklund and Mr. E. Wilkie for their assistance in constructing the apparatus used.

PHYSICAL REVIEW VOLUME 89, NUMBER 3 FEBRUARY 1, 1953

Z-Dependence of the Pair Production Cross Section at 1.33 and 2.62 Mev

IRVING E. DAYTON*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received October 21, 1952)

The Z-dependence of the pair production cross section has been measured at 1.33 Mev for Al, Cu, Sn, and Pb, and at 2.62 Mev for Be, C, Al, Cu, Sn, and Pb. A target is used which is thick for the positrons produced in it but thin for the incident gamma ray beam. The positrons stop in the target and are detected by observing their annihilation radiation with two NaI(Tl) scintillation counters in coincidence. At both energies the Z-dependence of the cross section is best represented by an equation of the form aZ^2+bZ^4. If it is assumed that the Born approximation calculations of the cross section give the correct value in the limit of low Z, then in lead at 2.62 Mev the measured cross section is 23 percent higher than the value calculated from the Born approximation, and 104 percent higher at 1.33 Mev. All these results are in good agreement with the exact numerical calculations of Jaeger and Hulme.

The present measurement shows that near threshold the pair production cross section in lead is considerably higher than the Born approximation calculation, whereas at energies between 17 Mev and 280 Mev the value has been measured to be 10 percent to 15 percent lower than that calculated from the Born approximation. The crossover appears to occur at around six Mev.

I. INTRODUCTION

IN the years since the discovery of the production of electron-positron pairs by gamma-rays, this process has frequently been studied using the 2.62 Mev gamma-ray from ThC″.[1] These experiments have served to establish the nature of the process and to give some information about the distribution of the particles in energy and angle. But since these measurements were based on at most a few hundred cloud-chamber pictures with one or perhaps two target materials, there is no accurate data on the total cross section for pair production, and until recently nothing on the Z-dependence of the total cross section at energies below 3 Mev.

The theory of pair production is summarized in Heitler.[2] It predicts that the total cross section for nuclear pair production will vary as Z^2. At the energies involved here, it is not necessary to include the effect of screening of the nuclear Coulomb field by the atomic electrons. The theory is calculated using the Born approximation, which is expected to fail for high Z and low gamma-ray energy. Jaeger and Hulme have made a number of exact numerical calculations of the pair production cross section.[3–5] They find that in lead the exact cross section is 25 percent higher than the Born approximation value at a gamma-ray energy of 5 mc^2 and about a factor of two higher at 3 mc^2. They also find[4] that at 3 mc^2 the Z-dependence of the cross section is best represented by an equation of the form aZ^2+bZ^4. The object of the present experiment is to check quantitatively these predicted deviations from the Born approximation theory, using gamma-rays of 2.62 Mev and 1.33 Mev.

While this experiment was in progress, indications of the predicted deviations from the Born approximation theory were obtained by Cleland, Townsend, and Hughes[6] using the 2.76-Mev gamma-ray from Mg^{24}. Hahn, Baldinger, and Huber[7] have recently reported an experiment similar to this one. Their results are in general agreement with ours and with the theoretical predictions.

General Considerations

At the gamma-ray energies used in this experiment, the pair spectrometer technique, which has been used at 17.6, 88, and 280 Mev[8–10] to study pair production,

* Now at Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

[1] For references to earlier work see: W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 201; K. H. Spring, *Photons and Electrons* (John Wiley and Sons, New York, 1950), Chap. V.

[2] See Heitler, reference 1, Chap. 4.

[3] J. C. Jaeger and H. R. Hulme, Proc. Roy. Soc. (London) **A153**, 443 (1936).

[4] J. C. Jaeger, Nature **137**, 781 (1936).

[5] J. C. Jaeger, Nature **148**, 86 (1941).

[6] Cleland, Townsend, and Hughes, Phys. Rev. **84**, 298 (1951).

[7] Hahn, Baldinger, and Huber, Helv. Phys. Acta **25**, 505 (1952).

[8] R. L. Walker, Phys. Rev. **76**, 1440 (1949).

[9] J. L. Lawson, Phys. Rev. **75**, 433 (1949).

[10] DeWire, Ashkin, and Beach, Phys. Rev. **83**, 505 (1951).

is not feasible because of the low energies of the electron and positron.

The basic idea of the experiment is to use a target which is thick for positrons produced in it but thin for the incident gamma-ray beam. The positrons stop in the target and are detected by observing in coincidence their two annihilation quanta. The experimental arrangement is shown in Fig. 1. The detectors were located in a plane through the center of the target and normal to the gamma-ray beam. One of the detectors was mounted so that it could be pivoted about the target in order that the detectors could be oriented at either 180° or 90° with respect to the target. The detectors were two scintillation counters using thallium activated NaI crystals mounted on RCA type 5819 photomultiplier tubes. After amplification, the pulse from each tube was passed through a differential discriminator whose high and low biases were set to pass only pulses corresponding to the photoline from annihilation radiation. This arrangement placed a very stringent requirement on coincidences counted and considerably reduced the background coincidence rate resulting from such events as Compton scattering from one crystal to the other. It eliminated false coincidences resulting from pulses on the ac supply line and to the Compton effect in the K shell of a heavy element. In the latter case, the lower bias will always reject pulses resulting from K x-rays.

With apparatus of the above design, there are a number of effects which must be understood before valid results can be obtained. These will be mentioned briefly here, and then considered in more detail in later sections. Most fundamental of these is the fact that the coincidence counting efficiency is not constant but is a function of the distance of the annihilating positron from the line joining the centers of the two counters. Closely involved with this effect is the fact that a positron may move an appreciable distance through the target between the time it is created and the time it annihilates. Absorption of the initial gamma-ray beam and of the annihilation radiation leaving the target must also be considered.

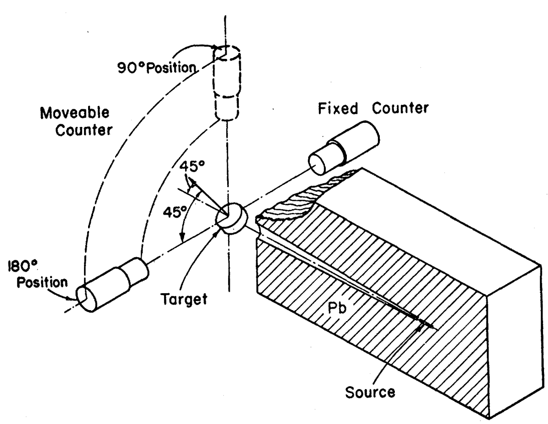

FIG. 1. Schematic drawing of the experimental arrangement. Most of the lead shielding is not shown.

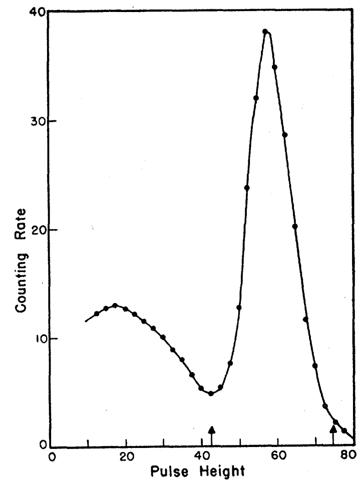

FIG. 2. Typical pulse-height spectrum from NaI (Tl) scintillation counter when exposed to annihilation radiation. Arrows show where upper and lower biases were set.

II. EXPERIMENTAL TECHNIQUE

Apparatus

The ThC″ source had a strength of about 60 mC and was in secular equilibrium. It was mounted 30 cm from the target behind 25 cm of lead shielding. The hole in the collimation tapered from $\frac{3}{16}$ in. in diameter at the source to 1 in. in diameter where the beam emerged. The Co^{60} source had a strength of about 2 curies, although a source of one-tenth that strength would have been adequate. It was mounted behind 75 cm of lead shielding.

Each NaI(Tl) crystal was 4 cm in diameter by 4 cm high, and was mounted in the conventional manner in mineral oil in a Lucite case which was cemented to the face of the photomultiplier tube. Each detector was 16 cm from the target.

The counters were calibrated with annihilation radiation before every run. A typical pulse-height spectrum is shown in Fig. 2. The full width of the photoline at half-maximum was about 25 percent in each counter. These counters were in use for a period of six months, and during that time a bias curve was taken at least once a week. No change in the resolution of either counter was observed during that time.

The electronics was quite simple and conventional. Each photomultiplier socket was mounted on a box containing only a voltage divider network and a few by-pass capacitors. Each box was connected to a linear amplifier whose output fed into the discriminators mentioned previously. The location of the high and low biases is indicated by the arrows in Fig. 2. The output pulses from the two discriminators were fed into a Rossi type coincidence circuit whose resolving time as measured with a sliding pulser was 0.7 μsec. The high voltage for the photomultiplier tubes was obtained from a highly stabilized supply.[11]

[11] Designed and built in this Laboratory by Leonard Walker.

The location of the dip between the photoline and the Compton edge in the pulse-height distribution gave a very sensitive way of monitoring the over-all gain of the system. It was found that a shift in over-all gain of 2 percent could be detected, and over a period of one or two weeks the change in gain never exceeded this amount. Over a period of several months the gain changed by less than 5 percent.

The targets were flat disks 2.0 in. in diameter. They were oriented so that the normal to the target made an angle of 45° with the direction of the beam and an angle of 45° with the line joining the two counters when they were in the 180° position. This orientation was chosen so that the distance through the target from any point in the target to the moveable counter would be the same whether that counter were in the 180° or the 90° position. Since the singles counting rate in each counter was the same whether the counters were in the 180° or the 90° position, the coincidence rate in the 90° position gave directly the background resulting from chance coincidences and also to such events as the double Compton effect where two truly coincident gamma-rays are produced with an isotropic distribution with respect to each other. At all times the singles rates in each counter were recorded in order to monitor the operation of the equipment, even though they were not needed in the analysis of the data.

Preliminary Measurements

Extensive survey measurements were made with the ThC″ source using six target materials, Be, C, Al, Cu, Sn, and Pb. Counting rate was measured as a function of thickness for 6 or 8 thicknesses up to about 2 g/cm^2. The original hope was that after applying corrections for the variation of detection efficiency over the target and the other corrections enumerated later on, the resulting curve would be a straight line whose slope could be interpreted as the rate of production of positrons in the given material. This program failed because of two effects which could be neither calculated nor measured.

Gamma-rays from the source produce pairs in the lead collimation. An unknown, but presumably constant, number of these positrons land on the front surface of the target and are counted. If the front of the target could be kept in the same place throughout the measurements, then this unknown number of positrons would be counted with the same efficiency and would not change the slope of the rate *versus* thickness curve.

A number of related effects are due to the previously mentioned fact that positrons produced in the target have a finite range, which may amount to several millimeters in the low density targets. This means, first, that a certain number of positrons made in the target are able to leave it and thus not be counted. This effect would not change the slope of the rate *versus* thickness curve (for target thicknesses greater than the maximum range of the most energetic positron produced) provided the back of the target were kept in the same position. In addition, the analysis of the data becomes complicated because positrons made at one point in the target have a distribution in range and are thus counted with varying efficiencies. An attempt was made to calculate this effect, but partly because of a lack of information about the distribution in range and scattering of the positrons from pair production at these energies, and partly because of the necessity of making simplifying mathematical assumptions, the attempt was not successful.

Experimental Procedure

The above difficulties could be overcome by insuring that all targets had identical geometry for the detection of positrons. This realization led to the following target design. Each element was made into a target 1.00 cm thick with a surface density of between 1.85 and 1.90 g/cm^2, which is the density of Be. In other elements this was accomplished by spacing eight or nine thin sheets with Lucite rings (except for carbon, where because of the low density the surface density was only 1.60 g/cm^2). A target holder was constructed which held the target in a sandwich between two Be slugs. For the ThC″ measurements the front slug was $\frac{1}{8}$ in. thick and the back slug $\frac{1}{4}$ in. thick. For measurements with the Co^{60} source the thicknesses of both Be slugs were halved. In both cases the rear slug could have been the same thickness as the front one if two slugs of the same thickness had been available. The target holder was arranged so that the center of the target slug was on the line joining the centers of the two crystals. We believe that a good part of the difference between our results and those of the Swiss group[7] at 2.62 Mev can be explained by their failure to consider the variation of detection efficiency over the target volume.

The effectiveness of this arrangement in eliminating trouble resulting from the motion of positrons in the target depends partly on the projected range of the positron, expressed in g/cm^2, being independent of Z. There is a scarcity of experimental data on this subject, but measurements by Trump, van de Graaff, and Cloud[12] can be interpreted to show that the range of monoenergetic electrons in lead differs from that in aluminum by 25 percent at most. Since the effect is small, this difference can be tolerated. The fact that the difference in range for different Z is not better known introduces an error of about 1 percent in the final result.

Other features of this target design are that whether or not a target slug is in place, all the positrons from the collimation land on the first Be slug and so are counted with constant efficiency. A few of the positrons which are made in the first Be slug leave it. When the target is in place, they stop near its front surface, and when it is not in place, they stop near the front surface

[12] Trump, van de Graaff, and Cloud, Am. J. Roentgenol. Radium Therapy **XLIII**, 728 (1940).

of the second Be slug. In either case, they are counted with the same efficiency because the variation in detection efficiency was measured to be symmetrical about the line joining the centers of the two crystals. The second Be slug also serves to stop all the positrons leaving the target. A few positrons are made in the second Be slug, but they provide only a constant background.

Measurements were taken with targets of Be, C, Al, Cu, Sn, and Pb with the ThC″ source and with targets of Al, Cu, Sn, and Pb with the Co^{60} source. In order that the relative counting rates observed be proportional to the pair production cross sections in the different target materials, it is important that the detection efficiency remain constant throughout the measurement. The excellent long time stability of the apparatus has already been described. In order to eliminate the effect of short time variations in detection efficiency, an additional precaution was taken. In a few hours data were taken on all target materials with the counters in both the 90° and 180° positions. The counting times were arranged so that the desired relative statistical accuracy was obtained for each material. This "cycle" was then repeated until the desired number of counts had been obtained. All the data was obtained in three runs each lasting about 30 hours. Thus, any short time variations in efficiency would affect all targets more or less equally and cancel out. In fact, it was never possible to detect any systematic changes in counting efficiency.

Counting rates with the counters in the 180° position varied from about 5 counts per minute to about 75 counts per minute. These rates could have been increased considerably by moving the counters closer to the target, but then the chance coincidences resulting from the increased singles rates would have become too numerous.

III. TREATMENT OF THE DATA

To obtain the net counting rate resulting from positrons produced in the target, for each Z the rate obtained with counters at 90° was subtracted from that obtained with counters at 180°. From this result was subtracted the counting rate with no target in the target holder, corrected for the annihilation radiation absorbed by the target when in place. The computation of the absorption coefficients is described below. For the ThC″ source, the statistical errors on the resulting net counting rates were about 3.6 percent for Be and C and 2 percent for the other targets. For the Co^{60} source the same errors varied from 4.9 percent for Al to 2.7 percent for Pb. The corrections enumerated below were then applied to these net counting rates.

Corrections

1. When a positron annihilates in the target, its annihilation radiation must pass through the target before reaching the counters. Some of the annihilation radiation will be absorbed in the target, and it is necessary to correct for this effect. Total absorption coefficients for 0.511-Mev gamma-rays were obtained by interpolating between values calculated by White.[13] These calculations have been well verified in this energy range by the accurate experiments of Colgate.[14]

Since the detector subtended a finite solid angle, it was possible for the annihilation radiation to undergo a small angle scattering in the target and still arrive at the detector and be counted. The correction to the Compton part of the cross section was made using the Klein-Nishina formula. Since the counters defined a cone of half-angle about 6°, the Compton contribution was reduced by 1 percent. Corrections to the coherent scattering cross section were made using graphs constructed by Colgate,[15] which were in turn based on the work of Franz.[16] Eighty percent of the coherent cross section was subtracted from Pb, and up to 100 percent at lower Z. The net result of these corrections was to reduce the total absorption cross section by amounts ranging from 1 percent at low Z up to 4.4 percent for Pb. Using these absorption cross sections, the corrections to the observed counting rates ranged between 30 percent and 40 percent, except for Pb where it was 70 percent.

2. Pairs produced in the field of an electron. The calculations of Borsellino[17] were used to correct for this effect. For 2.62-Mev gamma-rays, the correction amounted to 1.4 percent in Be and much less at higher Z. Since the threshold for this process is at 2 Mev, the effect was not present in the Co^{60} measurements.

3. Impurities in the targets. The Pb, Sn, and Cu targets were made from sheets which were also used by Walker[8] for his targets. These had been analyzed both chemically and spectroscopically and showed no impurities large enough to require a correction. The aluminum target was made from grade 2S sheet which contained, according to chemical analysis, 0.6 percent Fe. A correction of 1.2 percent was made for this impurity. According to the analysis supplied by the manufacturer with the Be slug, a 1.2 percent correction was required for the impurities it contained. However, Be is notoriously difficult to analyze, and it is possible that additional undetected impurities were present. The carbon target was made from pile graphite supplied by Brookhaven National Laboratory.

4. Absorption of the primary gamma-ray beam as it passes through the target. It was assumed that one interaction removed a quantum from the beam. The targets are thin enough so that this is well justified. Absorption coefficients computed by White[13] were used. The corrections were about 7 percent at 1.33 Mev and

[13] G. R. White, Nat. Bur. Standards Report No. 1003.
[14] S. A. Colgate, Phys. Rev. **87**, 592 (1952).
[15] S. A. Colgate, Cornell University Thesis (1952) (unpublished), App. II, p. 77.
[16] W. Franz, Z. Physik **98**, 314 (1935).
[17] A. Borsellino, Helv. Phys. Acta **20**, 136 (1947).

TABLE I. Relative cross section per atom for pair production by the 1.33- and 1.17-Mev gamma-rays from Co^{60}. The data have been normalized by fitting to a curve of the form $\sigma/Z^2=a+bZ^2$ and then setting $a=1$. On the assumption that the Born approximation is valid in the limit of low Z, the values of σ/Z^2 then give directly the departure of the cross sections from those predicted by the Born approximation. This normalization means that σ is expressed in arbitrary units.

Element	σ	σ/Z^2
Al	168.5± 8.28	0.997±0.049
Cu	934.4± 43.7	1.111±0.052
Sn	3735 ±142	1.494±0.057
Pb	13700 ±370	2.037±0.055

5 percent at 2.62 Mev, varying less than 1 percent from these values for all Z.

5. It is possible for positrons to annihilate in ways that do not give rise to two quanta 180° apart. These effects will be important in this experiment, only if they occur an appreciable fraction of the time and in addition have a Z-dependent cross section.

If a positron annihilates before becoming thermalized, then its two annihilation quanta will form an angle of less than 180° and will have, in general, different energies. Results given in Heitler[2] show that, practically independent of Z, only a small fraction of the positrons annihilate before reaching thermal velocities. This conclusion is supported by the experiment of DeBenedetti and co-workers[18] on the angular correlation of annihilation radiation. They find a small departure from 180° correlation between the two annihilation quanta which can be explained by the thermal motion of the positron and the electron with which it annihilates.

The ratio of the frequency of three-quantum annihilation to two-quantum annihilation has been calculated to be 1 to 370 in metals.[19] In addition, there is no reason for believing this ratio to be Z-dependent.

A positron may undergo one-quantum annihilation by uniting with a bound electron. This process is expected to be Z-dependent, since it depends on the strength of the binding of the electron to the nucleus. However, because of the electrostatic repulsion by the Coulomb field of the nucleus, a thermal positron cannot reach the inner electron shells. According to Jaeger and Hulme[20] the ratio of the probability of one-quantum annihilation to two-quantum annihilation in lead is of the order of 0.01.

6. The ThC″ source is supposed to contain other gamma-rays besides the one at 2.62 Mev, and these gamma-rays will produce pairs which will be recorded. If the pair production cross section could be represented by an equation of the form $\sigma(Z,E)=f(Z)\varphi(E)$, then no correction would be necessary since the other gamma-rays will produce the same fraction of the total number of pairs in all the targets. (The results given by the Born approximation calculation do satisfy the above relation, with $f(Z)=Z^2$ and $\varphi(E)$ given in Heitler.[2]) However, the calculations of Jaeger and Hulme[3–5] indicate that in the region from 3 Mev down this relation is apparently not satisfied.

Latyshev[21] has measured the gamma-ray spectrum from a ThC″ source and finds that for every 100 gamma-rays of 2.62 Mev, there are a total of 28 gamma-rays of five different energies between 1.0 and 2.2 Mev. However, Bell[22] recently pointed out that in a very pure ThC″ source there are no gamma-rays between 0.9 Mev and the principal gamma-ray at 2.62 Mev. He claims that the intermediate energy gamma-rays measured by Latyshev are the result of radium contamination of the source, which is very hard to remove chemically. Because of this fact, a rough measurement was made of the gamma-ray spectrum from our ThC″ source. A two-crystal Compton spectrometer of the type described by Hofstadter[23] was used.[24] The result was that for every 100 gamma-rays of 2.62 Mev our source had a total of 78 gamma-rays of three different energies between 1.0 and 2.2 Mev. Two pairs of lines observed by Latyshev were not resolved in this measurement, but that should not invalidate the measurement of their total intensity. Using this result, it was computed that 5 percent more positrons would be produced in lead than in low Z targets by these contaminating gamma-rays, and this correction was accordingly applied. In making these calculations it was assumed that the pair production cross section in lead is 24 percent higher than the Born approximation value at 2.62 Mev and 97 percent higher at 1.53 Mev.

For measurements taken with the Co^{60} source the situation is much more uncertain. Co^{60} emits two gamma-rays of equal strength of 1.33 and 1.17 Mev.[25]

TABLE II. Relative cross section per atom for pair production by 2.62-Mev gamma-rays. The data have been normalized by fitting the points for Al, Cu, Sn, and Pb to a curve of the form $\sigma/Z^2=a+bZ^2$ and then setting $a=1$. On the assumption that the Born approximation is valid in the limit of low Z, the values of σ/Z^2 then give directly the departure of the cross sections from those predicted by the Born approximation. This normalization means that σ is expressed in arbitrary units.

Element	σ	σ/Z^2
Be	17.21± 0.645	1.075±0.040
C	33.63± 1.19	0.934±0.033
Al	170.1 ± 3.36	1.006±0.020
Cu	866.1 ± 17.4	1.030±0.021
Sn	2702 ± 57.9	1.081±0.023
Pb	8262 ±167	1.229±0.025

[18] DeBenedetti, Cowan, Konneker, and Primakoff, Phys. Rev. **77**, 205 (1950).
[19] A. Ore and J. L. Powell, Phys. Rev. **75**, 1696 (1949).
[20] J. C. Jaeger and H. R. Hulme, Proc. Cambridge Phil. Soc. **32**, 158 (1936).
[21] G. D. Latyshev, Revs. Modern Phys. **19**, 132 (1947).
[22] P. R. Bell (private communication).
[23] R. Hofstadter and J. A. McIntyre, Phys. Rev. **78**, 619 (1950). See also James Draper, Cornell University thesis (1952) (unpublished).
[24] These measurements were made with the assistance of Dr. James Draper, who generously loaned his apparatus for the purpose.
[25] *Nuclear Data*, Natl. Bur. Standards Circular No. 499 (1950).

In order to eliminate the effect of the 1.17-Mev gamma-ray and obtain the relative cross sections at 1.33 Mev, it is necessary to interpolate between the exact point calculated by Jaeger and Hulme at 3 mc^2 and the threshold at 2 mc^2. This can be done in many different ways; there is no certain theoretical information on the shape of the exact cross section near threshold. Making widely different assumptions about this interpolation results in corrections for the 1.17-Mev gamma-ray ranging between plus 5 percent and minus 10 percent in Pb, with a value of zero fairly probable. Under these circumstances, it was decided to present the data without correcting for the presence of the 1.17-Mev gamma-ray, with the understanding that if this were done, the results would be changed little if any.

IV. RESULTS

The results are summarized in Tables I and II. The data are also exhibited in Figs. 3 and 4, where σ/Z^2 is plotted as a function of Z. If the Born approximation were valid, the points would lie on a horizontal line. The data have been fit by using least squares[26] to curves of the form $\sigma/Z^2=a+bZ^2$ and $\sigma/Z^2=a+bZ$. The points for Be and C at 2.62 Mev were disregarded in fitting the curves because of their larger statistical error and because the background was a fairly large fraction of the total counting rate. At both energies the first curve is the better fit, even though the second is by no means excluded. This agrees with the results of Jaeger[4] and Hahn, Baldinger, and Huber[7] mentioned in the Introduction. These results are in mild disagreement with those of Walker[8] who found at 17.6 Mev that his data was fit better by a curve of the form $\sigma/Z^2=a+bZ$, although the other possibility was not excluded. So far as we know, there is no general theoretical argument which tells what powers of Z might be expected in the formula for the exact cross section.

For the reasons given above, further analysis of the data is carried out using the curve $\sigma/Z^2=a+bZ^2$. For

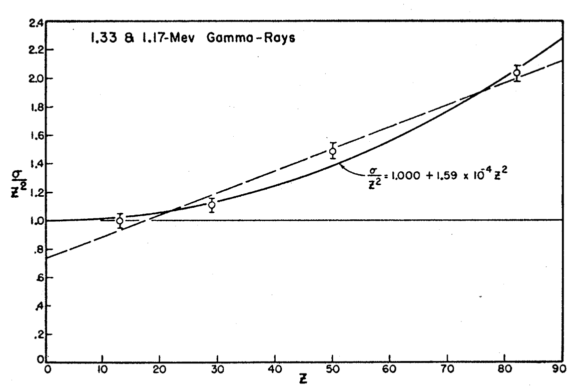

FIG. 3. Observed relative pair production cross sections for the 1.33- and 1.17-Mev gamma-rays from Co^{60}. Based on the data shown in Table I.

[26] Using the formulas given by R. T. Birge, Phys. Rev. **40**, 207 (1932).

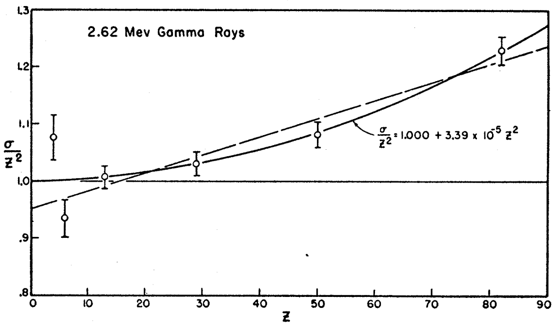

FIG. 4. Observed relative pair production cross sections for 2.62-Mev gamma-rays. Based on the data shown in Table II.

convenience the data are normalized so that $a=1$. The least squares fit then gives $b=1.59\times10^{-4}$ at 1.33 Mev and $b=3.39\times10^{-5}$ at 2.62 Mev. Since the Born approximation should give the correct result in the limit as Z approaches zero, this normalization of $a=1$ gives directly the departure of the measured cross sections from the Born approximation value. At 2.62 Mev the measured cross section in Pb is 23 percent higher than that predicted by the Born approximation. Jaeger and Hulme calculated that it should be 25 percent higher at 5 mc^2, and since they say that their calculations might be in error by as much as 10 percent, the calculations agree with experiment to well within the errors. In the case of the Co^{60} measurements, as pointed out above, comparison with theory is much more difficult. The measured cross section in Pb at 1.33 Mev is slightly more than twice the Born approximation value, and this is certainly not inconsistent with the exact calculation when the possible 5 percent error in the calculation and the slightly different energies of the measurement and calculation are taken into account.

Errors

The errors quoted above are only the statistical counting errors. There are, of course, additional uncertainties in the experiment, and it is necessary to evaluate their effect on the final result. Listed below are various possible sources of error with an estimate of their magnitude in percent.

Variation of range of electrons in different materials: 1.0.
Absorption of annihilation radiation in the target: 1.0 for Pb, 0.5 for others.
Pairs produced by other gamma-rays from ThC″ source: 0.5 for Pb, negligible for others.

All the other corrections to the data are so small that errors in them are quite negligible. Since this experiment was a relative not an absolute measurement, it was not necessary to know numerical values of many other quantities, but only to know that they stayed constant during the experiment.

Combining the above errors gives an error of about

TABLE III. Ratio of the pair production cross section in lead obtained either from experiment or more accurate calculations to that calculated using the Born approximation (Bethe-Heitler theory).

Energy (Mev)	Experimental ratio	Theoretical ratio	Reference
1.33	2.04 ±0.06	...	present work
1.53	...	1.97± 5%	3, 4
2.55	...	1.25±10%	5
2.62	1.23 ±0.025	...	present work
5.3	1.046±0.019	...	28
10.3	0.932±0.014	...	28
17.6	0.865±0.012	...	28
17.6	0.845±0.016	0.800	8, 27
88	0.896±0.014	0.882	9, 27
280	0.904±0.01	0.900	10, 27

1.5 percent in lead and 1.0 percent for other materials which must be combined with the counting errors. For measurements at 2.62 Mev, a conservative estimate of the combined errors would be 4 percent for Be and C and 3 percent for the other materials. As pointed out above, the uncertainty in the contribution of the 1.17-Mev gamma-ray to the Co^{60} data completely overshadows the effects listed here.

V. FAILURE OF THE BORN APPROXIMATION

In Table III are summarized both the ratios of the experimentally observed pair production cross sections in lead to those calculated using the Born approximation, and also the results of more exact calculations at various energies. The data in this table are displayed in Fig. 5. The dashed curve is that computed by Davies and Bethe[27] and is not expected to be valid at low energies. It can be seen that near the threshold the experimental cross sections are considerably larger than those given by the Born approximation, whereas at high energies they are slightly lower. Results recently reported from Case[28] serve to locate the cross over at about 6 Mev.

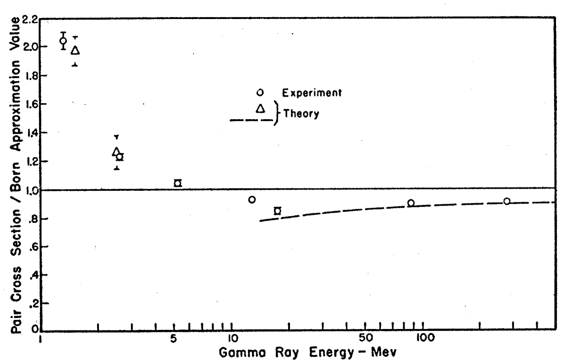

FIG. 5. Failure of the Born approximation calculations in lead. Based on data given in Table III. The dashed curve is based on the calculations of Davies and Bethe (see reference 27) and is not expected to be valid at low energies.

[27] H. Davies and H. A. Bethe, Phys. Rev. **87**, 156 (1952).

[28] Rosenblum, Shrader, and Warner, Jr., Phys. Rev. **88**, 612 (1952).

VI. NUMERICAL VALUES FOR THE CROSS-SECTIONS

In Table IV are tabulated values of the pair production cross section according to the Born approximation, and also "corrected" cross sections obtained by using the experimentally determined Z-dependence and assuming that the Born approximation gives the correct result in the limit of low Z. The Born approximation values are calculated from the formula given by Hough.[29] The corrected cross sections are calculated from the following formulas:

$$\sigma_{\text{corrected}} = \sigma_{\text{Born}}(1+1.59\times 10^{-4}Z^2) \text{ at } 1.33 \text{ Mev},$$

$$\sigma_{\text{corrected}} = \sigma_{\text{Born}}(1+3.39\times 10^{-5}Z^2) \text{ at } 2.62 \text{ Mev}.$$

VII. REANALYSIS OF COLGATE'S GAMMA RAY ABSORPTION DATA

The results given above can be used to reanalyze some of the recent accurate gamma-ray absorption measurements of Colgate.[14] The discussion which follows refers to Sec. VII of his paper. His method of analysis was to take one of the processes which contributes to gamma-ray absorption as unknown or poorly known, and to try to find out more about it by subtracting from the experimental cross sections theoretical values for the cross sections for all the other processes. At 1.33 Mev he takes the photoelectric effect as the unknown and subtracts from the experimental cross section theoretical values for the Rayleigh, Compton, and pair production cross sections. We have followed this procedure but have substituted what we believe to be more correct values for the pair production cross section. The resulting "remainder" cross section is then compared to the theoretical photoelectric effect cross section.

TABLE IV. Pair production cross sections (in barns). "Corrected" cross sections are obtained by using the experimentally determined Z-dependence and assuming that the Born approximation gives the correct result in the limit of low Z. The Born approximation values are calculated from the formula of Hough.[a]

Z	1.33 Mev		2.62 Mev	
	σ_{Born}	$\sigma_{\text{corrected}}$	σ_{Born}	$\sigma_{\text{corrected}}$
6	0.000588	0.000592	0.0136	0.0136
13	0.00276	0.00283	0.0640	0.0644
29	0.0137	0.0156	0.318	0.327
50	0.0409	0.0571	0.946	1.026
82	0.110	0.227	2.55	3.13
92	0.138	0.325	3.20	4.12

[a] See reference 29.

At 2.62 Mev Colgate took the pair production cross section as the unknown. We believe that it is now more appropriate to take the photoelectric cross section as unknown and have accordingly analyzed the data in the same way as the 1.33 Mev data. The results are given in Table V and compared with the theory in Fig. 6. The theoretical values are obtained by multiply-

[29] P. V. C. Hough, Phys. Rev. **73**, 266 (1948).

TABLE V. Reanalysis of Colgate's data on gamma-ray absorption at 1.33 and 2.62 Mev. The data given below are corrections to Tables X and XI of reference 14. At both energies $\sigma_{\text{remainder}}$ is computed by subtracting from the experimental cross sections the theoretical Compton, Rayleigh, and corrected pair production cross sections.

	1.33 Mev		2.62 Mev	
Z	σ_{pair} $\times 10^{24}$	$\sigma_{\text{remainder}}/Z^5$ $\times 10^{33}$	σ_{pair} $\times 10^{24}$	$\sigma_{\text{remainder}}/Z^5$ $\times 10^{34}$
6	0.000592	...	0.0136	...
13	0.00283	...	0.0644	...
29	0.0156	...	0.327	...
50	0.0571	1.124±0.06	1.026	2.85±0.64
78	0.196	0.944±0.024	...	...
82	0.227	0.957±0.011	3.125	2.78±0.081
83	0.236	0.929±0.010	3.217	2.90±0.076
92	0.325	0.861±0.006	4.124	2.98±0.24

ing the K shell cross sections of Hulme, *et al.*,[30] by the ratio σ_{K+L+M}/σ_K from the Stobbe theory (see reference 13, p. 15). The contribution of the higher shells obtained in this manner is probably more realistic than that obtained by the 5/4 rule. At 1.33 Mev the values of the remainder cross section are about 5 to 10 percent lower than the theoretical photoelectric cross section; at 2.62 Mev they are about 20 percent lower. This result could mean that the energy dependence of Hulme's curves at high energies is not correct.

The author is deeply indebted to Professor W. M. Woodward for his continued interest in this experiment and for his many fruitful suggestions. He also wishes to thank Professor P. Morrison, Professor H. A. Bethe, Dr. U. Fano, and Miss Gladys White for discussions on the theoretical aspects of the problem.

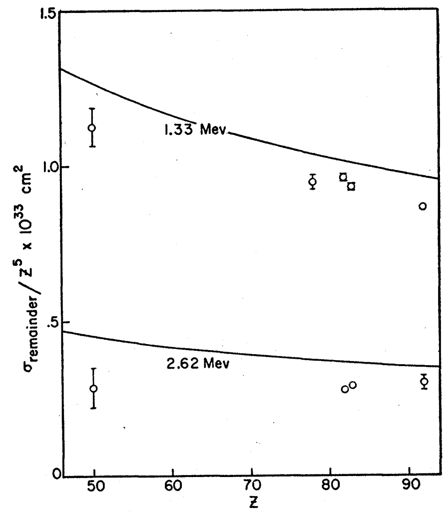

FIG. 6. Reanalysis of Colgate's data on gamma-ray absorption at 1.33 and 2.62 Mev. Curves are the theoretical photoelectric cross section/Z^5 for the respective energies. The points represent the experimental cross section minus the theoretical Compton, Rayleigh, and corrected pair production cross sections. Based on the data given in Table V.

APPENDIX

The value of 1.25 for the ratio of the cross section calculated by Jaeger and Hulme to that calculated from the Born approximation for lead at 5 mc^2 is obtained using the revised cross section given by Jaeger in reference 5 together with the Born approximation value calculated from Hough's formula.[29] There are numerous misprints and errors in the literature which could mislead one on this point. The values of the Born approximation cross section at 5.0 mc^2 and 5.2 mc^2 given in references 3 and 5 do not agree with other published values or with values obtained from Hough's formula. The same can be said for the Born approximation value at 5.2 mc^2 given at the top of page 259 of the second edition of Heitler. In addition, the exact value given in the same table has apparently been miscopied from the original reference. This latter error led Colgate, in plotting Fig. 8 of reference 14, to show the Jaeger and Hulme calculation as only 11 percent higher than the Born approximation value in lead at 2.62 Mev. It should also be noted that the value of the Born approximation cross section at 1.362 Mev quoted by Davisson and Evans[31] is 40 percent higher than the value computed from Hough's formula.

[30] H. R. Hulme *et al.*, Proc. Roy. Soc. (London) **A149**, 131 (1935).

[31] C. M. Davisson and R. D. Evans, Revs. Modern Phys. **24**, 79 (1952).