interesting that recently Sachs[4] suggested the necessity of the existence of such a coupling term to account for the data.

A full text of this work will be published elsewhere.

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Theory of the Microwave Permeability Tensor and Faraday Effect in Nonsaturated Ferromagnetic Materials

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THE microwave Faraday rotation in a ferromagnetic material is calculated below without invoking the restriction, implicit in Roberts'[1] and Hogan's[2] adaptation of Polder's[3] theory, that the material be magnetized to saturation. Under certain conditions the rotation θ is shown to be proportional to the static magnetization M throughout the magnetization curve or hysteresis loop of the material.

If the magnetic inhomogeneities are larger than the domain wall thickness, the exchange torque may be neglected at points not contained within walls, so that at such points the time-dependent saturation magnetization and effective field vectors are related by

$$\partial \mathbf{M}_s(t)/\partial t = \gamma \mathbf{M}_s(t) \times \mathbf{H}(t), \tag{1}$$

where the damping torque is omitted for simplicity, and $ge/2mc$ is denoted by γ. Letting $\mathbf{H}(t)=\mathbf{H}_0+\mathbf{h}(t)$, $\mathbf{M}_s(t)=\mathbf{M}_s+\mathbf{m}(t)$, and $\mathbf{m}(t)\times h(t)\approx 0$, one obtains with $\exp(i\omega t)$ type time-dependence

$$\mathbf{m}=(\gamma/i\omega)(H_0\mathbf{m}-M_s\mathbf{h})\times\mathbf{u}, \tag{2}$$

where $\mathbf{u}$ is a time-independent unit vector defined by $\mathbf{M}_s=M_s\mathbf{u}$. The vector $\mathbf{H}_0=H_0\mathbf{u}$ is an effective, local, static field and represents the resultant of the applied, anisotropy, and magnetic interaction fields at the equilibrium orientation defined by $\mathbf{M}_s(t)=\mathbf{M}_s$. (Although H_0 is not calculable, it may be characterized by the energy increase $M_sH_0\varphi^2/2$ caused by a small deflection φ of $\mathbf{M}_s(t)$ relative to $\mathbf{M}_s$.) The vectors $\mathbf{m}$, $\mathbf{h}$, $\mathbf{u}$, and the scalar H_0 are functions of position.

Assume that $H_a\gamma \ll \omega$, where H_a is some applied field capable of saturating the material and corrected, if necessary, for demagnetization due to the shape of the specimen. Since $H_0 \lesssim H_a$, it follows that $H_0\gamma \ll \omega$. Assume, in addition, that $4\pi M_s\gamma \ll \omega$. Equation (2) then leads to

$$\mathbf{b}\equiv\mathbf{h}+4\pi\mathbf{m}\approx\mathbf{h}+i(4\pi M_s\gamma/\omega)\mathbf{h}\times\mathbf{u}, \tag{3}$$

where H_0 does not occur explicitly.

Consider a polycrystalline material containing randomly oriented crystallites and let it be statically magnetized to some definite state. The internal dynamic demagnetizing field, which contributes to $\mathbf{h}$, may be sufficiently random so that the probability of $\mathbf{h}$ having any specified value at a given point is independent of the direction of $\mathbf{u}$ at that point. This lack of correlation justifies the equation

$$\langle \mathbf{h}\times\mathbf{u}\rangle_{\mathrm{Av}}=\langle\mathbf{h}\rangle_{\mathrm{Av}}\times\langle\mathbf{u}\rangle_{\mathrm{Av}}. \tag{4}$$

The symbol $\langle\ \rangle_{\mathrm{Av}}$ is introduced to indicate an average[4] taken over a volume whose linear dimensions are large compared to domain sizes but small compared to a distance L in which $\langle\mathbf{h}\rangle_{\mathrm{Av}}$ changes appreciably; clearly $L\lesssim\lambda/4$, where λ is the wavelength in the material. Letting $\mathbf{u}=\mathbf{i}\alpha_1+\mathbf{j}\alpha_2+\mathbf{k}\alpha_3$, where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors along the x, y, z coordinate axes, respectively, and noting that in a polycrystal magnetized parallel to $\mathbf{k}$ the relations $\langle\alpha_1\rangle_{\mathrm{Av}}=\langle\alpha_2\rangle_{\mathrm{Av}}=0$ and $\langle\alpha_3\rangle_{\mathrm{Av}}=M/M_s$ are valid, one obtains from Eqs. (3) and (4)

$$\langle b_x\rangle_{\mathrm{Av}}=\langle h_x\rangle_{\mathrm{Av}}+i(4\pi M\gamma/\omega)\langle h_y\rangle_{\mathrm{Av}}, \tag{5a}$$

$$\langle b_y\rangle_{\mathrm{Av}}=-i(4\pi M\gamma/\omega)\langle h_x\rangle_{\mathrm{Av}}+\langle h_y\rangle_{\mathrm{Av}}, \tag{5b}$$

$$\langle b_z\rangle_{\mathrm{Av}}=\langle h_z\rangle_{\mathrm{Av}}. \tag{5c}$$

In general, domain wall displacements may also contribute to the (diagonal) components of the tensor relating $\langle\mathbf{b}\rangle_{\mathrm{Av}}$ and $\langle\mathbf{h}\rangle_{\mathrm{Av}}$, but if such contributions are neglected, and the other assumptions of Eqs. (5) apply, then these equations (or their obvious generalizations for finite damping)[5] should be used instead of Polder's[3] tensor for the Faraday effect and other applications.

If $\langle\mathbf{b}\rangle_{\mathrm{Av}}$, $\langle\mathbf{h}\rangle_{\mathrm{Av}}$, and $\langle\mathbf{E}\rangle_{\mathrm{Av}}$ are proportional to $\exp(i\omega t-pz)$, Maxwell's curl equations in an unbounded, nonconducting, medium lead to

$$\langle\mathbf{b}\rangle_{\mathrm{Av}}=(p^2c^2/\omega^2\epsilon)(\langle h_z\rangle_{\mathrm{Av}}\mathbf{k}-\langle\mathbf{h}\rangle_{\mathrm{Av}}), \tag{6}$$

where $\epsilon=\epsilon_1-i\epsilon_2$ is the dielectric constant. Combining Eqs. (5) and (6) one obtains two circularly polarized waves described by $p_\pm^2=-(\omega^2\epsilon/c^2)(1\mp 4\pi M\gamma/\omega)$. With the abbreviation $\epsilon_{\mathrm{eff}}=(|\epsilon|+\epsilon_1)/2$, it follows that

$$\theta=(z/2)[\mathrm{Im}(p_-)-\mathrm{Im}(p_+)]\approx(2\pi\gamma\epsilon_{\mathrm{eff}}^{\frac{1}{2}}/c)zM, \tag{7}$$

a result derived by others[1,2] for $M=M_s$ and not, as they believed, for an arbitrary M.

The published data[1,2] suggest that the predicted proportionality of θ and M may apply even under wave guide conditions provided M is uniform throughout the sample. In the case of a thin disk possessing a large demagnetizing factor in the z direction, of course, no remanence (or hysteresis) of θ or M is to be expected.

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[4] It was pointed out previously [See G. T. Rado, Revs. Modern Phys. (to be published)] that the validity of such an average is a prerequisite for the interpretation of the measured complex initial permeability of polycrystalline ferrites.
[5] If the damping term $-\eta\{[\mathbf{H}(t)\cdot\mathbf{M}_s(t)]\mathbf{M}_s(t)/M_s^2-\mathbf{H}(t)\}$ is included in the right-hand side of Eq. (1), and it is assumed that $\eta'\equiv 4\pi\eta/\omega\ll 1$, then the nondiagonal components of the tensor in Eqs. (5) remain unchanged; the diagonal components become $\mu_{11}=\mu_{22}=1-i\eta'$ and $\mu_{33}=1-i\eta'[1-(M/M_s)^2]$. The Faraday rotation, Eq. (7), remains unchanged.

The Half-Life of Ca^{45}

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THE half-life of Ca^{45} has been estimated by Walke, Thompson, and Holt[1] to be 180 ± 10 days, and by Matthews and Pool[2] to be 152 days. The discrepancy between these values seemed to warrant the redetermination described below.

The decay of 2.3 g of calcium carbonate which had been irradiated for about a day in the Harwell pile, was followed with a G.E.C. end-window G-M counter, type EHM2. Day to day variations in the response of this counter (which, however, never exceeded 2 percent) were corrected for by counting a standard uraninite source. The active material was contained in a brass source holder sealed off with thin cellophane to prevent any change or losses. The activity fell from 3934 counts/min to 1668 counts/min in 202 days. A least squares semilogarithmic plot of the decay gave a value of 163.5 days for the half-life, with a calculated maximum error of ±4 days.

Since an interval of over 100 days was allowed to elapse between the irradiation of the material and the first measurement, any short-lived radioisotopes produced during the activation will have almost completely disappeared. As regards long-lived contaminants, a consideration of the analysis of the calcium carbonate shows that the greatest contribution will be made by S^{35}. Even this, however, will be less than 0.1 percent of the calcium activity. The