interesting that recently Sachs[4] suggested the necessity of the existence of such a coupling term to account for the data.

A full text of this work will be published elsewhere.

The author wishes to thank Prof. L. Infeld for many valuable suggestions and helpful criticism.

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Theory of the Microwave Permeability Tensor and Faraday Effect in Nonsaturated Ferromagnetic Materials

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THE microwave Faraday rotation in a ferromagnetic material is calculated below without invoking the restriction, implicit in Roberts'[1] and Hogan's[2] adaptation of Polder's[3] theory, that the material be magnetized to saturation. Under certain conditions the rotation θ is shown to be proportional to the static magnetization M throughout the magnetization curve or hysteresis loop of the material.

If the magnetic inhomogeneities are larger than the domain wall thickness, the exchange torque may be neglected at points not contained within walls, so that at such points the time-dependent saturation magnetization and effective field vectors are related by

$$\partial \mathbf{M}_s(t)/\partial t = \gamma \mathbf{M}_s(t) \times \mathbf{H}(t), \tag{1}$$

where the damping torque is omitted for simplicity, and $ge/2mc$ is denoted by γ. Letting $\mathbf{H}(t)=\mathbf{H}_0+\mathbf{h}(t)$, $\mathbf{M}_s(t)=\mathbf{M}_s+\mathbf{m}(t)$, and $\mathbf{m}(t)\times h(t)\approx 0$, one obtains with $\exp(i\omega t)$ type time-dependence

$$\mathbf{m}=(\gamma/i\omega)(H_0\mathbf{m}-M_s\mathbf{h})\times\mathbf{u}, \tag{2}$$

where $\mathbf{u}$ is a time-independent unit vector defined by $\mathbf{M}_s=M_s\mathbf{u}$. The vector $\mathbf{H}_0=H_0\mathbf{u}$ is an effective, local, static field and represents the resultant of the applied, anisotropy, and magnetic interaction fields at the equilibrium orientation defined by $\mathbf{M}_s(t)=\mathbf{M}_s$. (Although H_0 is not calculable, it may be characterized by the energy increase $M_sH_0\varphi^2/2$ caused by a small deflection φ of $\mathbf{M}_s(t)$ relative to $\mathbf{M}_s$.) The vectors $\mathbf{m}$, $\mathbf{h}$, $\mathbf{u}$, and the scalar H_0 are functions of position.

Assume that $H_a\gamma \ll \omega$, where H_a is some applied field capable of saturating the material and corrected, if necessary, for demagnetization due to the shape of the specimen. Since $H_0 \lesssim H_a$, it follows that $H_0\gamma \ll \omega$. Assume, in addition, that $4\pi M_s\gamma \ll \omega$. Equation (2) then leads to

$$\mathbf{b}\equiv\mathbf{h}+4\pi\mathbf{m}\approx\mathbf{h}+i(4\pi M_s\gamma/\omega)\mathbf{h}\times\mathbf{u}, \tag{3}$$

where H_0 does not occur explicitly.

Consider a polycrystalline material containing randomly oriented crystallites and let it be statically magnetized to some definite state. The internal dynamic demagnetizing field, which contributes to $\mathbf{h}$, may be sufficiently random so that the probability of $\mathbf{h}$ having any specified value at a given point is independent of the direction of $\mathbf{u}$ at that point. This lack of correlation justifies the equation

$$\langle \mathbf{h}\times\mathbf{u}\rangle_{\mathrm{Av}}=\langle\mathbf{h}\rangle_{\mathrm{Av}}\times\langle\mathbf{u}\rangle_{\mathrm{Av}}. \tag{4}$$

The symbol $\langle\ \rangle_{\mathrm{Av}}$ is introduced to indicate an average[4] taken over a volume whose linear dimensions are large compared to domain sizes but small compared to a distance L in which $\langle\mathbf{h}\rangle_{\mathrm{Av}}$ changes appreciably; clearly $L\lesssim\lambda/4$, where λ is the wavelength in the material. Letting $\mathbf{u}=\mathbf{i}\alpha_1+\mathbf{j}\alpha_2+\mathbf{k}\alpha_3$, where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors along the x, y, z coordinate axes, respectively, and noting that in a polycrystal magnetized parallel to $\mathbf{k}$ the relations $\langle\alpha_1\rangle_{\mathrm{Av}}=\langle\alpha_2\rangle_{\mathrm{Av}}=0$ and $\langle\alpha_3\rangle_{\mathrm{Av}}=M/M_s$ are valid, one obtains from Eqs. (3) and (4)

$$\langle b_x\rangle_{\mathrm{Av}}=\langle h_x\rangle_{\mathrm{Av}}+i(4\pi M\gamma/\omega)\langle h_y\rangle_{\mathrm{Av}}, \tag{5a}$$

$$\langle b_y\rangle_{\mathrm{Av}}=-i(4\pi M\gamma/\omega)\langle h_x\rangle_{\mathrm{Av}}+\langle h_y\rangle_{\mathrm{Av}}, \tag{5b}$$

$$\langle b_z\rangle_{\mathrm{Av}}=\langle h_z\rangle_{\mathrm{Av}}. \tag{5c}$$

In general, domain wall displacements may also contribute to the (diagonal) components of the tensor relating $\langle\mathbf{b}\rangle_{\mathrm{Av}}$ and $\langle\mathbf{h}\rangle_{\mathrm{Av}}$, but if such contributions are neglected, and the other assumptions of Eqs. (5) apply, then these equations (or their obvious generalizations for finite damping)[5] should be used instead of Polder's[3] tensor for the Faraday effect and other applications.

If $\langle\mathbf{b}\rangle_{\mathrm{Av}}$, $\langle\mathbf{h}\rangle_{\mathrm{Av}}$, and $\langle\mathbf{E}\rangle_{\mathrm{Av}}$ are proportional to $\exp(i\omega t-pz)$, Maxwell's curl equations in an unbounded, nonconducting, medium lead to

$$\langle\mathbf{b}\rangle_{\mathrm{Av}}=(p^2c^2/\omega^2\epsilon)(\langle h_z\rangle_{\mathrm{Av}}\mathbf{k}-\langle\mathbf{h}\rangle_{\mathrm{Av}}), \tag{6}$$

where $\epsilon=\epsilon_1-i\epsilon_2$ is the dielectric constant. Combining Eqs. (5) and (6) one obtains two circularly polarized waves described by $p_\pm^2=-(\omega^2\epsilon/c^2)(1\mp 4\pi M\gamma/\omega)$. With the abbreviation $\epsilon_{\mathrm{eff}}=(|\epsilon|+\epsilon_1)/2$, it follows that

$$\theta=(z/2)[\mathrm{Im}(p_-)-\mathrm{Im}(p_+)]\approx(2\pi\gamma\epsilon_{\mathrm{eff}}^{\frac{1}{2}}/c)zM, \tag{7}$$

a result derived by others[1,2] for $M=M_s$ and not, as they believed, for an arbitrary M.

The published data[1,2] suggest that the predicted proportionality of θ and M may apply even under wave guide conditions provided M is uniform throughout the sample. In the case of a thin disk possessing a large demagnetizing factor in the z direction, of course, no remanence (or hysteresis) of θ or M is to be expected.

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[4] It was pointed out previously [See G. T. Rado, Revs. Modern Phys. (to be published)] that the validity of such an average is a prerequisite for the interpretation of the measured complex initial permeability of polycrystalline ferrites.
[5] If the damping term $-\eta\{[\mathbf{H}(t)\cdot\mathbf{M}_s(t)]\mathbf{M}_s(t)/M_s^2-\mathbf{H}(t)\}$ is included in the right-hand side of Eq. (1), and it is assumed that $\eta'\equiv 4\pi\eta/\omega\ll 1$, then the nondiagonal components of the tensor in Eqs. (5) remain unchanged; the diagonal components become $\mu_{11}=\mu_{22}=1-i\eta'$ and $\mu_{33}=1-i\eta'[1-(M/M_s)^2]$. The Faraday rotation, Eq. (7), remains unchanged.

The Half-Life of Ca^{45}

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THE half-life of Ca^{45} has been estimated by Walke, Thompson, and Holt[1] to be 180 ± 10 days, and by Matthews and Pool[2] to be 152 days. The discrepancy between these values seemed to warrant the redetermination described below.

The decay of 2.3 g of calcium carbonate which had been irradiated for about a day in the Harwell pile, was followed with a G.E.C. end-window G-M counter, type EHM2. Day to day variations in the response of this counter (which, however, never exceeded 2 percent) were corrected for by counting a standard uraninite source. The active material was contained in a brass source holder sealed off with thin cellophane to prevent any change or losses. The activity fell from 3934 counts/min to 1668 counts/min in 202 days. A least squares semilogarithmic plot of the decay gave a value of 163.5 days for the half-life, with a calculated maximum error of ±4 days.

Since an interval of over 100 days was allowed to elapse between the irradiation of the material and the first measurement, any short-lived radioisotopes produced during the activation will have almost completely disappeared. As regards long-lived contaminants, a consideration of the analysis of the calcium carbonate shows that the greatest contribution will be made by S^{35}. Even this, however, will be less than 0.1 percent of the calcium activity. The

Harwell impurity specification and the linearity of the decay curve confirm the absence of appreciable amounts of such contaminants.

A more detailed account of this work will be published elsewhere.

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The Mass Difference $Si^{27}-Al^{27}$†

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THE $Al^{27}(p,n)Si^{27}$ threshold has been found to be 5.819±0.010 Mev using the 5.5-Mev electrostatic generator at Oak Ridge. From this the calculated value of the mass difference $Si^{27}-Al^{27}$ is 0.00519 amu and the Si^{27} disintegration energy is 3.81±0.01 Mev.

In each magnet cycle the calibration of the magnet was effected by locating in order the $F^{19}(p,\alpha\gamma)O^{16}$ levels at 1.355 Mev and 1.381 Mev[1] with mass two beam, the $Al^{27}(p,n)Si^{27}$ threshold, and the $F^{19}(p,\alpha\gamma)O^{16}$ level at 0.669 Mev[1] with mass three beam. The gamma-rays were detected with a NaI crystal counter and the neutrons with a "Bonner-Butler" type[2] counter.

Previously reported values for the $Al(p,n)$ threshold were 6.1±0.1 Mev,[3] and 5.93 Mev.[4] These measurements were done with cyclotrons using essentially a stacked-foil technique. The maximum positron energy from Si^{27} disintegration has been reported as 3.54±0.01 Mev,[5] 3.74,[6] 3.48±0.10 Mev,[7] and 3.6 Mev.[4]

† This document is based on work performed for the U. S. Atomic Energy Commission at Oak Ridge National Laboratory.

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The Ground State Hyperfine Structure and Nuclear Magnetic Moment of Praseodymium

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THE hyperfine structure of the ground state of praseodymium has been studied by the atomic beam magnetic resonance method. Transitions between magnetic levels belonging to the same total quantum number F and between magnetic levels belonging to different F have been observed. From the former transitions and the previously known nuclear spin value of 5/2, the total electronic angular momentum of the atom in the ground state has been found to be 9/2 and the g_J value of this state to be 0.727±0.005. Since this value of the Landé g factor is exactly that of a $^4I_{9/2}$ state in Russell-Saunders coupling, it seems most likely that the atomic ground state of Pr is a $^4I_{9/2}$. This state is also, according to Hund's rules, the most probable ground state of the $4f^36s^2$ configuration which has been predicted for this element.[1]

Of the five hyperfine intervals which exist for the ground state of Pr with $I=5/2$ and $J=9/2$, only two have been observed, *viz.*, the intervals $F=4\rightarrow F=3$ and $F=3\rightarrow F=2$. Their values are 3708.10±0.05 Mc/sec and 2782.20±0.05 Mc/sec, respectively. Attempts to obtain other intervals were unsuccessful, probably because of insufficient microwave power in the transition region. If it is assumed that the hyperfine levels of the $^4I_{9/2}$ state are unperturbed by neighboring states, the observed intervals may be expressed in terms of two constants A and B which measure, respectively, the interaction between the nuclear magnetic moment and the electrons, the interaction between the nuclear electric quadrupole moment and the electrons.[2] We find

$$A=+926.11\pm0.09 \text{ Mc/sec},$$
$$B=-12.9\pm1.0 \text{ Mc/sec},$$
$$B/A=-0.014.$$

The sign of A has been found to be positive by an analysis of the trajectory of the atoms in the apparatus, following the method of Davis, Feld, Zabel, and Zacharias.[2]

A deviation from the interval rule in hyperfine structure may arise from perturbations by neighboring states as well as from a nuclear electric quadrupole moment. In the case of Pr, however, an estimate of the perturbing effect of the $^4I_{11/2}$ state lying about 1450 cm^{-1} above the ground state[3] indicates that the perturbation is not sufficient to account for the observed deviation.

From the magnetic dipole interaction constant A, the contribution a_{4f} due to a single f electron has been calculated by the method of Goudsmit.[4] We obtain

$$a_{4f}=0.7626A(f^3, {}^4I_{9/2})=706.25 \text{ Mc/sec}.$$

From a_{4f} the nuclear gyromagnetic ratio is calculated from the relation

$$a_{4f}=\frac{g_I}{1836}\frac{R\alpha^2(Z-\sigma)^3}{n^3l(l+\frac{1}{2})(l+1)},$$

using $n=4$ and $\sigma=35.5$. This value of the screening constant σ is calculated from the fine structure multiplet of the ground state of neodymium.[5] A value nearly the same as this is also obtainable from the fine structure of Pr I as deduced from Pr II.[3] We find $g_I=1.53$ and hence

$$\mu=+3.8 \text{ nm}.$$

As a check, this crude method of evaluating the magnetic moment has been applied to the configurations $3d4s^2$, $3d^34s^2$, $3d^94s^2$ of Sc I, Cu I, V I, respectively, for which the magnetic moments are accurately known, and the errors in the calculated moments have been found to be from 10 to 15 percent. Therefore the accuracy of the above value of μ is probably no better than 10 percent. It is interesting to note the close agreement between this value and that calculated by Brix[6] from Pr II. This value also lends support to the view in the independent particle picture of nuclei that the nuclear moments of Pr arise mainly from a $d_{5/2}$ proton.

A detailed report of the experiment and calculations, together with an estimate of the nuclear quadrupole moment, will be given in a later paper.

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[3] An estimate of the fine structure splitting of the $f^3s^2\,{}^4I$ multiplet of Pr I may be obtained from the known levels of the f^3s, $^{3,5}I$ multiplet of Pr II in the manner illustrated for Nd in reference 5.
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