This agrees with the expression obtained by other authors.[10] Using the numerical value for $k_0(2, 0)$ as calculated by Bethe *et al.*,[10] the $2S$ state of hydrogen is shifted by 994.82 Mc/sec. Table III does not, however, predict the known additional displacement of the $2P_{\frac{1}{2}}$ state of hydrogen by 4.00 Mc/sec.

It is hoped that in future publications we may be able to demonstrate the utility of Tables I and II in beta-decay and meson theories.

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The Diffraction of Strong Shock Waves*

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The pseudostationary flow field resulting from diffraction of a strong shock in air over a convex corner has been investigated with the shock tube and interferometer. Only when the shock has turned nearly 180°, is it observed to become vanishingly weak. A potential flow theory for part of the field is inadequate to predict the extent of a Prandtl-Meyer expansion around the corner. Viscous forces produce a boundary layer which causes the flow to separate at the corner with significant changes in the density field as a result.

INTRODUCTION

THE diffraction of a plane shock wave by a convex corner is one example of the general problem of interacting shock and rarefaction waves. Since no unit of length is given for the flow, the pattern remains similar to itself in time, i.e., is pseudostationary, and may be described by the variables x/t and y/t. Like the complementary problem of Mach reflection, the nonlinear nature of the fluid mechanical equations present such formidable difficulties that no complete solution of the problem has been found. Jones, Martin, and Thornhill,[1] in a recent paper, have shown that a part of the flow field may be readily obtained when the incident shock is strong enough for the flow behind it to be supersonic. Previously, Lighthill[2] linearized the flow equations by retaining only terms of first order in the angle of the corner and calculated the pressure on the wall and the shape of the diffracted shock. This solution applies to shocks of any strength as long as the angle is sufficiently small. Previous experiments in this shock tube[3] established the validity of Lighthill's solution. Keller[4] has computed the density field for an acoustic wave rounding a 90° corner which is in good agreement with experimental data obtained for very weak shocks by White. The purpose of this paper will be to present the results of experiments with strong shocks (such that the flow behind them is supersonic) and any corner angle for comparison with the theory of reference 1 and as a guide to further theoretical work.

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[1] Jones, Martin, and Thornhill, Proc. Roy. Soc. (London) **A209**, 238 (1951).

[2] M. J. Lighthill, Proc. Roy. Soc. (London) **A198**, 454 (1949).

[3] Fletcher, Taub, and Bleakney, Revs. Modern Phys. **23**, 271 (1951).

[4] Mathematics Research Group Rept. EM-43, New York University, unpublished.

THEORETICAL CONSIDERATIONS

Consider a shock wave of velocity V advancing into still air in which the velocity of sound is a_1. According to one-dimensional shock theory, the flow velocity u_2 behind the shock is

$$u_2/a_1 = 2(M - 1/M)/(\gamma+1), \tag{1}$$

where M is the shock Mach number V/a_1, and the local velocity of sound a_2 is

$$a_2/a_1 = \{[2\gamma M^2 - \gamma + 1][2/M^2 + \gamma - 1]\}^{\frac{1}{2}}/(\gamma+1). \tag{2}$$

The flow behind the shock becomes supersonic when $u_2/a_2 = 1$ or for air with $\gamma = 1.4$ when $M = 2.068$.

When the incident shock passes the corner, a rarefaction wave advances back through the moving air with the velocity a_2, but the air itself is carried along with the velocity u_2. As a result, only the region below OAE in Fig. 1 will know of the existence of the corner. The strength of the diffracted shock EF varies in some unknown way so that the boundary conditions are difficult to establish.

A part of the flow can still be found, however. Jones, Martin, and Thornhill show that a Prandtl-Meyer expansion at the corner may be terminated in either one of two ways: (a) a uniform flow parallel to the wall, or (b) uniform flow parallel to a line on the other side of which some other flow maintains equal pressure, i.e., a slip stream. They predict that transition to the second case will occur when the Prandtl-Meyer flow expands to the ambient pressure p_1. The region below the slip stream would then be at rest at pressure p_1.

The radial extent of the Prandtl-Meyer flow may be found from the characteristics in the x/t, y/t system. One set consists of course of the radial lines through O. The other family must be found numerically. The

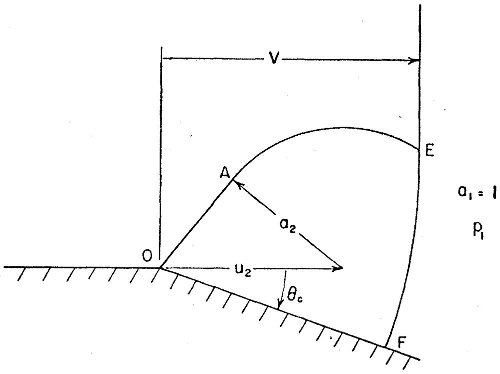

FIG. 1. Schematic drawing of shock diffraction pattern.

characteristic which passes through the point A was calculated for $M=3$ and is plotted in Fig. 2 as the dotted curve ABD. This provides an outer limit on the Prandtl-Meyer expansion. It will be seen that shocks appear which end the potential flow sooner.

EXPERIMENTAL RESULTS

Three models were made having corner angles $\theta_c=35°$, 90°, and 160°. In each case, the model completely spanned the four-inch width of the shock tube. The maximum distance past the corner which the shock could travel with the entire diffraction field still in view was about 2.5 in. Since the theory and operation of the tube have been described already,[5] no detailed explanation will be given here. Plane shock waves striking the model are photographed through a Mach-Zehnder interferometer with the aid of a 1-μsec spark triggered from an adjustable delay circuit. Thus any stage in the development of the flow could be recorded. This was especially significant in checking the assumption that the entire process was pseudostationary.

Figure 2 shows a typical density pattern obtained from an interferogram with a shock Mach number of 3.10. Numbers on the contours give the ratio of the density to the density ahead of the incident shock. The two discontinuities marked S may be either shocks or slip streams. Since we cannot decide which they are from these experiments alone, we shall arbitrarily refer to them as shocks. A small region of low density is observed below the slip stream (denoted by the dashed line) indicating that a vortex is shed from the corner. For comparison, the boundaries of the flow predicted theoretically for $M=3$ are shown by the dotted lines. OB is the final expansion wave which produces plane parallel flow at the ambient pressure in region OBD.

The influence of the corner angle on the flow pattern may be seen from Fig. 3 in which the shock and slip stream positions are drawn for three different corners with the same shock strength. For each case the pattern was found to grow similar to itself over a range of times varying by a factor of 5.

Only when the diffracted shock has turned nearly 180° does it become vanishingly weak. In the other cases the end moves forward to meet the wall normally. Except where the wall interferes directly, the subsidiary shocks S are nearly indistinguishable in the three experiments. An important result of this comparison is the observation that the slip stream angle and, therefore, the pressure to which the flow expands depend on the corner angle. This comes about through the action of viscosity in forming a boundary layer along the surface upstream from the corner. The vorticity so generated is shed from the corner in a sheet which then curls up to form the vortex already referred to in Fig. 2. Any such disturbance can make its presence felt in the entire region below the slip stream since there the air is nearly at rest.

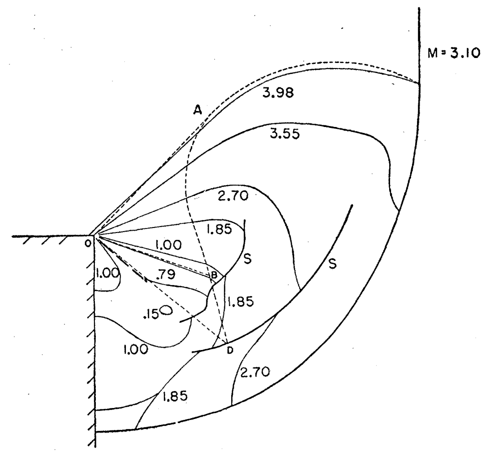

FIG. 2. Density field resulting from diffraction of a shock moving with a Mach number $M=3.10$ over a 90° corner. The dotted lines are the boundaries of the flow predicted theoretically for $M=3$.

Figure 4 shows the experimentally determined slip stream angle for each of the three corners as a function of M. The theoretical curve is obtained by computing the angle of turn necessary to expand the high pressure air behind the incident shock to ambient pressure. By coincidence, a corner angle of about 77° would give excellent agreement with the theory for all M between 2.06 and 3.2.

To study further the origin of viscous effects, the plate upstream from the corner was lengthened from 2 in. to 7 in. No changes could be detected. It appears, therefore, that the boundary layer thickness at the corner in these experiments was determined by the time since the flow was initiated by arrival of the shock rather than by the distance from the leading edge of the plate.

No theory is available for the growth of the boundary layer in the flow set up by a shock, but an order of

[5] Bleakney, Weimer, and Fletcher, Rev. Sci. Instr. **20**, 807 (1949).

magnitude for the thickness of such a layer may be obtained by applying the result for a plate suddenly set in motion.[6] The thickness is given by $2(\nu t)^{\frac{1}{2}}$, where ν is the kinematic viscosity and t is the elapsed time. For the earliest picture, where $t = 10$ μsec, this gives 0.007 inch. The surprising result is that such an exceedingly thin boundary layer causes the flow to

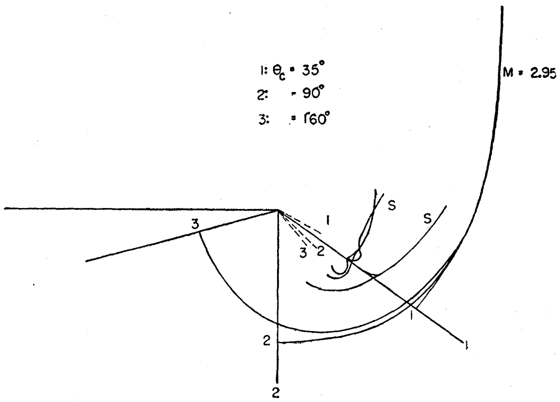

FIG. 3. Influence of corner angle on shock and slip stream position for $M = 2.95$.

separate at the corner and depart significantly from the predictions of potential flow theory.

CONCLUSIONS

The strength of a plane strong shock diffracted at a convex corner decreases monotonically from the point

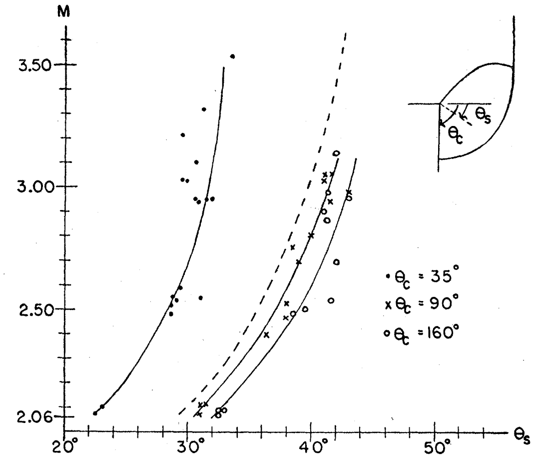

FIG. 4. Slip stream angle plotted as a function of M for each of the three corners. The dotted curve shows the theoretical dependence obtained by Thornhill.

of intersection with the leading edge of the rarefaction wave and reaches vanishing strength only in the neighborhood of a 180° turn. Predictions from potential theory concerning the flow field are only qualitatively correct in air. Viscous forces produce a boundary layer which causes the flow to separate at the corner with resulting modifications in pressure distribution. The location of the final wave in the Prandtl-Meyer expansion, therefore, becomes dependent upon the corner angle as well as the shock strength. Two subsidiary discontinuities occur whose positions are remarkably stable with respect to changes in corner angle and shock strength.

[6] W. F. Durand, *Aerodynamic Theory* (J. Springer, Berlin, 1934), Vol. 3, p. 65.