The Excited States of the N^{13} Nucleus*

HERBERT L. JACKSON† AND AARON I. GALONSKY
University of Wisconsin, Madison, Wisconsin

(Received July 28, 1952)

The properties of the excited states of N^{13} were reinvestigated by applying dispersion theory to the $C^{12}(p,p)C^{12}$ differential cross section reported in the preceding paper. The quantity and accuracy of the data were sufficient to permit extraction of the experimental S wave phase shift up to 1.4 Mev. The remainder of the data was fitted by successive readjustment of the level parameters.

The present analysis corroborates the previous momentum and parity assignments but gives somewhat different values for the resonant energies and widths. The level assignments together with the revised widths and resonant energies are as follows: The 0.46-Mev scattering anomaly is due to a virtual $(j=1/2, +)$ level in the compound nucleus N^{13} with $E_r=2.369$ Mev and $\Gamma=31$ kev. The 1.7-Mev scattering anomaly is the result of two virtual levels, a $(j=3/2, -)$ level with $E_r=3.511$ Mev and $\Gamma=55$ kev and a $(j=5/2, +)$ level with $E_r=3.558$ Mev and $\Gamma=61$ kev. E_r and Γ are calculated in the center-of-mass system and E_r is measured from the ground state of N^{13}. These new values for E_r and Γ agree quite well with those obtained from the $C^{12}(p,\gamma)N^{13}$ reaction provided it is assumed that the $(j=5/2, +)$ level does not participate appreciably in the capture process. The values of the reduced widths of the levels indicate that the two even-parity levels arise mainly from single particle excitation, but that the odd-parity level involves appreciable excitation of the core.

THE purpose of this paper is to present the information about the excited states of N^{13} obtained from the analysis of the $C^{12}(p,p)C^{12}$ differential cross section reported in the preceding article. For the most part, this analysis is similar to that of the original data of Goldhaber and Williamson,[1,2] but because the more recent measurements include data at four scattering angles and are of relatively high accuracy, the widths and resonant energies of the excited levels as determined in this investigation are more reliable than those previously proposed. Unlike the earlier values, they agree with those obtained from the $C^{12}(p,\gamma)N^{13}$ reaction.

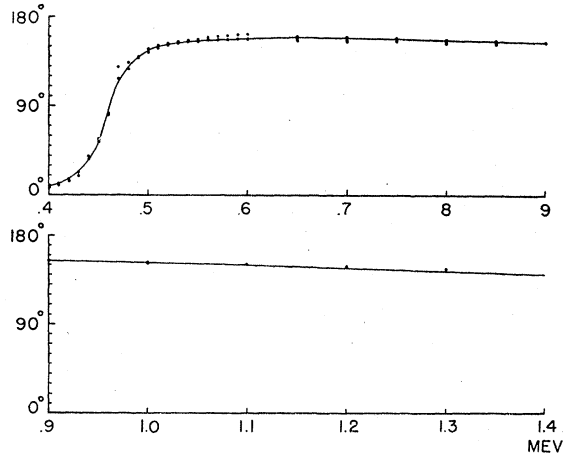

FIG. 1. The $C^{12}(p,p)C^{12}$ S wave phase shift as a function of energy. The points are the values of δ_0 extracted from the experimental cross section. The solid line is the phase shift calculated from the parameters in Table I.

I. METHOD

As the dispersion theory formalism used in this analysis and the usual methods of applying it are already described in the literature,[2–5] they will not be repeated in detail here. Instead this section will merely summarize the procedures which are of especial importance in the present work.

Whenever practicable, a dispersion theory analysis should begin with the extraction of the phase shifts of each partial wave from the experimental cross section. Once these are known as explicit functions of the bombarding energy, the task of identifying the parameters characteristic of the energy levels present is relatively simple and their numerical values reasonably dependable.

In dealing with the present data, we were able to obtain the S wave phase shift as a function of energy up to 1.4 Mev. Beyond this point, however, the contribution of higher momentum components could no longer be ignored at any of the scattering angles studied. As both the P and D waves appeared to enter in approximately equal strengths, further attempts at

TABLE I. Parameters of the first three excited states in N^{13}.

Level:	$a=1.45(12^{\frac{1}{3}}+1^{\frac{1}{3}})\times10^{-13}$ cm $(j=1/2, +)$	$(j=3/2, -)$	$(j=5/2, +)$
E_p(Mev)[a]	0.461	1.698	1.748
Γ(kev)[a]	34	60	66
E_r(Mev)	2.369	3.511	3.558
Γ(kev)	31	55	61
E_λ(Mev)	0.951	3.516	3.612
γ_λ^2(Mev-cm)	7.58×10^{-13}	0.440×10^{-13}	2.92×10^{-13}
$\gamma_\lambda^2\div\hbar^2/\mu a$	0.81	0.047	0.31

[a] Laboratory system. All other numbers apply to the center-of-mass system.

* Work supported by the AEC and the Wisconsin Alumni Research Foundation.

† Now at the University of Basel, Basel, Switzerland.

[1] G. Goldhaber and R. M. Williamson, Phys. Rev. **82**, 495 (1951).

[2] H. L. Jackson and A. I. Galonsky, Phys. Rev. **84**, 401 (1951).

[3] C. L. Critchfield and D. C. Dodder, Phys. Rev. **76**, 602 (1949).

[4] R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. **84**, 18 (1951).

[5] L. J. Koester, Phys. Rev. **85**, 643 (1952).

explicit extraction were abandoned as unrewarding, and the analysis was continued by successive readjustment of the level parameters just as in the analysis[2] of the Goldhaber-Williamson data.

Whenever the elastic scattering cross section involves only Rutherford and S wave effects, the partial wave expansion of the differential cross section[2] can be written in the two-dimensional vector form $\mathbf{X}=\mathbf{R}+\mathbf{S}$, where

$$|\mathbf{X}|=k(d\sigma/d\omega)^{\frac{1}{2}},$$

$$\mathbf{R}=-\tfrac{1}{2}\eta\csc^2\theta/2\exp(i\eta\ln\csc^2\theta/2),$$

and

$$\mathbf{S}=\sin\delta_0 e^{i\delta_0}.$$

The only unknown quantities in the equation are δ_0 and the phase of $\mathbf{X}$, the latter being of no interest. The solution for the unknowns is straightforward, both graphically and analytically. In general, the solution at any one scattering angle gives two values for the phase shift. The one which is physically meaningful can usually be identified by comparing the solutions obtained at different scattering angles.

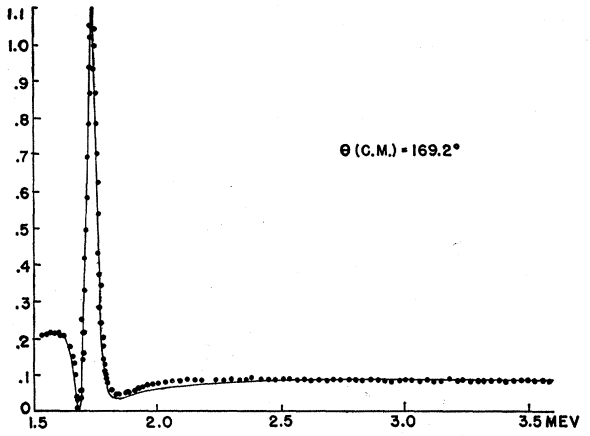

FIG. 2. Comparison of the calculated and experimental cross sections at 169.2 degrees center-of-mass. The points are the experimental cross sections reported in the preceding paper. The solid line is the cross section calculated from the parameters in Table I together with the modifications of δ_0 and δ_2^- discussed in the text. Vertical scale is in barns per steradian c.m.

II. RESULTS

According to the analysis of the Goldhaber-Williamson data,[2] the 0.46-Mev scattering anomaly in the $C^{12}(p,p)C^{12}$ reaction is due to a $(j=1/2,+)$ level in N^{13} and the 1.7-Mev scattering anomaly is due to two levels about fifty kev apart. These two levels were classified as $(j=3/2,-)$ and $(j=5/2,+)$, the latter lying higher in energy. The present analysis corroborates the momentum and parity assignments of these levels but gives somewhat different values for their widths and resonant energies. The remainder of this section will present only those results which are different from or supplementary to the previous work.

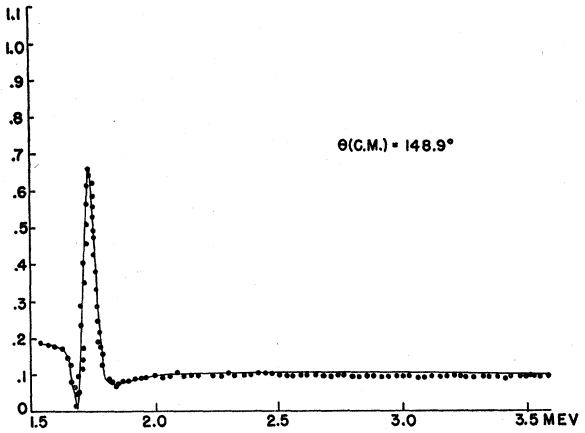

FIG. 3. Comparison of the calculated and experimental cross sections at 148.9-degrees c.m. The points are the experimental cross sections reported in the preceding paper. The solid line is the cross section calculated from the parameters in Table I together with the modifications of δ_0 and δ_2^- discussed in the text. Vertical scale is in barns per steradian c.m.

The points in Fig. 1 are the values of the S wave phase shift determined from the experimental cross section. Below 1.2 Mev the effects of the higher momentum phase shifts appear to be less than the statistical uncertainty of the experimental cross sections. Between 1.2 and 1.4 Mev, the values for δ_0 obtained from the data at different scattering angles no longer agree, thus indicating the presence of higher momentum components. The points plotted in the figure above 1.2 Mev are those obtained from the data at 127.8 degrees. The cross section at this angle is believed to be relatively free from the effects of higher momentum components because the P and D vectors at this angle happen to be rather small in magnitude and are so oriented as to have little effect. The solid line in the

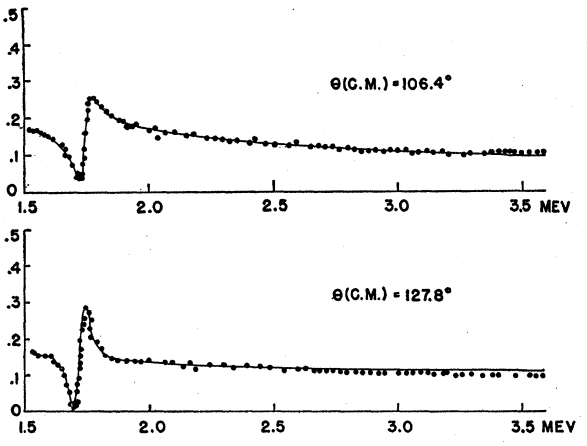

FIG. 4. Comparison of the calculated and experimental cross sections at 127.8-degrees and 106.4-degrees c.m. The points are the experimental cross sections reported in the preceding paper. The solid lines are the cross sections calculated from the parameters in Table I together with the modifications of δ_0 and δ_2^- discussed in the text. Vertical scale is in barns per steradian c.m.

figure is the S wave phase shift calculated from the parameters in Table I.

The final values of the parameters characteristic of the $(j=3/2, -)$ and the $(j=5/2, +)$ levels are listed in Table I and the cross section calculated from these values is shown by the solid lines in Figs. 2, 3, and 4. The calculated cross sections also include modifications in the values of δ_0 and δ_2^- to be discussed below. The points in these figures are the experimentally determined cross sections reported in the preceding paper.

As the figures show, the fit is quite good except at the largest scattering angle. In this curve discrepancies exist both below the first minimum and above the principal maximum of the 1.7-Mev anomaly. Investigation has shown that it is possible to reduce the disagreement at this scattering angle without seriously disturbing the fit elsewhere, but to do so would require a number of simultaneous readjustments of the S, P, and D waves. We believe that further work of this sort with the present data is not worthwhile, mainly because the required readjustment in any one phase shift would apparently not amount to more than one or two degrees at any given energy. This being the case, little change would be introduced into the resonant energy (where the resonant part of the phase shift is equal to ninety degrees) or into the width of the level at half-maximum. Furthermore, the observed cross section almost certainly differs from the true cross section because of isotopic contamination. The $C^{13}(p,\gamma)N^{14}$ reaction has several resonances between 1.0 and 2.2 Mev,[6] some of them quite broad, which undoubtedly give rise to corresponding anomalies in the elastic scattering. If the maximum of any such anomaly is equal to or greater than one barn, and if it occurs at an energy where the $C^{12}(p,p)C^{12}$ cross section is low, then the resulting increase in the observed yield could easily amount to twenty percent or more. Thus, it is by no means certain that further refinement of the fit would result in any actual improvement in the accuracy of the values of the level parameters.

The discrepancy between the calculated and experimental cross sections happens to occur at 169.2 degrees rather than at some other angle because we worked mostly at the lower scattering angles while fitting the data over the region of the 1.7-Mev anomaly. As a result, the effects of errors both in the phase shifts and in the value of the experimental cross section would tend to accumulate at the unfitted scattering angle, 169.2 degrees.

This procedure of fitting at the lower angles was adopted because of certain properties of the incoherent scattering. As is clear from the expansion of $d\sigma/d\omega$,[2] the incoherent term (that is, the term beginning with the factor $\sin^2\theta/k^2$) involves only P and higher waves, is increasingly important for scattering angles nearer ninety degrees, and makes its principal contribution when $|\delta_l^+ - \delta_l^-| \sim \pi/2$. Thus, over the maximum of the 1.7-Mev anomaly, which is due to resonances in the $P_{\frac{3}{2}}$ and $D_{\frac{5}{2}}$ waves, the value of the cross section at angles near ninety degrees is relatively more sensitive to small changes in δ_1^+ and δ_2^+. This increased sensitivity results because these phase shifts, together with δ_1^- and δ_2^-, affect both the coherent and incoherent parts of the scattering while the Rutherford and S wave effects are confined to the coherent part.

The S wave phase shift as calculated from the parameters given in Table I does not permit an acceptable fit of the experimental data above 1.4 Mev. Trial and error readjustments of δ_0 showed that a positive increment varying linearly with energy from 0° at 1.2 Mev to 8° at 3.6 Mev added to the calculated S wave phase shift was satisfactory. The fit is further improved by setting δ_2^- equal to zero over all energies up to 3.6 Mev. The value of this latter phase shift, calculated from the expression $\delta_2^- = -\arctan F_2/G_2|_a$ is about zero below 1.7 Mev and decreases to −5° at 3.6 Mev. The calculated cross sections shown in the figures include the just mentioned deviations of δ_0 and δ_2^- from the values given by the dispersion theory formula.

III. DISCUSSION

An interpretation of the results of analyses of this kind requires consideration of the assumptions entering the dispersion theory formalism.[7] The basic hypothesis upon which the theory rests is that the Hamiltonian for the scattering system is known for all space excepting a finite region within which the particles interact strongly. The treatment begins by surrounding this part of space with a closed surface which in practice is usually a sphere of radius a. In the present development of the theory, a must be large enough to permit description of the system outside the sphere by a single-particle Hamiltonian of known form but is otherwise arbitrary. As thus introduced, the radius a has no direct connection with the physical processes under study.

In order that the remainder of this discussion may be explicit, we shall assume that the formalism has been specialized for the elastic scattering of protons from spinless nuclei and that the single-particle wave functions in the outer region are expanded in spherical harmonics in the usual way.[3]

Since the Hamiltonian is known in the outer region, the wave functions are determined explicitly in this region except for an arbtrary phase shift $\delta_l^{\pm}$ in each radial component. The value of each of these phase shifts is fixed by the requirement that the external wave function join the internal wave function smoothly on the surface of the sphere which separates the inner and outer regions. However, no solution for the internal wave functions is practicably obtainable because of

[6] J. D. Seagrave, Phys. Rev. **85**, 197 (1952). Levels are reported at 1.16, 1.25, 1.76, and 2.10 Mev with widths of 6, 500, 2.1, and 45 kev, respectively.

[7] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

the complexity of the system. Consequently, the phase shifts must be determined experimentally and used to infer information about the internal wave functions. The value of dispersion theory lies in the fact that it provides relationships between the phase shifts of the external wave functions and certain constants E_λ and γ_λ characteristic of the internal wave functions χ_λ. In general, the number of χ_λ and, hence, the number of pairs of $(E_\lambda, \gamma_\lambda)$ associated with each phase shift is infinite. Hence, in its general form, the dispersion theory cannot be applied to any specific scattering process because knowledge of any phase shift $\delta_l^{\pm}$ over a finite energy range is insufficient to determine the infinite set of $(E_\lambda, \gamma_\lambda)$ pairs associated with it. Application of the theory to specific nuclear processes is possible only if there exists an energy range within which the number of $(E_\lambda, \gamma_\lambda)$ pairs appreciably affecting the value of $\delta_l^{\pm}$ is finite—and for practical purposes—small. Obviously the validity of this condition depends upon, among other things, the choice of a used to bound the interior region, because not only the values of E_λ and γ_λ but also the extent in energy of the influence of each such pair depends upon the magnitude of a. Usually, the larger the value of a, the greater the number of $(E_\lambda, \gamma_\lambda)$ pairs which must be introduced to explain the behavior of $\delta_l^{\pm}$.

The special form of dispersion theory used in this analysis is usually called the one-level approximation. This means that the expression for $\delta_l^{\pm}(E_\lambda, \gamma_\lambda)$ is derived on the assumption that not more than one energy level, and hence not more than one $(E_\lambda, \gamma_\lambda)$ pair affects the value of $\delta_l^{\pm}$ in the energy range under consideration. Whether this assumption is ever even approximately appropriate to the description of a scattering process is still debated, but if so, the most favorable circumstance is that the separation of successive levels of given parity and angular momentum in the compound nucleus be much larger than their widths both within and near the energy region under study. Whenever this condition holds, it is not unreasonable to suppose that only one level contributes to the phase shift at a given value of the bombarding energy.

This assumption, together with the requirements of continuity on the surface of the sphere separating the inner and outer regions, leads to the expression

$$\delta_l^{\pm}=\left[-\arctan\frac{F_l}{G_l}+\arctan\frac{k\gamma_\lambda^2/A_l^2}{E_\lambda+\Delta_\lambda-E}\right]_{r=a}$$

where F_l and G_l are the regular and irregular radial Coulomb wave functions, respectively, E_λ and γ_λ^2 are, respectively, the characteristic energy and reduced width of the level $k=p/\hbar$ in center-of-mass units,

$$\Delta_\lambda=\left[-\frac{k\gamma_\lambda^2}{\rho}\left(\frac{\rho}{A_l}\frac{dA_l}{d\rho}+l\right)\right]_{\rho=ka}$$

where $\rho=kr$, $A_l^2=F_l^2+G_l^2$, E is the bombarding energy of the incident particle.

From the above expression for $\delta_l^{\pm}$ it is at once clear that the non-Coulomb scattering has been mathematically divided into two parts. The first, represented by the term $[-\arctan(F_l/G_l)]_a$, happens to be mathematically equal to the phase shift which would be produced by a hard-sphere potential of range a. The second term may be called the resonant part of the phase shift since its principal contribution occurs in the vicinity of the energy $E_r=E_\lambda+\Delta_\lambda$.

Therefore, any cross section that can be fitted by the present formalism may be thought of as owing to three causes, namely: (a) a Coulomb potential Ze^2/r, (b) a hard sphere potential of range a, and (c) a resonant interaction characterized by the parameters E_λ and γ_λ. However crude this model may be, it at least has the advantage of providing a simple mental picture of the process and of permitting the cross section to be completely described over the observed energy range by $4x+1$ numerical constants, x being the number of energy levels present. These constants are E_λ, γ_λ, parity, and j for each level, and the parameter a.

The one-level approximation imposes restrictions upon the acceptable numerical values of a which do not appear in the general formulation of the theory. This situation arises from the fact that we have excluded the possibility of introducing an unlimited number of $(E_\lambda, \gamma_\lambda)$ pairs to account for the behavior of the phase shift. Since only one such pair is allowed, a reasonable fit can be achieved only for a limited range of a values. To illustrate the point, let us consider the phase shift shown in Fig. 1. This phase shift is a slowly and monotonically decreasing function of energy except for the rather rapid increase of about 180 degrees near 0.46 Mev. We will now show that the off-resonance behavior of the phase shift determines the appropriate value of a.

Since the term $-\arctan F/G$ decreases monotonically with increasing energy, the positive increase in the phase shift must be attributed to the resonant term, $\arctan(k\gamma_\lambda^2/A_l^2)/(E_\lambda+\Delta_\lambda-E)$. The value of this term depends upon E_λ, γ_λ, and also upon a which appears implicitly in the quantities Δ_λ and A_l^2. However, over the energy range of a narrow resonance, the values of Δ_λ and A_l^2 are practically independent of energy regardless of the value of a. Such being the case, it is obvious from the mathematical form of the resonant term, that E_λ is fixed by the value of the bombarding energy for which $\delta_l^{\pm}+\arctan(F_l/G_l)=90°$ and γ_λ is determined by the rapidity with which $\delta_l^{\pm}$ increases near this energy. Thus, the acceptable values of E_λ and γ_λ depend almost entirely upon the behavior of $\delta_l^{\pm}$ near resonance. Consequently these two parameters are not available for fitting the off-resonance behavior of the phase shift. Since the only other parameter at our disposal is a itself, it follows that, in the single level approximation, a becomes a measure of the strength of the off-resonant interaction between the target nucleus and the incident proton.

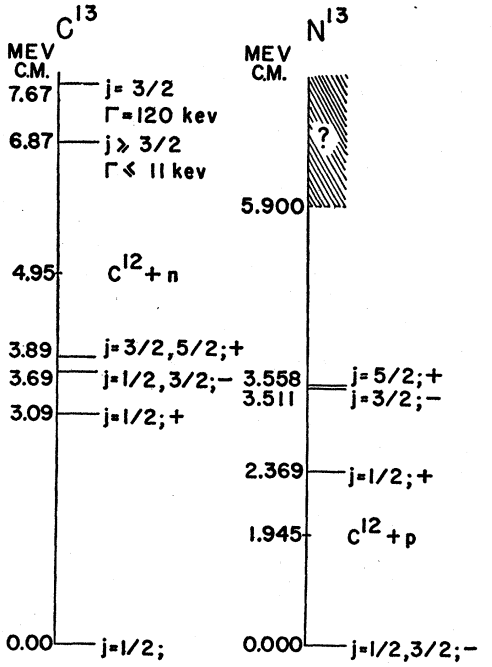

FIG. 5. Comparison of the energy levels in N^{13} and C^{13}. The characteristics of the first three levels in C^{13} were obtained by Rotblat from the $C^{12}(d,p)C^{13}$ stripping reaction. Those of the two highest levels were obtained by Bockelman *et al.* from the $C^{12}(n,n)C^{12}$ total cross section. The characteristics of the excited states in N^{13} are the results of the present analysis.

IV. CONCLUSION

The necessity for the alterations introduced into δ_0 and δ_2^- may be "accounted for" in a variety of ways, but since the alterations are positive and increase with energy, it is plausible to assume that they reflect the influence of higher resonances.‡ In the case of the S wave, any high energy resonance could be very broad because the barrier factor is relatively small. Consequently the low energy tail of any such resonance could be appreciable over an extensive range of energy.

On the basis of Mayer's hypotheses[8] a $(j=3/2,+)$ level is expected to lie a few Mev above the $(j=5/2,+)$ level. Although the position and width of this level are as yet unknown, reasonable estimates show that it is possible for its low energy effect to extend into the observed region and so give the required positive increment to δ_2^-.

The widths and resonant energies of the proposed levels as obtained from the present analysis agree satisfactorily with those found from the study of the $C^{12}(p,\gamma)N^{13}$ reaction. For the lower capture level, Fowler and Lauritsen[9] obtain the values $E_p=0.456\pm0.002$ Mev and $\Gamma=35$ kev. Our values are $E_p=0.461$ Mev and $\Gamma=34$ kev. Since the error in locating the resonant energy may easily be as much as 3 kev, the agreement is satisfactory.

For the upper capture level, Van Patter[10] obtains the values $E_p=1.697\pm0.012$ Mev and $\Gamma=74\pm9$ kev, while Seagrave[11] gets the values $E_p=1.698\pm0.005$ Mev and $\Gamma=70\pm10$ kev. These values of the resonant energy are in excellent agreement with our value of 1.698 Mev for the $(j=3/2,-)$ level. Our value for the width of this level is 60 kev. This value is somewhat smaller than those obtained from the $C^{12}(p,\gamma)N^{13}$ reaction, but because of the uncertainties both in our analysis and in the just-cited experimental results, the discrepancy is probably not significant. In any event, the inference is that the $(j=5/2,+)$ level makes relatively little, if any, contribution to the gamma-yield.

Figure 5 shows the known energy levels in N^{13} and C^{13} up to 8 Mev. In N^{13} the region above 5.9 Mev is unexplored. The parities and possible momentum values of the first three excited states in C^{13} are those obtained by Rotblat[12] from the $C^{12}(d,p)C^{13}$ stripping reaction. The values of the parameters of the two highest levels are those of Bockelman *et al.*[13]

As the figure shows, the parities and momenta of the corresponding levels agree in so far as they are known, thus lending support to the hypothesis of equal *nn* and *pp* forces.

The ratio $\mu a\gamma_\lambda^2/\hbar^2$ for a given level is usually regarded as a measure of the degree to which that level arises from the excitation of a single nucleon. Qualitatively, the numerical value of this ratio should be about one for single-particle levels, and much less for levels due to multiparticle excitation. As shown in Table I, the values of this ratio are 0.81, 0.047, and 0.31 for the $(j=1/2,+)$, $(j=3/2,-)$ and the $(j=5/2,+)$ levels in N^{13}, respectively. On the basis of these numbers, it is probable that the even parity levels arise from the excitation of the outer nucleon and the odd parity level involves excitation of the core.

The arguments concerning the uniqueness of the level assignment based on the analysis of the Goldhaber-Williamson data,[2] of course, remain valid. The fact that this level assignment also explains the behavior of the cross section as a function of the scattering angle is also strong evidence of its correctness.

‡ *Note added in proof.*—See discussion of T. Teichman and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

[8] M. G. Mayer, Phys. Rev. **78**, 16 (1950).

[9] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **76**, 314 (1949).

[10] D. M. Van Patter, Phys. Rev. **76**, 1264 (1949).

[11] J. D. Seagrave, Phys. Rev. **84**, 1219 (1951).

[12] J. Rotblat, Nature **167**, 1027 (1951), and Phys. Rev. **83**, 1271 (1951).

[13] Bockelman, Miller, Adair, and Barschall, Phys. Rev. **84**, 69 (1951).